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Prong Analysis of Multiple Production of Pions at Bevatron Energy

According to a Statistical Theory

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ABSTRACT

A detailed analysis of p-p collisions at 6.2 GeV is made according to a Fermi-type statistical theory, assuming the existence of a N- π isobar. Energy spectra of particles and relative probabilities of two-, four- and six-prong events are computed.

The particle numbers agree with experiment, but the computed inelasticity is 50 % too high. The calculated mean proton energies (280 MeV) are much lower than the experimental ones, the calculated mean pion energies somewhat higher (360 MeV).

A model is then proposed in which "central collisions" are treated according to the statistical theory and "peripheral" collisions according to a simple two-centre model. The relative probability of the two kinds of collisions is adjusted to give a good fit to the inelasticity.

It is then possible to have agreement of the average number of pions, the ratio of two-to four-to six-prong events and the average energies with the measured values of these quantities.

It was tried to introduce also a $\pi\pi$ isobar into the model. This changes the results, but present experiments cannot rule out or confirm this hypothesis, because of lack of precision. The model predicts also angular distributions and some angular correlation.

Prong Analysis of Multiple Production of Pions at Bevatron Energy
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1. Introduction.

In a recent paper ¹⁾ R. Hagedorn has applied a statistical theory of particle production to events resulting from p-p collisions at Bevatron energies. The results, as far as the average number of produced pions is concerned, agree well with experiment ^{2),3),4)}. It would, however, be much desirable to have further points of comparison, as e.g. spectra of particles coming from events with the same multiplicity. In the statistical theory particles with the same mass have the same form of spectrum, for a given type of event. All pions (of charge +, 0, -) produced for example in a collision of the type $p+p \rightarrow 2N+5\pi$ will show the same spectrum. There are now many different possibilities to distribute the charges over 2 nucleons and 3 pions :

pp + - 0
 0 0 0
pn + + -
 + 0 0
nn + + 0

One sees that the same type of event (in the example : production of three mesons) will appear to the observer in many different ways. For instance as events with two protons, one proton, no proton, or as event with 2 prongs, 4 prongs, 6 prongs. Other types of events (e.g. 4 meson production) which give different spectra will appear also to the observer as 2, 4 or 6 prong events. As it is almost impossible in most cases to ascertain the number of neutrals in a star, one cannot compare directly the theory and the experiment.

However, because of isospin conservation, it is possible to compute the relative probabilities of different charge-states for events with a given multiplicity. One can therefore compute the relative weight of 2, 4, 6 prong stars for each multiplicity. The statistical theory, in turn, gives probabilities and spectra for every multiplicity. Combining the two informations, it is possible to derive the spectra of protons and charged pions in 2, 4, 6 or 8 prong events, and thus arrive at data which are directly comparable with experiment.

A detailed comparison with the experimental data shows then an important discrepancy : the inelasticity predicted by the theory is 75%, the experimental figure is 49%. We have tried to explain this by treating the glancing p-p collisions with a so-called "two-centre" model, and using the statistical model only for the central collisions. The ratio of central to glancing collisions is introduced as a parameter.

The success of the $\overline{J\pi}$ -isobar in the statistical theory of $\overline{p}$ -p annihilation has led us to consider a $\overline{J\pi}^*$ also in the meson production from p-p collisions. The result is that this does not contradict the experimental data; these are however too inaccurate at present to allow definite conclusions to be drawn about the $\overline{J\pi}^*$ in these phenomena.

Calculation of charge distribution in the statistical theory.

The formula - which was explained in ¹⁾ and ⁵⁾ - giving the probability of an event of type b is

$$P_b = W_{\alpha, \beta, \gamma}^{(T)} \left\{ \frac{\prod (2s_i + 1)^{j_i}}{\prod N_i!} \right\}_b \Omega^{n-1} \varphi_b^*(E, m_1, \dots, m_n; p=0) \quad (1)$$

where the event b is characterised by stating the number N_i of particles with mass m_i , spin s_i and isospin t_i . The coefficient $W_{\alpha, \beta, \gamma}$ depends on the total isospin and on the t_i ; $W_{\alpha, \beta, \gamma}$ is equal to the number of

independent isospin eigenfunctions to a total isospin T that can be built from α isospin $\frac{1}{2}$ particles β isospin 1 and γ isospin $\frac{3}{2}$ particles. Tables of $W_{\alpha,\beta,\gamma}$ have been published ^{6),7)}.

We see that b stands for a whole set of events, which can be distinguished from each other by different charge distributions of the particles. One may ask e.g. for the probability that in an event of type $b = 2$ nucleons + n mesons one will find a proton, a neutron and numbers n_+, n_0, n_- of π^+, π^0, π^- respectively. This probability is obtained by replacing in (1) the coefficient $W_{2,n,0}$ by another coefficient

$$\frac{1}{2}, \frac{1}{2}, 1, \dots, 1 \cdot P_{\frac{1}{2}, -\frac{1}{2}; n_+, n_0, n_-}^1$$

The coefficients $\dots P^{(1)}$ have been derived and computed for all interesting cases ⁸⁾. The sum of all the $\dots^* P$ over all possible charges (lower indices) is equal to $W_{2,n,0}$, so that $(\dots^* P^1 \dots W_{\alpha,\beta,\gamma})$ is the relative probability of a given charge state in events of type b .

A slight complication arises in our problem because of the production of nuclear isobars ^{*)} which decay fast and are observed as nucleons and pions, whose spectrum is different from the spectrum of the directly produced nucleons and pions. For instance, the events where 2 nucleons and 3 pions are produced come from three different types b of collisions, viz.

Because N^* has isospin $3/2$, some charge states of systems with one or more N^* give rise to several charge states of the end-products, according to Table 1 :

) The introduction of a N^ is discussed in ⁹⁾ and ⁵⁾. It corresponds to the $3/2 \ 3/2$ resonance in π -N scattering. Its mass is therefore taken as $1.31 M_N$ with $T = 3/2$, $J = 3/2$.

TABLE 1

t_3 of N^*	decay products	branching ratio
3/2	$\pi^+ + p$	1
1/2	$\pi^+ + n$	1/3
	$\pi^0 + p$	2/3
-1/2	$\pi^0 + n$	2/3
	$\pi^- + p$	1/3
-3/2	$\pi^- + n$	1

So, having calculated the probability of charge states for each type b (which involve 0, 1 or 2 N^*), one has to go a step further and compute from them the probabilities for a given observable charge state (i.e. $2N$ and $n\pi$). This has been described elsewhere⁵⁾ but for the sake of clarity we shall treat the above example in detail. We shall denote by

$w_{b,\pi}(\xi)$ the spectrum of the pions produced directly in an event b
 $w_{b,\pi^*}(\xi)$ " " " " " " by N^* " " " "
 $w_{b,N}(\xi)$ " " " " nucleons " directly " " " "
 $w_{b,N^*}(\xi)$ " " " " " " by N^* " " " "

We number the events (2) by $b=0, b=1, b=2$, the $w_{b,\dots}$ are normalised in such a way that

$$P_b = W_{\alpha,\beta,\gamma} \int_0^{\xi_{\max}} w_b(\xi) d\xi$$

The charge states for the $b=0,1,2$ are given in Table 2.

TABLE 2

b	states with isobars				observable end states					serial number of end states
	t_3		$\dots^*_{\text{P}}(1)$	nucleons direct	nucleons from N	π direct	π from N	weight of state		
	baryons	pions								
0	1/2 1/2	+ 0 -	2.400	p p		+ 0 -		2.400	1	
		0 0 0	0.300	p p		0 0 0		0.300	2	
	1/2 -1/2	+ 0 0	2.100	n p		+ 0 0		2.100	3	
		+ + -	3.000	n p		+ + -		3.000	4	
	-1/2 -1/2	+ + 0	1.200	n n		+ + 0		1.200	5	
1	3/2 1/2	0 -	0.800	p	p	0 -	+	0.800	6	
		0 0	0.350	n	n	0 0	+	0.350	7	
			1.000	n	p	+ -	+	1.000	8	
	1/2 1/2	0 0	0.250	p	p	0 0	0	0.167	9	
				p	n	0 0	+	0.083	10	
		+ -	0.600	p	p	+ -	0	0.400	11	
				p	n	+ -	+	0.200	12	
		1/2 -1/2	+ 0	1.200	n	p	+ 0	0	0.800	13
				n	n	+ 0	+	0.400	14	
	-1/2 -1/2	+ +	0.300	n	p	+ +	-	0.100	15	
				n	n	+ +	0	0.200	16	
	-3/2 1/2	+ +	0.500	p	n	+ +	-	0.500	17	
	2	3/2 1/2	-	0.600		p p	-	+ 0	0.4	18
					p n	-	+ +	0.2	19	
					p p	0	+ -	0.2	20	
					p n	0	+ 0	0.4	21	
3/2 -3/2		+	1.000		p n	+	+ -	1.000	22	
1/2 1/2		0	0.200		p p	0	0 0	0.089	23	
					p n	0	+ 0	0.089	24	
					n n	0	+ +	0.022	25	
					p p	+	0 -	0.133	26	
					p n	+	0 0	0.267	27	
1/2 -1/2		+	0.600		p n	+	+ -	0.067	28	
					n n	+	+ 0	0.133	29	

Suppose one wants to compute the spectra of pions from two prong events with one proton. One selects therefore end states with one proton and one charged pion.

For $b=0$ this is state $n^r : 3$
 $b=1$ these are states $n^r : 7, 10, 13$
 $b=2$ " " " $n^r : 21, 24, 27$

$w_{0,\pi}$	is therefore multiplied with the weight	2.100
$w_{1,\pi}$	" " " " " "	0.800
$w_{2,\pi}$	" " " " " "	0.267
$w_{1,\pi'}$	" " " " " "	(0.350 + 0.083)
$w_{2,\pi'}$	" " " " " "	(0.400 + 0.089)

and all these weighted spectra are added, together with the other weighted spectra from end states with 1, 2, 4, 5 ... pions. These other weights are obtained in a similar way, by combining the coefficients $\dots^*P'$ from ⁸⁾ with the known branching ratios of the N^* .

In practice the spectra for the events labelled b (i.e. phase space calculations) are computed by an electronic computer ¹⁰⁾, and the calculation outlined in the previous paragraph is done electronically too ¹¹⁾. The calculation has been organised in such a way that the end states are distinguished first by the number of charged particles (prongs). For a given number of prongs one distinguishes further between stars with 0, 1 or 2 protons, and in each case the spectrum of the $\pi^\pm$ and the p is computed ^{*)}. Once these spectra are known, it is easy to obtain any other kind of spectrum, to compare with experiment. E.g. the total p -spectrum, or the spectrum of $\pi^\pm$ in 4 prong events, etc.

The multiplicities and average energies are displayed in Tables 3 and 4. An analysis which can be compared directly with experiment ²⁾ has been carried out, and is displayed in Figures 1 - 12.

^{*)} For conciseness they are not given here, but they are available upon request.

In all these calculations strange particle production has been neglected. This is justified, we think, because of the rather small probability with which K-meson events appear. According to the statistical theory ¹⁾ we neglect 7.2% of all reactions. It would of course be possible to include the strange particles in the prong analysis. This would, however, involve much more calculations, while it is unnecessary to obtain a comparison with experiment. Indeed it should be possible reject from the statistics most of the stars leading to a V-particle.

Influence of a strong $\bar{\pi} - \pi$ interaction.

It has been assumed that a strong $\bar{\pi} - \pi$ interaction exists which would lead to a pronounced resonance in the $\bar{\pi} - \pi$ scattering ¹²⁾. If the width of the resonance is sufficiently small, one could introduce this state of two pions as another isobar, $\bar{\pi}^*$ say, in the same manner as the $\bar{\pi} - N$ resonance was introduced as a N^* .

Such a $\bar{\pi}^*$ gives quite good results in a statistical theory of $\bar{p} - p$ annihilation ¹³⁾, and it seemed therefore interesting to investigate its effect also in meson production from $p - p$ collisions.

This computation was done on exactly the same lines as those outlined above for the N^* isobar. All possible reactions leading to any combination of nucleons, N^* 's, pions and $\bar{\pi}^*$'s were listed, the corresponding phase space volumes computed, and a prong analysis made, which was then combined with the analysis of the events without $\bar{\pi}^*$.

For the $\bar{\pi}^*$ we took, as in ¹³⁾, :

mass = 4 pion masses, $T = 1$ and $J = 1$.

As a consequence the desintegration modes of the $\bar{\pi}^*$ are always unique, with a Q-value of 2μ .

$$\begin{aligned}\pi^{*+} &\rightarrow \pi^+ + \pi^0 \\ \pi^{*0} &\rightarrow \pi^+ + \pi^- \\ \pi^{*-} &\rightarrow \pi^0 + \pi^-\end{aligned}$$

TABLE 3

Probabilities of different prong numbers in inelastic p-p collisions at 6.2 GeV (neglecting K-meson production), for different values of the interaction volume Ω ; $\Omega_0 = \frac{4\pi}{3} \left(\frac{h}{\mu c}\right)^3$, $L = \frac{2M}{E}$. The results are given :

- a) assuming a pion-nucleon isobar N^*
 b) assuming a pion-nucleon isobar N^*
 and a pion-pion isobar π^*

number of prongs	probability with N^*			probability with N^* and π^*		
	$\Omega = 2L \Omega_0$	$\Omega = L \Omega_0$	$\Omega = \frac{1}{2}L \Omega_0$	$\Omega = 2L \Omega_0$	$\Omega = L \Omega_0$	$\Omega = \frac{1}{2}L \Omega_0$
2	0.356	0.434	0.502	0.215	0.255	0.303
4	0.575	0.526	0.476	0.643	0.635	0.616
6	0.068	0.040	0.022	0.142	0.110	0.081
8	$3 \cdot 10^{-4}$	$7 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	$7 \cdot 10^{-4}$	$4 \cdot 10^{-4}$

TABLE 4

Average particle numbers $\langle n \rangle$ and average kinetic energies $\langle \xi_{kin} \rangle$ (in units 0.938 MeV) of nucleons and pions from 6.2 GeV p-p collisions in different types of events for $\Omega = \frac{2M}{E} \Omega_0$.

Type of event			Assuming N^*				Assuming N^* and π^*			
number of prongs	number of protons	number of $\pi^\pm$	$\langle n \rangle_p$	$\langle \xi_{kin} \rangle_p$	$\langle n \rangle_{\pi^\pm}$	$\langle \xi_{kin} \rangle_{\pi^\pm}$	$\langle n \rangle_p$	$\langle \xi_{kin} \rangle_p$	$\langle n \rangle_{\pi^\pm}$	$\langle \xi_{kin} \rangle_{\pi^\pm}$
2	2	0	0.117	405		496	0.0315	405		
	1	1	0.289	333	0.289	496	0.160	323	0.160	430
	0	2			0.182	435			0.161	350
4	2	2	0.446	285	0.446	424	0.445	264	0.445	349
	1	3	0.282	225	0.847	347	0.378	204	1.14	289
	0	4			0.107	370			0.137	269
6	2	4	0.0671	173	0.134	284	0.160	164	0.319	249
	1	5	0.00638	133	0.0319	225	0.030	126	0.150	216
	0	6			$2.28 \cdot 10^{-4}$	176			$2.05 \cdot 10^{-3}$	201
8	2	6	$1.41 \cdot 10^{-4}$	103	$4.22 \cdot 10^{-4}$	176	$1.34 \cdot 10^{-3}$	89	$4.0 \cdot 10^{-3}$	185
	1	7	$8.91 \cdot 10^{-7}$	84	$6.23 \cdot 10^{-6}$	136	$2.4 \cdot 10^{-7}$	84	$1.7 \cdot 10^{-6}$	136

TABLE 5

Average particle numbers $\langle n \rangle$ and average kinetic $\langle \xi_{\text{kin}} \rangle$ energies (in units ~ 0.938 MeV) of nucleons and pions from 6.2 GeV p-p collisions, averaged over all events.

	Assuming N^*		Assuming N^* and π^*	
	$\langle n \rangle$	$\langle \xi_{\text{kin}} \rangle$	$\langle n \rangle$	$\langle \xi_{\text{kin}} \rangle$
neutrons	0.792	289	0.794	240
π^0	0.959	385	1.23	301
nucleons (p and n)	2.00	288	2.00	240
pions (π^- , π^0 , π^+)	3.00	385	3.65	302
protons	1.21	287	1.21	240
$\pi^\pm$	2.04	388	2.51	302
prongs (p and $\pi^\pm$)	3.25	350	3.72	282

Conclusions from the purely statistical model.

The figures which can at present be compared most easily with experiment are the average pion number and the probability for different prongs; these are found in Table 5 and Table 3. Comparing with the experimental numbers²⁾ it seems that the assumption of a N^* (without π^*) yields the right average pion number of 3.0 and a good fit to the prong-distribution, within the experimental errors.

However, the average energy of the pions, in this case, is too high. We compute 360 MeV (kinetic energy), and the measured value is 205. The discrepancy is in the other direction for the protons. We compute, for the 2-prong events, an average proton energy of 353 MeV, and the measured value is about 550.

These two facts are reflected in the degree of inelasticity K , defined as the ratio of the average total energy carried off by the pions to the total available kinetic energy in the C.M. system

$$K_{\text{computed}} = 0.74 \quad (\text{on the assumption of a } N^*)$$

$$K_{\text{measured}} = 0.49 \pm 0.05$$

If we turn now to the other assumption (existence of N^* and π^*), it appears that the average pion number 3.65 is too high and the prong distribution gives too many 4-prong events and not enough 2-prong events.

The average pion kinetic energy of 283 comes nearer to the measured values, but is still too high. The proton kinetic energy of 225 MeV gets still worse than in the no- π^* case. The degree of inelasticity we find is

$$K_{\text{computed}} = 0.76$$

and consequently neither of the two versions can explain the experiments.

The effect of peripheral collisions

The comparison of the computed values with the measured ones strongly suggest that the assumptions of the purely statistical theory are not justified, especially the assumption of a statistical distribution of the available energy between the outgoing particles. The nucleons seem to retain on the average a substantial portion of their initial energy, and the full kinetic energy of the colliding protons is not available to the pions.

At this point one should recall that only a fraction of all collisions are central collisions, for which the statistical model could be justified. In the glancing collisions it is not possible for the perturbation - which occurs at the edge of the nucleon - to spread over the two nucleons before the bulk of them separates again.

One would think much more, for such glancing collisions, of a so-called two-centre model in which the two nucleons are excited and which, after separating, radiate off mesons. Such a model yields automatically a high elasticity and a low multiplicity, and is so to say complementary to the purely statistical theory which should be able to treat only the central collision adequately.

Peripheral collisions, which yield only one or two mesons, and involve mostly the meson cloud of the nucleons should most adequately be treated by meson theory. What we shall do here is only to use a crude model, so as to obtain a qualitative estimate of the peculiarities of such collisions.

In such a two-centre model one has to assume a law for the excitation of the two centres. In the present case of glancing collisions, where only a small part of the available energy could be converted to excitation energy of the nucleon, this excitation should be governed again entirely by the predominance of the $3/2 - 3/2$ resonance at low relative π -N energies.

We shall make the working hypothesis that this excitation consists mainly of exciting one or both nucleons to isobars N^* , which then fly off along approximately the same direction as the incoming protons.

We shall simplify still further, and consider all collisions to be of three kinds :

- a) "central collisions" where the statistical theory applies and the whole kinetic energy is available for particle production;
- b) "peripheral collision", where one of the nucleons is excited into an isobar;
- c) "peripheral collision" where both nucleons are excited.

We repeat that we consider this whole argument only as a leading to a working hypothesis which will have to be justified by its consequence.

According to this model all quantities such as energy spectra, prong distributions, etc. should be calculated by taking a weighted average of the quantities predicted by the statistical model and those predicted by the "two centre" model.

One could then by comparison with some experimental numbers - the prong distribution say - derive the ratios of central collision to peripheral $N + N^*$ to peripheral $N^* + N^*$ collision.

With those numbers one should then be able, if the working hypothesis is any good, to derive the inelasticity and the spectra, and even to state something about angular distribution.

Amount of peripheral collision.

The reactions $N + N^*$ and $N^* + N^*$ lead to the prong distribution given in table 6.

TABLE 6

Prong distribution in peripheral collisions

		$N + N^*$	$2 N^*$
2 prongs	p p	0.1666	0.1777
	n p	0.8333	0.5777
	n n	-	0.0444
4 prongs	p p		0.2

If we call p_0 the probability to have a central collision

p_1 " " " " " peripheral " of type $N+N^*$
 p_2 " " " " " " " " " 2 N^*

and f_2, f_4, f_6 the experimentally measured frequencies of 2, 4 or 6-prong events, we have then the following equation for p_0, p_1 and p_2 :

$$\left. \begin{array}{l} 0.434 \\ 0.255 \end{array} \right\} p_0 + p_1 + 0.8 p_2 = f_2$$

$$\left. \begin{array}{l} 0.526 \\ 0.635 \end{array} \right\} p_0 + 0.2 p_2 = f_4 \quad (3)$$

$$\left. \begin{array}{l} 0.040 \\ 0.110 \end{array} \right\} p_0 = f_6$$

where the upper coefficients in the first column refer to the N^* statistical theory, the lower ones to the $(\pi^* + N^*)$ theory with $\Omega = L \Omega_0$.

The inelasticity and the average number of charged particles can now be computed, from table 5 and table 7.

TABLE 7

Average number $\langle n \rangle$ and average kinetic energy $\langle \epsilon \rangle$ (in units ~ 0.938 MeV of particles from glancing collision.

Channel	$\langle n \rangle_{p, \text{direct}}$	$\langle \epsilon \rangle_{p, \text{direct}}$	$\langle n \rangle_{p, \text{decay}}$	$\langle \epsilon \rangle_{p, \text{decay}}$	$\langle n \rangle_{\pi^*}$	$\langle \epsilon \rangle_{\pi^*}$
$N + N^*$	0.25	975	0.917	725	0.8333	325
$N^* + N^*$	-	-	1.333	650	1.0667	300

The probabilities for 2, 4, 6-prong events, the inelasticity K and the number of charged particles C are plotted as functions of p_0 in Figs. 13-16. These figures represent the equation (3). As a function of p_0 various observables are plotted, p_1 or p_2 set to zero as a parameter. Then the measured values of the observables are inserted and the values of p_0 compatible with the experimental error are indicated by dotted lines. One sees that the ratio p_1/p_2 does not have much effect on the result, but that the important quantity is p_0 , i.e. the fraction of central collision.

The published measurements of f_2, f_4, f_6 have rather wide errors, and it will be better to try to fit first the inelasticity and the prong number, which are better measured^{2), 3), 4)}.

It appears at once that, due to the effect of the peripheral collisions, it is possible to fit C and K simultaneously, with or without the π^* -hypothesis, indicating a p_0 around 0.5.

With this value of p_0 more or less fixed, it appears then that f_2 and f_4 can be fitted within the limits of error of the Seattle measurements²⁾, only if the π^* is assumed. With the Bern measurement⁴⁾, the fit can be achieved with and without the π^* hypothesis.

Further information is contained in the spectra. In Table 8 are shown the calculated mean energies of protons and pions, under the assumption $p_0 = 0.5$ $p_1 = p_2 = 0.25$.

This is to be compared with the experimental numbers given in Table 9.

TABLE 8

Average particle numbers $\langle n \rangle$ and average kinetic energies $\langle \epsilon \rangle$
 (in units 0.938 MeV) of nucleons and pions from 6.2 GeV p-p collisions
 for $\Omega = \frac{2M}{E} \Omega_0$, and for equal numbers of central and peripheral
 collisions ($p_0 = 0.5$ $p_1 = p_2 = 0.25$)

Type of event			Assuming N^*				Assuming N^* and π^*			
N^r of prongs	N^r of p	N^r of $\pi^\pm$	$\langle n \rangle_p$	$\langle \epsilon \rangle_p$	$\langle n \rangle_{\pi^\pm}$	$\langle \epsilon \rangle_{\pi^\pm}$	$\langle n \rangle_p$	$\langle \epsilon \rangle_p$	$\langle n \rangle_{\pi^\pm}$	$\langle \epsilon \rangle_{\pi^\pm}$
2	2	0	0.230	573	-	-	0.188	721	-	-
	1	1	0.449	601	0.449	367	0.435	639	0.435	336
	0	2	-	-	0.093	432	-	-	0.083	349
4	2	2	0.320	395	0.320	367	0.329	381	0.319	334
	1	3			as in table 4					
	etc				"					
average over all events			1.23	502	1.50	364	1.23	479	1.73	304

TABLE 9

Average numbers and kinetic energies of protons and charged pions (experimental)

Energy unit of 0.938 MeV

	mean prong	n^r	mean $\mathcal{E}_p$	mean $\mathcal{E}_{\pi^\pm}$	mean $\mathcal{E}_{\pi^\pm}$
Seattle measurements 2)	-	-	$550 \pm ?$	1.9 ± 0.3	252 ± 21
Bombay " 3)	-	-	636 ± 96	1.51 ± 0.18	309 ± 48
Bern 4)	2.8	-	-	-	-

It seems, from a comparison of the experiments and the theory, that the present model, with perhaps a more careful choice of the values of Ω and p_0 , could very well predict all such features of the production process that are concerned with quantities averaged over all angles (i.e. mean particle numbers and spectra).

Angular distributions

In deriving the statistical model Hagedorn¹⁴⁾ has shown that it makes sensu stricto no prediction about angular distribution, but is only expected to yield correctly all quantities averaged over angles.

The model we use here does, however, make definite predictions about the angular distribution of the particles from peripheral collisions.

If we assume that the isobars move along the axis defined by the incoming protons before they decay isotropically with respect to their own rest-system, the resulting p and $\bar{p}$ will have a definite correlation between their energy and angle in the C.M.S. It turns out that this correlation is nearly the same in the two cases, where either one or two N^* are produced in a collision.

In Fig. 17 this correlation is given for the case, where the N^* has the direction $\theta = 0^\circ$. As with equal probability the N^* will have the direction $\theta = 180^\circ$ this relation has to be reflected with respect to the plane $\theta = 90^\circ$.

While a resulting nucleon will move in a direction of at most 11° with respect to the incoming nucleon, the resulting pion will have a less peaked angular distribution as shown in Figs. 18 and 19.

We could now make the additional assumption that the central collisions yield approximately isotropic distributions.

This isotropy is suggested by the experimental distribution of 6-prong events²⁾ which, at least in our model, are entirely due to central collisions.

Combining this isotropic distribution with the computed anisotropy of the peripheral collisions one is able to predict the angular distribution of product particles as well as the average dependence of the energy spectra.

If one considers a fixed angle, in addition to the continuous spectrum originating from central collisions a line spectrum will arise as a consequence of the angle-energy-correlation shown in Fig. 17. Of course this line spectrum will have a large width due to the energy width of the pion-nucleon-isobar. As an example we show in Fig. 20 the combined energy spectrum for $\theta = 0^\circ$ (or $\theta = 180^\circ$).

In comparing with experiment one should bear in mind the two special assumptions made in deriving the figures of this section :

- a) isotropy of the products of central collisions;
- b) direction of the two centres in peripheral collisions = direction of incoming protons.

These assumptions are of a less convincing nature than the others made in this theory and should be checked experimentally. The study of angular correlations (cf. next section) in certain types of events allows to study peripheral collisions and to check the isobar model independently of the assumptions just mentioned.

Angular correlation

Charge analysis of peripheral collisions shows that the channel with the doubly-charged isobar will be relatively probable. Because this isobar is decaying into a proton and a π^+ one will find an angular correlation between these particles. For those two-prong events resulting in proton and π^+ a detailed analysis gives

$$g(\theta)d\cos\theta = \frac{\{0.08 p_0 + [0.75 p_1 + 0.4 p_2] \mathcal{C}(\theta) + [0.083 p_1 + 0.177 p_2] \mathcal{C}(\pi - \theta)\} d\cos\theta}{0.16 p_0 + 0.833 p_1 + 0.577 p_2} \quad (4)$$

where $g(\theta)$ is the probability density for the angle between the π^+ and the direction of the isobar from which it originates. Because the angle between the isobar and the proton into which it decays is at most 11° the above formula is approximately correct if θ is interpreted as the angle between p and π^+ .

The function $\mathcal{C}(\theta)$ is given in Fig. 22 while $g(\theta)$ is shown in Fig. 21 for $p_0 = \frac{1}{2}$, $p_1 = p_2 = \frac{1}{4}$. In deriving (4) we assumed that those two-prong events which originate in central collisions will exhibit no angular correlation, as they are produced mainly by other channels. It is interesting to note that this angular correlation is independent of the assumption that the isobars fly away along the collision line. This is due to the fact that in two centre collisions both centres must have opposite directions. Even if it will not be possible to distinguish between protons and pions one should find a similar angular correlation between the couples of charged particles in two-prong events. The main difference will be an increase of the function $g(\theta)$ between $\cos\theta = -1$ and $\cos\theta = -0.95$ due to couples of proton tracks.

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Fig.1 2 prong events, p spectrum

Fig. 2 2 prong events, $\pi^\pm$ spectrum

Fig.3 2 prong events, $p+\pi^\pm$ spectrum

Fig.4 4 prong events, p spectrum

Fig. 5 4 prong events, $\pi^\pm$ spectrum

Fig. 6 4 prong events, $p+\pi^\pm$ spectrum

Fig. 7 6 prong events, p spectrum

Fig.8 6 prong events, $\pi^{\pm}$ spectrum

Fig.9 6 prong events, $p+\pi^\pm$ spectrum .

Fig. 10 All events, p spectrum .

Fig. 11 All events, $\pi^{\pm}$ spectrum.

Fig.12 All events, $\pi^{\pm} + p$ spectrum.

Fig.13 Inelasticity (K) and number of charged pions as a function of the proportion of central collisions, p_0 (N^* , $n_0\pi^*$ hypothesis).

Fig.14 Frequencies of 2,4 and 6 prong events as a function of the proportion of central collisions.
 N^* , no π^* hypothesis.

Fig 15 Inelasticity (K) and number of charged pions as a function of the proportion of central collisions, p_0 . (π^* , N^* hypothesis)

Fig.16 Frequencies of 2,4 and 6 prong events as a function of the proportion of central collisions N^*, π^* Hypothesis.

Fig.17 Energy angle correlation for π' s.

Fig.18 Angular distribution of charged π 's produced in peripheral collisions through N^*+N channel $\int f(\theta) d \cos \theta = 0,833$

Fig.19 Angular distribution of charged π 's produced in peripheral collisions through $N^* + N^*$ channel. $\int f(\theta) d \cos \theta = 1.065$

Fig.20 Composed energy spectrum for charged π' 's
at $\theta=0^\circ$ (with $p_0 = 1/2$, $p_1 = p_2 = 1/4$)

This spectrum for charged π' 's is composed of the continuous spectrum from central collisions and the "line"-spectrum from peripheral collisions for $\theta=0^\circ$. The relative number of charged π' 's from central collisions is only 36% at this angle.

Fig. 21 Probability density $g(\theta)$ for the angle between the

p and the π^+ in 2 prong events (with $p_0 = 1/2$, $p_1 = p_2 = 1/4$)

This angular correlation function $g(\theta)$ gives the relative probability of finding a π^+ between $\cos \theta$ and $\cos \theta + d \cos \theta$, θ being the angle between the π^+ and the proton in two prong events.

Fig. 22

The function $C(\theta)$ is obtained by calculating the probability of finding the π between $\cos \theta$ and $\cos \theta + d\cos \theta$ in the centre of mass system, assuming that the N^* decays isotropically with respect to its own rest system. The direction of the N^* is 0° . As the velocity of the CMS is comparable within the velocity of the π in the N^* system most π 's fly into the forward direction.