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The effect of a pion-pion interaction on
low-energy meson-nucleon scattering II

by

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Abstract

Using the formalism developed in a previous paper we investigated the consequences of a pion-pion interaction on low energy pion nucleon phase shifts. By confining our considerations to the isotopic spin flip combination of waves we are able to isolate the effects due to a pion - pion interaction in the $T = 1$ state. We find that such an interaction is definitely necessary to give agreement with experiment and that a simple resonance in the $J = 1, T = 1$ state gives a good fit to the data.

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also given in this section. Finally we give a qualitative discussion to illustrate the assumptions of the theory.

II. Representation for the Invariant Amplitudes

We have shown in I that for low energies and momentum transfers the four invariant amplitudes $A^{(\pm)}$ and $B^{(\pm)}$ may be represented as follows

$$\begin{aligned}
 A^{(+)}(s, \bar{s}, t) &= \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} \left\{ \alpha_1^{(+)}(s') + t \alpha_2^{(+)}(s') \right\} \left\{ \frac{1}{s'-s} + \frac{1}{s'-\bar{s}} \right\} + \frac{1}{\pi} \int_{4\mu^2}^{\infty} \frac{\rho^{(+)}(t')}{t'-t} + C_A^{(+)} \\
 A^{(-)}(s, \bar{s}, t) &= \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} ds' \left\{ \alpha_1^{(-)}(s') + t \alpha_2^{(-)}(s') \right\} \left\{ \frac{1}{s'-s} - \frac{1}{s'-\bar{s}} \right\} + \frac{(s-\bar{s})}{\pi} \int_{4\mu^2}^{\infty} dt' \frac{\sigma^{+}(t')}{t'-t} \\
 B^{(+)}(s, \bar{s}, t) &= g_r^2 \left(\frac{1}{M^2-s} - \frac{1}{M^2-\bar{s}} \right) + \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} ds' \left\{ \beta_1^{(+)}(s') + t \beta_2^{(+)}(s') \right\} \left\{ \frac{1}{s'-s} - \frac{1}{s'-\bar{s}} \right\} \\
 &\quad + \frac{(s-\bar{s})}{\pi} \int_{4\mu^2}^{\infty} dt' \frac{\sigma^{+}(t')}{t'-t} \\
 B^{(-)}(s, \bar{s}, t) &= g_r^2 \left(\frac{1}{M^2-s} + \frac{1}{M^2-\bar{s}} \right) + \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} ds' \left\{ \beta_1^{(-)}(s') + t \beta_2^{(-)}(s') \right\} \left\{ \frac{1}{s'-s} + \frac{1}{s'-\bar{s}} \right\} \\
 &\quad + \frac{1}{\pi} \int_{4\mu^2}^{\infty} dt' \frac{\sigma^{(-)}(t')}{t'-t} + C_B^{(-)}
 \end{aligned} \tag{1}$$

processes. We shall assume that for pion-nucleon scattering the $(3,3)$ resonance gives the only important contribution to our integrals although corrections to this assumption are discussed in section IV. We also suppose that the π - π scattering is dominated by a resonance in the $T = 1, J = 1$ state.

With these assumptions the spectral functions are given, with the notation of I, by

$$\begin{aligned}
 \alpha_1^{(+)}(s') &= \frac{4\pi}{3} \left[\frac{3(W'+M)}{(E'+M)} + \frac{(W'-M)}{(E'-M)} \right] \frac{\sin^2 \delta_{33}(q')}{q'} \\
 \alpha_2^{(+)}(s') &= 4\pi \left[\frac{W'+M}{E'+M} \right] \frac{1}{2q'^2} \frac{\sin^2 \delta_{33}(q')}{q'} \\
 \alpha_1^{(-)}(s') &= -\frac{1}{2} \alpha_1^{(+)}(s') \\
 \alpha_2^{(-)}(s') &= -\frac{1}{2} \alpha_2^{(+)}(s') \\
 \beta_1^{(+)}(s') &= \frac{4\pi}{3} \left[\frac{3}{E'+M} - \frac{1}{E'-M} \right] \frac{\sin^2 \delta_{33}(q')}{q'} \\
 \beta_2^{(+)}(s') &= 4\pi \left[\frac{1}{E'+M} \right] \frac{1}{2q'^2} \frac{\sin^2 \delta_{33}(q')}{q'} \\
 \beta_1^{(-)}(s') &= -\frac{1}{2} \beta_1^{(+)} \\
 \beta_2^{(-)}(s') &= -\frac{1}{2} \beta_2^{(+)} \\
 \rho^{(-)}(t') &= \frac{3\pi}{p'^2} \left(\frac{M}{\sqrt{2}} \operatorname{Im} f_{-}(t') - \operatorname{Im} f_{+}(t') \right) \\
 \rho^{(+)}(t') &= 0 \\
 \sigma^{(+)}(t') &= 0 \\
 \sigma^{(-)}(t') &= \frac{12\pi}{\sqrt{2}} \operatorname{Im} f_{-}(t')
 \end{aligned} \tag{2}$$

Assuming also a narrow width for the (3,3) resonance we may write

$$\sin^2 \delta_{33}(q') = \frac{4}{3} f^2 q_r^3 \pi \delta(\omega_r - \omega') \quad (5)$$

For the $\pi + N \rightarrow \pi + N$ channel if one makes the above assumptions and sets the $\pi-\pi$ interaction equal to zero (and, therefore, $\rho^{(-)} = \sigma^{(-)} = 0$) one obtains a set of equations for the partial wave amplitudes which reduce in the static limit to those obtained by CGLN.⁽⁶⁾

III. s- and p- waves

S waves; $C_A^{(+)}$, $C_B^{(-)}$, and C_1 are parameters of our theory which must be determined from experiment. If we confine ourselves to the isotopic spin-flip amplitude $A^{(-)}$ and $B^{(-)}$, i.e. the $(T = \frac{1}{2}) - (T = \frac{3}{2})$ combination of amplitudes, our results are independent of $C_A^{(+)}$. From our representation, (1) we obtain the following expressions for the s-wave isotopic spin-flip amplitude, [Eqs. (5.7), (5.1) and (5.14) of I].

$$f_s^{(\frac{1}{2})} - f_s^{(\frac{3}{2})} = -\frac{q}{2} \frac{M}{W} \frac{\omega}{2q^2} C_1 \log\left(1 + \frac{4q^2}{t_r}\right) + b\omega \quad (6)$$

where b is given by

$$b = \frac{C_B^{(-)}}{4\pi} + \frac{2}{9} f^2 \quad (7)$$

IV. Rescattering Corrections and Higher Waves

Let us now consider the possible corrections to the theoretical curves given in the preceding section. In writing down expressions for the s- and p- waves we have taken the static limit both for simplicity and because of the large errors on the experimental phase shifts at higher energies. The relativistic corrections to this limit could, however, be taken into account in a straightforward way. As calculated by Hamilton and Spearman, the relativistic corrections to the nucleon terms for s- waves are fairly small and a rough estimate of the corrections to the pion terms seems to indicate that up to energies of the order of 400 MeV, relativistic corrections will in general be less than 20%.

A more important correction, on the other hand, arises when one takes into account the absorptive parts of partial waves other than f_{33} . If one includes this rescattering but neglects crossed terms owing to the large size of the denominators one is led to integral equations for each amplitude $f_{\ell\pm}$ of the type

$$f_{\ell\pm} = f_{\ell\pm}^0 + \frac{q^{2\ell}}{\pi} \rho \int_1^\infty \frac{\sin^2 \delta_{\ell\pm}(\omega')}{q'^{2\ell+1}(\omega' - \omega)} d\omega' \quad (8)$$

Here $f_{\ell\pm}^0$ is the sum of the nucleon and pion terms plus terms in $C_A^{(+)}$ and $C_B^{(-)}$ for s waves. The best treatment of these equations would be to attempt to solve them as integral equations (for example by means of the N/D method) and such a programme is at present under consideration. Here we shall adopt the alternative procedure and evaluate the rescattering terms in Eq. (9) using the experimental values for the phase shifts. Hamilton and Spearman have

fit experiment. Because of the 600 MeV resonance in the D_{13} wave the rescattering correction is in this case sizeable and with its inclusion we are able to fit the data with the same C_1 as for the S waves i.e. $C_1 = -1.0$. The corresponding curves are shown in Fig. 3.

$$f_{D\frac{5}{2}}^{(\frac{1}{2})} - f_{D\frac{5}{2}}^{(\frac{3}{2})}$$

is almost zero experimentally and any attempt to fit the data would depend on a good evaluation of the nucleon term in this energy region, which we feel we are not able to make.

F waves

Experimentally there seems to be evidence for the resonance around 900 MeV to be in the $F_{\frac{5}{2}}^{\frac{1}{2}}$ wave and although there are no F wave phase shifts with which to compare our theory we might hope to be able to show that the theory gives the right conditions for a resonance to occur in that state. This is indeed so. The expressions for the F waves from the nucleon term may be written

$$\begin{aligned}\delta_{F\frac{5}{2}}^{\frac{1}{2}} &= \lambda_F \left[2 + \frac{320}{9} \left(\frac{\omega}{\omega_r + \omega} \right)^3 \right] \\ \delta_{F\frac{5}{2}}^{\frac{3}{2}} &= -\lambda_F \left[1 - \frac{80}{9} \left(\frac{\omega}{\omega_r + \omega} \right)^3 \right] \\ \delta_{F\frac{7}{2}}^{\frac{1}{2}} &= -\lambda_F \left[12 - \frac{32}{3} \left(\frac{\omega}{\omega_r + \omega} \right)^3 \right] \\ \delta_{F\frac{7}{2}}^{\frac{3}{2}} &= \lambda_F \left[6 + \frac{8}{3} \left(\frac{\omega}{\omega_r + \omega} \right)^3 \right]\end{aligned}\quad (9)$$

with

$$\lambda_F = \frac{1}{105} \frac{f^2}{M^2} \frac{q^7}{\omega^3} \quad (10)$$

V. Discussion

We have seen in the preceding sections that experiments on pion nucleon scattering show definitely the existence of a pion-pion interaction in the $T = 1$ state. Moreover we have shown that a simple resonance in the $J = 1, T = 1$ state is sufficient to give a rather good fit to the experimental data considered. The width at half maximum of this resonance (with $C_1 = -1.0$) is $.9\mu$.

The fact that our curve does not pass through the experimental S wave points in Fig. 1 around $q = 1.5$ may be due to two causes. Either our theory is deficient and it is necessary to have a pion-pion interaction of a more complicated kind or it may be that the experimental values of the phase shifts at this energy are incorrect. Although α_3 may be well determined by doing π^+p scattering, α_1 must be determined by doing π^-p scattering where the isotopic spin states are mixed and is therefore subject to more uncertainty. Experiments with good statistics and a careful phase shift analysis in this region would be extremely useful to clear up this point.

If one also wishes to look in a similar manner for evidence of a pion-pion interaction in even J states the appropriate combination of pion nucleon isotopic spins is the (+) state i.e. $(T = \frac{1}{2}) + 2(T = \frac{3}{2})$. Here one is immediately confronted with the problem of a reliable determination of the rescattering corrections which in this case add together instead of cancelling as for the (-) states. This is a difficult task since the corrections are principal value integrals over experimental phase shifts with appreciable errors. A more hopeful procedure is to attempt to solve the integral equations indicated in Eq. (9).

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Figure Captions

- Fig. 1 : For several values of C_1 and α we plot the theoretical curves for the isotopic spin flip S waves. Experimental points from references 7, 8, 9.
- Fig. 2 : The nucleon term is plotted together with the pion term for $C_1 = -1.0$. The total is also shown. Experimental points from references 7, 8, 9.
- Fig. 3 : The nucleon and pion ($C_1 = -1.0$) terms are plotted together with the total and the total corrected for rescattering. Experimental points from reference 9.
- Fig. 4 : Taking $(\sin 2\alpha_1 - \sin 2\alpha_3)$ from theory and α_3 from theory and α_3 from experiment α_1 is plotted. Experimental points from references 7, 8, 9.

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Fig 1

Fig 2

Fig. 3.

Fig 4