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Investigation of the possibility of a $\pi - \Lambda$ resonance
in the case of different parities of Λ and Σ .⁺)

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The existence of a resonant state in pion-hyperon scattering has recently been confirmed experimentally ¹⁾. Due to its importance in the physics of the strongly interacting particles, various theoretical previsions on its nature and its implications were attempted before its experimental discovery ²⁾. These were based on various assumptions on the nature and the symmetries of the elementary interactions. In particular, Amati-Stanghellini-Vitale studied the problem, applying the no-crossing variational method, within the framework of the fixed-source theory ³⁾. From the beginning, global symmetry was postulated, at least as a guiding principle, but possible small differences of the coupling constants f_{Λ} and f_{Σ} were taken into account, along with the Λ - Σ mass difference. These authors predicted two resonant states for $J=3/2$, with $T=1$ and $T=2$, respectively. However, at present it seems that the observed resonance is characterised by $J=\frac{1}{2}$. Such a resonance can only be explained by these authors taking a large difference between f_{Λ} and f_{Σ} . This means a strong deviation from global symmetry. In such a situation one should introduce two cut-off parameters into the theory instead of one. Moreover, the possibility of opposite parities of Λ and Σ has to be taken into account.

In this paper we study the consequences of the hypothesis of opposite parities, using the same technique. Because a characteristic feature of this technique is the fact that, together with p-waves, s-waves (and no higher ones) enter into picture, one may hope to find resonances in $J=\frac{1}{2}$ states.

Although this technique certainly does not provide for a complete description of the scattering processes, it will at least lead to valuable qualitative understanding of the scattering in the low-energy region.

The Λ -parity is assumed to be +1. The interaction Hamiltonian is $H_{sc} + H_v$ with

$$H_{pv} = \sqrt{4\pi} f_{\Sigma} \sum_{k, \alpha, m} C(k) \sigma_m \left[a_{\alpha, m}(k) T_{\Sigma}^{\alpha} + a_{\alpha, m}^{\dagger}(k) T_{\Sigma}^{+\alpha} \right]$$

$$H_{sc} = \sqrt{4\pi} f_{\Lambda} \sum_{k, \alpha} D(k) \left[a_{\alpha}(k) + a_{\alpha}^{\dagger}(k) \right] T_{\Lambda}^{\alpha}$$

(units $\hbar = c = \mu = 1$),

where $a_{\alpha}^{\dagger}(k)$ is the creation operator for a pion with momentum k , isotopic spin index α , in an s-wave, while $a_{\alpha, m}^{\dagger}(k)$ is the creation operator for a pion with momentum k , isotopic spin index α , in a p-wave, m specifying the third component of its orbital angular momentum ($m=1, 2, 3$).

$$D(k) = \frac{kv(k)}{\sqrt{\pi\omega}}$$

$$C(k) = \frac{k^3 v(k)}{\sqrt{3\pi\omega}}$$

($v(k)$ = cut-off function)

T_{Λ}^{α} and T_{Σ}^{α} are analogous to the τ -operators in the pion-nucleon interaction. They are explicitly given by:

$$T_{\Lambda}^{\alpha} = \sum_{\alpha}^{+} \Lambda + \Lambda^{+} \sum_{\alpha}$$

$$T_{\Sigma}^{\alpha} = \frac{\epsilon_{\alpha\beta\gamma}}{2} \sum_{\beta}^{+} \sum_{\gamma}$$

$\epsilon_{\alpha\beta\gamma}$ = Levi-Civita tensor

The scattering states are classified according to the total isotopic spin T , angular momentum J and parity P . In the one meson approximation they are represented by the general formula:

$$|\pi_q^Y\rangle = \sum_{q''} \left[a_{q''}^+ |\Lambda\rangle X_1(q, q'') + a_{q''}^+ |\Sigma\rangle X_2(q, q'') \right] + |\Lambda\rangle c_0 + |\Sigma\rangle c_1$$

where q comprises all relevant indices.

The $\Lambda - \Sigma$ mass difference Δ is taken into account explicitly. The other parameters, on which our results will depend, are the renormalized coupling constant f_Λ and the product $f_\Sigma^2 I$ with renormalized coupling constant f_Σ , and I depending on one cut-off.

The various coefficients X and C are determined, using a well-known variational procedure ^{2), 3)}. The X -functions must satisfy integralequations (or systems of coupled integral equations), which can be solved in an elementary way, using given boundary conditions, which describe the initial and final states of the scattering problems. The possibility of a resonance is determined by vanishing of the denominators of the X -functions.

Omitting the tedious calculations which are necessary to arrive at expressions for these denominators, the final results are given here for all scattering states.

T	J	P	state vector	denominator
0	$\frac{1}{2}$	+	$ \Lambda\rangle + \pi\Sigma\rangle_s$	$\begin{vmatrix} 1+3f_\Lambda^2\omega\rho_+^*(\omega-\Delta) & -f_\Lambda^2(\omega-2\Delta)\rho_+^*(\omega-\Delta) \\ 3f_\Lambda^2\omega\rho_+^*(\omega-\Delta) & 1-f_\Lambda^2(\omega-2\Delta)\rho_+^*(\omega-\Delta) \end{vmatrix}$
		-	$ \pi\Sigma\rangle_p$	$1-2f_\Sigma^2(\omega-\Delta)I$
	$\frac{3}{2}$	+	—	
		-	$ \pi\Sigma\rangle_p$	$1+4f_\Sigma^2(\omega-\Delta)I$
	$\frac{1}{2}$	+	$ \pi\Lambda\rangle_p + \pi\Sigma\rangle_s$	$1-6f_\Lambda^2f_\Sigma^2\omega^2I\rho_+^*(\omega-\Delta)$
		-	$ \Sigma\rangle + \pi\Lambda\rangle_s + \pi\Sigma\rangle_p$	$\begin{vmatrix} 1-f_\Lambda^2(\omega+\Delta)\rho_+^*(\omega) & f_\Lambda^2(\omega-\Delta)\rho_+^*(\omega) & \sqrt{2}f_\Lambda f_\Sigma(\omega-\Delta)\rho_+^*(\omega) \\ -f_\Lambda^2(\omega+\Delta)\rho_+^*(\omega) & 1+f_\Lambda^2(\omega-\Delta)\rho_+^*(\omega) & \sqrt{2}f_\Lambda f_\Sigma(\omega-\Delta)\rho_+^*(\omega) \\ 0 & 3\sqrt{2}f_\Lambda f_\Sigma(\omega-\Delta)I & 1+7f_\Sigma^2(\omega-\Delta)I \end{vmatrix}$
1	$\frac{3}{2}$	+	$ \pi\Lambda\rangle_p$	—
		-	$ \pi\Sigma\rangle_p$	$1-2f_\Sigma^2(\omega-\Delta)I$
	$\frac{1}{2}$	+	$ \pi\Sigma\rangle_s$	$1+f_\Lambda^2(\omega-2\Delta)\rho_+^*(\omega)$
		-	$ \pi\Sigma\rangle_p$	$1+f_\Sigma^2(\omega-\Delta)I$
	$\frac{3}{2}$	+	—	
		-	$ \pi\Sigma\rangle_p$	$1-2f_\Sigma^2(\omega-\Delta)I$
2	$\frac{1}{2}$	+	$ \pi\Sigma\rangle_s$	$1+f_\Lambda^2(\omega-2\Delta)\rho_+^*(\omega)$
		-	$ \pi\Sigma\rangle_p$	$1+f_\Sigma^2(\omega-\Delta)I$
	$\frac{3}{2}$	+	—	
		-	$ \pi\Sigma\rangle_p$	$1-2f_\Sigma^2(\omega-\Delta)I$

with the following definitions:

$$I = \frac{1}{3\pi} \int d\omega' \frac{k'^3 v^2(k')}{\omega'^2(\omega' - \omega)}$$

$$\rho_{\pm}^{\pm}(\omega) = \frac{1}{\pi} \int d\omega' \frac{k' v^2(k')}{(\omega' \pm \Delta)^2(\omega' - \omega)}$$

$$\rho_{+}^{-}(\omega) = \frac{1}{\pi} \int d\omega' \frac{k' v^2(k')}{(\omega'^2 - \Delta^2)(\omega' - \omega)}$$

Before examining these results, we want to stress two peculiar aspects, due to the introduction of opposite $\Lambda - \Sigma$ parities:

- a) The doubling of the number of possible states as a consequence of parity conservation, makes the one meson approximation rather poor ^{*)}. For example in the state $T=1$, $J=3/2$, $P=+$ no scattering occurs.
- b) The presence of s-waves leads to integrals of the type $\rho(\omega)$ which depend strongly on ω in the low-energy region, in contrast to integrals of the type $I(\omega)$, for which the effective range approximation is justifiable. However, the ρ 's practically do not depend on the cut-off. This eliminates one of the two cut-off parameters in the theory. This can be considered as a typical advantage of the hypothesis of opposite parities, within the framework of our method.

Before discussing the main features of the pion-hyperon scattering in our model, as far as resonances are concerned, the most important results are comprised in the following tables:

^{*)} It is felt therefore that elaborate calculation of branching ratios and the width of resonances in this model makes little sense.

J	T	P	possible resonant state
$\frac{1}{2}$	0	-	$X_0^*(\Sigma)$
$\frac{1}{2}$	1	+	$Y_{(+)}^*(\Lambda, \Sigma)$
$\frac{1}{2}$	1	-	$Y_{(-)}^*(\Lambda, \Sigma)$
$3/2$	1	-	$X_1^*(\Sigma)$
$3/2$	2	-	$X_2^*(\Sigma)$

possible resonant state	ω	f_Λ^2	$f_\Sigma^2 I$
$X_1^*(\Sigma)$	$1 < \omega < 2.5$		≥ 0.25
$Y_{(+)}^*(\Lambda, \Sigma)$	$1.5 < \omega < 2.5$	> 1.5	≤ 0.1
$Y_{(-)}^*(\Lambda, \Sigma)$	$1 < \omega < 2.4$	> 1.5	

Although one might expect that the state $T=0$, $J=\frac{1}{2}$, $P=+$ also would give a resonance, it can be proved that in the energy region, $1.5 < \omega < 2.4$, the denominator can only vanish for $f_\Lambda^2 < 0$. Then one is left with five possible resonant states, of which the three states denoted by $X_1^*(\Sigma)$ have the same location on the ω -axis, which does not depend on $f_\Lambda^2 I$. However, either $f_\Sigma^2 I$ has to take on a value which is larger than the corresponding quantity for π -N scattering¹⁾ or this location is at high energy, $\omega > 2.5$, where the method becomes doubtful.

The only states which can have a resonance in $\pi - \Lambda$ as well as $\pi - \Sigma$ scattering at low energies are just the remaining possibilities $Y_{(+)}^*$ and $Y_{(-)}^*$ characterised by $J=\frac{1}{2}$ and $T=1$. At this stage one should examine whether both resonances occur for the same values of the parameters ω , f_{Λ}^2 and $f_{\Sigma}^2 I$ or not. However, from the resonance-equation only necessary conditions can be derived suitably and as is shown in the last table the conditions do not demand elimination of any of the two possibilities. Subsequently the special case is considered where the observed $\pi - \Lambda$ resonance is identified with $Y_{(-)}^*$ and it is found that in this case $Y_{(+)}^*$ can occur only at high energy: $\omega_+ > 2.5$.

In somewhat more details:

As far as $Y_{(+)}^*$ is concerned it is evident that the requirements for f_{Λ}^2 and $f_{\Sigma}^2 I$ are not independent. Because f_{Σ} is analogous to the pion-nucleon coupling constant, it is reasonable to suppose $f_{\Sigma}^2 I \leq 0.1$ which leads to $f_{\Lambda}^2 > 1.5$. If on the contrary $f_{\Sigma}^2 I > 0.1$, then for existence of $Y_{(+)}^*$, f_{Λ}^2 should be ≤ 1.5 , which excludes existence of $Y_{(-)}^*$.

On the other hand, analysis of the resonance-equation for the state $Y_{(-)}^*$ leads to the following results:

The equation can be written in the form:

$$-F(f_{\Sigma}^2 I, \omega) \cdot L(f_{\Lambda}^2, \omega) = Q(f_{\Lambda}^2, \omega)$$

where the relevant properties of the functions F , L and Q are as follows:

- F is not negative on the intervals $1 < \omega \leq 2.4^{*)}$ and $0 \leq f_{\Sigma}^2 < \infty$.
- L is linear in f_{Λ}^2 : $L = 1 - f_{\Lambda}^2 (\omega + \Delta) \rho_+^+(\omega)$. This function increases as function of ω and is positive on the sub. a) mentioned ω -interval, if $f_{\Lambda}^2 \leq 1.5$. For $1.5 < f_{\Lambda}^2 < 4$ L passes through the ω -axis, the zero is called $\omega(f_{\Lambda}^2)$.

*) This is the low energy region for which the numerical calculations have been performed.

c) Q is quadratic in f_{Λ}^2 and can be written in the form

$$Q = \{1 + f_{\Lambda}^2 g(\omega)\} L(f_{\Lambda}^2, \omega) + f_{\Lambda}^4 h(\omega)$$

where $h(\omega) > 0$ and $g(\omega) > -0.5$ on the mentioned ω -interval.

This function is positive definite on the interval $1.2 < \omega < 2.3$.

From these properties it follows that a resonance is not admitted on the interval $1 < \omega \leq 2.4$ if $f_{\Lambda}^2 \leq 1.5$. If $1.5 < f_{\Lambda}^2 < 4$ the equation can be satisfied for appropriate sets of values of $f_{\Lambda}^2, f_{\Sigma}^2, \omega$. A slightly more severe condition for f_{Λ}^2 follows from $L < 0$ (on $1.2 < \omega < 2.3$), or $\omega < \omega(f_{\Lambda}^2)$. For instance in $\omega = 1.9$, which energy corresponds to the observed resonance energy¹⁾, one finds thus $f_{\Lambda}^2 > 2.5$. It is emphasized here, that these requirements for f_{Λ}^2 are independent of the value of f_{Σ}^2 , provided that $f_{\Sigma}^2 \neq 0$.

In the physically possible limiting case $f_{\Sigma}^2 = 0$ (we know from the existence of hyperfragments that f_{Λ}^2 must be $\neq 0$), all indicated resonances disappear, except $Y_{(-)}^*$. For this state the resonance equation reduces to $Q(f_{\Lambda}^2, \omega) = 0$. Consequently ω cannot belong to the interval $1.2 < \omega < 2.3$. This being in contradiction to experimental evidence, one would conclude $f_{\Sigma}^2 \neq 0$.

As concerns the relative magnitude of f_{Λ}^2 and f_{Σ}^2 , it is noticed that on account of the different nature of scalar and pseudo-vector interaction, no direct comparison between the two coupling constants can be made. Therefore it would be important to have an independent estimation, for instance for f_{Λ}^2 , resulting from the study of hyperfragments.

Finally it is mentioned that some support can be given to the speculation that the resonance is actually $Y_{(-)}^*$ which means $P = -$. Decay of $Y_{(-)}^*$ is then possible into $(\pi \Lambda)_s$ and $(\pi \Sigma)_p$, of which the

first state is favored, for energy and angular momentum reasons. There is experimental confirmation of such a preference ¹⁾. Taking this for granted, ω_- can be taken equal to 1.9, corresponding to the observed energy, which yields "resonating" values of f_{Λ}^2 and f_{Σ}^2 1. These values are used to calculate ω_+ ; the result has already been mentioned.

We arrive at the following conclusions:

- 1) Opposite parities of Λ and Σ do not allow for a $\pi-\Lambda$ resonance in a $T=1, J=3/2$ state, but are consistent with $T=1, J=1/2$ for such a resonance. No other $\pi-\Lambda$ resonances appear. ^{*)}
- 2) If the observed resonance is actually $Y_{(-)}^*$, than $Y_{(+)}^*$ eventually can occur at an energy $\omega_+ > 2.5$.

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^{*)} In ref. ⁴⁾ an analysis is made of both possibilities for the relative $\Lambda-\Sigma$ parity. For odd relative parity no $\pi-\Lambda$ resonance is found at all, but three $\pi-\Sigma$ resonances occur in the same T, J -states as our X_i^* . The differences with our results are due to the very crude approximation of taking the integrals of the type $\rho(\omega)$ independent of ω and Δ .

R E F E R E N C E S

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