HIGH-ENERGY BEHAVIOR OF CROSS SECTIONS IN QUANTUM ELECTRODYNAMICS

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(Received October 10, 1960)

If one wishes to detect a possible breakdown of quantum electrodynamics at small distances by comparing high-energy scattering experiments with theoretical predictions available to some given order in α , the following question arises: What is the magnitude of the radiative corrections beyond that order? One knows indeed that terms of the type $(\alpha/\pi)^n \ln^{2n}(q^2/m^2)$, where q is the invariant momentum transfer and m the electron rest mass, are likely to occur in a series expansion in α . In other words, one is presented with corrections which are possibly expressed as a series expansion in the effective parameter $(\alpha/\pi) \ln^2(q^2/m^2)$. Such a situation actually obtains in scattering processes of a spin $\frac{1}{2}$ particle by a neutral vector meson field with nonvanishing rest mass, and in quantum electrodynamics in several first order corrections to cross sections, evaluated in the laboratory system.¹ The importance of the possible existence of such $(\alpha/\pi)^n \times \ln^{2n}(q^2/m^2)$ terms is made apparent by considering an experiment,² in which $q^2 = 10^6 m^2$, yielding $\ln(q^2/m^2) \cong 14$, so that $\ln^2(q^2/m^2) \cong 200$ would make the convergence of the perturbation expansion very doubtful – to say the least.

We have now investigated in detail the problem, whether the effective expansion parameter is indeed $(\alpha/\pi) \ln^2(q^2/m^2)$, and we conclude that this is not the case but that the series of radiative corrections to scattering cross sections, evaluated in the c.m. system, is rather an expansion in terms of an effective parameter which is at most

$$(\alpha/\pi) \ln^{4/3}(q^2/m^2). \quad (1)$$

The available first order calculations related to various processes described in the c.m. system^{2,3} agree with the above theorem on the maximum power of $\ln(q^2/m^2)$. However, one could not from these first order calculations alone conclude as to whether the maximum power involved is n or $2n - 1$, as these are equal for $n = 1$. Our theorem provides the means to rule out – among other things – the occurrence of $(\alpha/\pi)^n \ln^{2n-1}(q^2/m^2)$ terms, which could invalidate the meaning of first order calculations due to a breakdown of the expansion.

Our proof is based on a method similar to that

of Sudakov.⁴ The variables of integration, k_i , in momentum space are first resolved into components parallel and perpendicular to the “plane” determined by the initial and final electron momenta p and p' . It is next shown that the $(2n)$ th order Spence functions,⁵ to which one is led in evaluating the n th order correction to the elastic scattering cross section, originate from soft-photon contributions exclusively. Cutting off the photon momenta in the infrared can be achieved by forbidding that both components along p and p' are small at the same time.⁶ The $(2n)$ th order Spence functions thus factorize into bilinear products of n th order Spence functions involving q^2/m^2 and n th order Spence functions involving the infrared cutoff. The latter, as is well known, can be eliminated rigorously in terms of the maximum energy loss ΔE (in the c.m. system), involved in the corresponding inelastic processes with emission of soft real photons and electron pairs, under the condition that ΔE is small compared to the c.m. energies E of the scattered particles.⁷

The n th order Spence functions are asymptotically equal to $\ln^n$ for large arguments. But contributions of the form $\ln^n(E/\Delta E)$ are harmless, because they can be summed in closed form, if $\Delta E/E \ll 1$, i.e., when $\ln(E/\Delta E)$ is large.⁸ On the other hand, it just happens that in the case of realistic high-energy experiments, ΔE is not expected to be much smaller than E , so that $\ln(E/\Delta E)$ will amount only to a few units.² Therefore, the only large quantity that will occur to any order in α is $(\alpha/\pi) \ln(q^2/m^2)$. The exponent $4/3$ in (1) stems from the fact that for corrections involving closed electron loops the analysis is much more complicated, and $4/3$ is the upper bound for the exponent that could be derived with a reasonable amount of work. It is very likely, of course, that actually $(\alpha/\pi) \ln(q^2/m^2)$ simply occurs. The methods to prove this are very intricate and would not be justified considering that an expansion in terms of (1) “converges” rapidly enough for energies higher than any energy realizable in the foreseeable future.⁹

The fact that at most only $(\alpha/\pi)^n \ln^{4/3 n}(q^2/m^2)$ terms can occur makes it now possible to decide whether or not standard quantum electrodynamics

remains valid at the high energies available nowadays, because it shows that the first order terms of the perturbation expansion are meaningful and reliable. Moreover, for the presently proposed experiments, the calculation of the leading corrections in first order in α to $\sigma_{el} + \sigma_{soft}$ is quite sufficient for the precision needed. Of course, in addition to soft photons and pairs, a large amount of hard photons and pairs may be emitted along the directions of the incoming and outgoing momenta. Thus the cross sections should be written as

$$\sigma = \sigma_{el} + \sigma_{soft} + \sigma_{hard} + \sigma_{hard+soft}. \quad (2)$$

But the cross section for hard photon and pair emission, σ_{hard} , is very sensitive to the experimental geometry and must be calculated separately for every particular experimental design.¹⁰

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⁹For instance, for $q = 100$ Bev, (1) yields roughly 1/6.

¹⁰The last term in (2) gives the correction to σ_{hard} due to emission of soft photons and pairs. The result obtained above for $\sigma_{el} + \sigma_{soft}$ applies also for $\sigma_{hard} + \sigma_{hard+soft}$ and thus for the complete cross section.