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Effect of a Pion-Pion Scattering Resonance on
Low Energy Meson-Photoproduction

by

M. Gourdin, D. Lurié and A. Martin

CERN - Geneva

ABSTRACT

The influence of the two and three pion intermediate states in the channel $\gamma \pi \rightarrow \bar{N}N$ on photoproduction is studied.

Application of the Cini-Fubini version of the Mandelstam representation to the amplitude $H^{(0)}$ (isoscalar photon) permits the evaluation of the contribution of a $J = I = 1$ pion pion resonance in terms of the isovector nucleon form factors and of the quantities describing the $\gamma + \pi \rightarrow \pi + \pi$ amplitude. In the limit of a very narrow resonance the result can be obtained by introducing an isovector vector boson coupled to $\gamma \pi$ and to $\bar{N}N$. This contribution can be merely added to the previous evaluations of $H^{(0)}$, since rescattering corrections are negligible at low energies.

Arguments are given to show that the only important three-pion state is a resonant $I = 0$ $J = 1$ state which is crudely described by an isoscalar vector boson. Some of the parameters entering in this calculation can be obtained from experimental information on the isoscalar nucleon form factors. Though the three pion contribution to $H^{(+)}$ (isovector photon) is very simple, the final result is obtained in a more complicated way because rescattering corrections are important. Through crossing, $H^{(-)}$ (isovector photon) is also affected.

It seems that one of the most interesting quantities to consider, in order to compare theory and experiment, is the $d\sigma^+/d\sigma^-$ ratio and its dependence on energies and angles.

I. Introduction

A good deal of interest has recently been devoted to the study of the pion pion interaction¹⁾. Although no direct evidence for such an interaction has as yet been found, indications of its existence have appeared, for example, in the analysis of single pion production in $\bar{\pi}$ -p collisions^{2),3)} and of $\bar{N}N$ annihilation^{4),5)}.

More specifically, recent work by Frazer and Fulco⁶⁾ has indicated that the experimental data on the isovector part of the nucleon electromagnetic form factors may be satisfactorily fitted by assuming a pion pion scattering resonance in the $I = 1$ $J = 1$ state. In order to obtain additional confirmation of this hypothesis as well as further information regarding the values of the parameters characterizing the $J = I = 1$ resonance, it is of interest to consider the effect of this resonance on various observable processes. This has already been done for low energy pion-nucleon scattering⁷⁾ and is currently being done for nucleon-nucleon scattering⁸⁾. The present paper is concerned with the effect of this pion pion resonance on pion photoproduction on nucleons^(*). It is hoped, furthermore, that the existing discrepancies between the experimental data for this process and the theoretical predictions of Chew, Low, Goldberger and Nambu⁹⁾ could be adequately explained by taking the pion pion resonance into account.

In Chapter II we recall the kinematics and write down the fundamental invariant forms as given by CGLN. Section III is concerned with the two and three pion intermediate states in the $\gamma\pi \rightarrow \bar{N}N$ reaction channel. In Chapter IV the unitarity condition in this channel is applied to the isoscalar amplitudes and leads to expressions for their imaginary parts in terms of the $\gamma\pi \rightarrow \pi\pi$ amplitude and the imaginary parts of the isovector nucleon form factors. In section V the expressions for the isoscalar amplitudes resulting from the Cini-Fubini form of the Mandelstam representation

(*) Similar work has been carried out independently by Munczek at the University of Roma, and by J.S. Ball (thesis) at the Lawrence Radiation Laboratory.

combined with unitarity in channel III are written down under the assumption that the $I = \frac{1}{2}$ pion-nucleon phase shifts may be neglected.

In section VI it is shown that a simple model involving a $J = I = 1$ intermediate particle or "Bipion" can reproduce the main features of the dispersion treatment of the preceding section. An analogous model, involving a $J = 1$ $I = 0$ intermediate particle or "Tripion" is applied in section VII to the isovector amplitudes. In the hope of obtaining a rough estimate of some of the parameters characterizing this particle, the same model is applied to the isoscalar nucleon form factors. Finally a programme for the comparison of this theory with experiment is indicated in section VIII.

II. Decomposition in scalar amplitudes

In this section we recall the spin and isotopic spin structure of the S matrix element for photomeson production. We use the same notations as CGLN.

We first define three invariant scalar quantities:

$$s_1 = -(k + p_1)^2$$

$$s_2 = -(k - p_2)^2$$

$$t = -(p_1 - p_2)^2$$

where on the mass shell:

$$s_1 + s_2 + t = 2M^2 + \mu^2.$$

The various four momenta are defined in Figure 1.

Fig. 1

The S matrix element can be written as:

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \int_4 (k + p_1 - q - p_2) \frac{1}{(2\pi)^6} \frac{M}{2(k_q^0 p_1^0 p_2^0)^{\frac{1}{2}}} T_{fi} \quad (1)$$

where the fields are normalized in the volume $(2\pi)^3$.

Because of the non-conservation of isotopic spin, one must introduce three invariant quantities for the photoproduction of one π meson with isospin α :

$$\mathcal{C}_\alpha^+ = \int_3 \alpha \quad \mathcal{C}_\alpha^- = \frac{1}{2} [\tau_\alpha, \tau_3] \quad \mathcal{C}_\alpha^{(0)} = \tau_\alpha \quad (2)$$

The first two correspond to the isovector part of the electromagnetic current and the latter to the isoscalar part.

As shown by CGLN, the requirements of Lorentz and gauge invariance lead to four fundamental forms which may be chosen to be:

$$\begin{aligned} M_A &= i \gamma_5 (\gamma e)(\gamma k) \\ M_B &= 2i \gamma_5 [(e \cdot P)(q \cdot k) - (e \cdot q)(P \cdot k)] \\ M_C &= \gamma_5 (\gamma \cdot e)(q \cdot k) - (\gamma \cdot k)(q \cdot e) \\ M_D &= 2 \gamma_5 (\gamma \cdot e)(P \cdot k) - (\gamma \cdot k)(e \cdot P) - iM(\gamma \cdot e)(\gamma \cdot k) \end{aligned} \quad (3)$$

where e is the photon polarization and P the combination

$$P = \frac{p_1 + p_2}{2}$$

Writing now T_{fi} as

$$T_{fi} = \bar{u}_f(p_2) \left[\mathcal{C}_\alpha^{(+)} H^+ + \mathcal{C}_\alpha^{(-)} H^- + \mathcal{C}_\alpha^{(0)} H^0 u_i(p_1) \right] \quad (4)$$

where $u(p)$ is a Dirac spinor normalized according to $\bar{u}u = 1$, each function H may be decomposed as follows:

$$H = A(s_1 s_2 t) M_A + B(s_1 s_2 t) M_B + C(s_1 s_2 t) M_C + D(s_1 s_2 t) M_D \quad (5)$$

Invariance of the T matrix under the exchange of the two nucleons yields the following crossing properties: the scalar functions $A^{(+,0)}$, $B^{(+,0)}$, $C^{(-)}$, $D^{(+,0)}$ are even under the transformation $s_1 \rightleftharpoons s_2$ $t \rightleftharpoons t$ whereas $A^{(-)}$, $B^{(-)}$, $C^{(+,0)}$, $D^{(-)}$ are odd.

III. Intermediate states for the reaction $\gamma + \pi \rightarrow N + \bar{N}$

The combination of the Mandelstam representation with the unitarity condition for the reaction $\gamma + \pi \rightarrow N \bar{N}$ will enable us to take into account the effect of resonant $\pi\pi$ interaction on the invariant photo-meson production amplitudes. Let us consider specifically the two- and three-pion intermediate states^(*) which occur in the unitarity condition.

a) Two-pion state

In a previous paper¹⁰⁾, it was shown that for the process $\gamma + \pi \rightarrow \pi + \pi$ only the isoscalar part of the photon contributes to the reaction amplitude. It follows that only the isoscalar amplitudes $A^{(0)}$, $B^{(0)}$, $C^{(0)}$ and $D^{(0)}$ will be affected. We note that the next highest state which contributes to these amplitudes is a four pion state (owing to G-invariance).

The two pions are in a $I = 1$ isospin state and their total angular momentum J is odd. As a result the existence of a $\pi\pi$ interaction in

(*) The one-pion intermediate state has already been included in the Born terms given in CGLN in order to ensure gauge invariance.

the S state¹¹⁾ will not affect the amplitudes under consideration. We will assume a $I = J = 1$ $\pi\pi$ resonance and neglect all other angular momentum states.

b) Three-pion state

It can be seen from G invariance that only the isovector part of the photon current contributes to the $\gamma + \pi \rightarrow \pi + \pi + \pi$ reaction amplitude. Here the three pions have either $I = 0$ or $I = 1$. In the case where $I = 0$ only the amplitudes associated with $\mathcal{C}_\alpha^{(+)}$ are involved whereas for $I = 1$ only those associated with $\mathcal{C}_\alpha^{(-)}$ are affected.

It has been shown by Chew and Mottelson¹²⁾ that in an $I = 0$ $J = 1$ three-pion state all three pairs of pions may be in an $I = J = 1$ state. Conversely, if one requires that all three pairs of pions be in an $I = J = 1$ state, it can be shown that their total isospin and total angular momentum will be $I = 0$ and $J = 1$. The assumption made above of a strong pion-pion resonance leads us to retain only the three pion state characterized by these quantum numbers. As a result, the three pion state will affect only the amplitudes $A^{(+)}$, $B^{(+)}$, $C^{(+)}$ and $D^{(+)}$.

IV. Unitarity for the isoscalar amplitudes

In the photoproduction channel, it is well known that the phases of the multipole amplitudes are given by the pion-nucleon scattering phase shifts in the corresponding quantum states. As the isoscalar amplitudes lead to a final state where $I = \frac{1}{2}$, and as the corresponding phase shifts are small, one may, at low energy, neglect the imaginary part of $A^{(0)}$, $B^{(0)}$, $C^{(0)}$ and $D^{(0)}$ following CGLN. At higher energies, of course, (above the $3/2$ $3/2$ resonance, for example) such an approximation is no longer valid.

We now consider the reaction $\gamma + \pi \rightarrow N + \bar{N}$. Introducing the centre of mass variables:

$$k = (\vec{K}, K) \quad q = (\vec{K}, -\omega_K) \quad p_1 = (-\vec{p}, -E) \quad p_2 = (\vec{p}, -E)$$

and setting $e_4 = 0$, $e \cdot K = 0$ by a suitable choice of gauge, we can write the isoscalar amplitude as a matrix in the nucleon spin space as follows:

$$\begin{aligned} T = & i \frac{K}{M} (A + tB)(\vec{e} \cdot \vec{p}) + \frac{2KE}{M} C(\vec{\sigma}, \vec{e}, \vec{p}) + (A - \frac{t}{2M} D)(\vec{\sigma}, \vec{e}, \vec{K}) + \\ & + \frac{1}{M(M+E)} (A + 2ED)(\vec{\sigma} \cdot \vec{p})(\vec{e}, \vec{K}, \vec{p}) \end{aligned} \quad (6)$$

where the index (0) has been dropped; A, B, C and D are functions of $t = 4E^2$ and $\cos \theta = \frac{\vec{K} \cdot \vec{p}}{Kp}$.

The unitarity condition in this channel may be written as

$$\langle f | T - T^\dagger | i \rangle = \sum_n \langle f | T | n \rangle \langle n | T^\dagger | i \rangle. \quad (7)$$

In the energy region $(2\mu)^2 \leq t \leq (2M)^2$ equation (7) is to be understood as an analytic continuation of the unitarity condition from the region of physical energies¹³⁾. Retaining only the two-pion contributions to the RHS of equation (7), one can express the imaginary parts^(*) of A, B, C and D in terms of the $\Upsilon + \pi \rightarrow \pi + \pi$ and $\pi + \pi \rightarrow N + \bar{N}$ reaction amplitudes given in references^{10), 7)}.

We note, however, that the same $I = 1 \quad J = 1$ two pion intermediate state occurs in the expression of the imaginary part of the isovector nucleon form factors¹⁴⁾. As the same $\pi + \pi \rightarrow N + \bar{N}$ amplitude appears therefore in both processes, we can eliminate this amplitude and re-express the imaginary parts of A, B, C and D in terms of the imaginary parts of the isovector nucleon form factors. More precisely the $\Upsilon \rightarrow N + \bar{N}$ vertex function may be written in the centre of mass system of the $N\bar{N}$ pair:

$$\begin{aligned} \frac{E}{M} e_\mu \langle \bar{N}N | j_\mu | 0 \rangle = & \chi_N^* \left\{ \frac{E}{M} (\vec{\sigma} \cdot \vec{e}) [F_1^V(t) + 2M F_2^V(t)] + \right. \\ & \left. + \frac{1}{M(E+M)} (\vec{\sigma} \cdot \vec{p})(\vec{e} \cdot \vec{p}) [2E F_2^V(t) - F_1^V(t)] \right\} \chi_{\bar{N}} \end{aligned} \quad (8)$$

(*) The phases of M_A, M_B, M_C and M_D have been chosen so that the imaginary parts of A, B, C and D will appear in the LHS of equation (7).

where we have used the same notations as for the $\gamma + \pi \rightarrow N + \bar{N}$ reaction. The invariant functions $F_1^V(t)$ and $F_2^V(t)$ are the ordinary isovector form factors for the nucleon. On the other hand, the $\gamma \rightarrow \pi_\ell + \pi_m$ vertex and the $\gamma + \pi_{(6), (14), (10)} \rightarrow \pi_\ell + \pi_m$ reaction amplitude are given respectively by

$$2\omega_q e_\mu \langle \pi_\ell \pi_m | j_\mu | 0 \rangle = e \varepsilon_{3lm} \vec{e} \cdot \vec{q} F_\pi(t) \quad (9)$$

where the pion form factor $F_\pi(t)$ satisfies $F_\pi(0) = 1$, and:

$$2\omega_q (2\omega_K)^{\frac{1}{2}} \langle \pi_\ell \pi_m | j_\mu | \pi_n \rangle e_\mu = \frac{e \wedge \varepsilon_{lmn}}{\mu^3} \frac{1}{\sqrt{2}} \omega_q(\vec{e}, \vec{K}, \vec{q}) F_\pi(t) \phi(t) \quad (10)$$

where $\phi(t)$ is a real smooth function considered in reference¹⁰⁾ and $\wedge$ a dimensionless "coupling constant" corresponding to the normalization $\phi(0) = 1$. The formal substitution

$$\vec{e} \rightarrow \vec{e} \times \vec{K}$$

allows one to establish a correspondence between equations (9) and (10) and equations (6) and (8) and to write:

$$\begin{aligned} \text{Im } A &= \frac{1}{4\sqrt{2}} \frac{\wedge}{\mu^3} t \phi(t) \text{Im } F_2^V(t) \\ \text{Im } B &= -\frac{1}{4\sqrt{2}} \frac{\wedge}{\mu^3} \phi(t) \text{Im } F_2^V(t) \\ \text{Im } C &= 0 \\ \text{Im } D &= -\frac{1}{4\sqrt{2}} \frac{\wedge}{\mu^3} \phi(t) \text{Im } F_1^V(t) \end{aligned} \quad (11)$$

V. Mandelstam representation for the isoscalar amplitudes

In this section we again consider only the amplitudes $A^{(0)}$, $B^{(0)}$, $C^{(0)}$ and $D^{(0)}$ for which we assume Mandelstam representations:

$$\begin{aligned} \text{Born terms} + \frac{1}{\pi^2} \int_{(M+\mu)^2}^{\infty} \int_{(2\mu)^2}^{\infty} dx dy \frac{p(x,y)}{y-t} \left[\frac{1}{x-s_1} + \varepsilon \frac{1}{x-s_2} \right] \\ + \frac{1}{\pi^2} \int_{(M+\mu)^2}^{\infty} \int_{(M+\mu)^2}^{\infty} dx dy \frac{\sigma(x,y)}{(x-s_1)(y-s_2)} \end{aligned} \quad (12)$$

where the Born terms may be deduced from those given in CGLN and where, as a result of crossing, $\varepsilon \sigma(xy) = \sigma(yx)$ and $\varepsilon = 1$ for A, B and D and $\varepsilon = -1$ for C. It is easily seen that owing to the isoscalar nature of the photon current, one may apply the Cini-Fubini arguments¹⁵⁾ and write the representation (12) in one-dimensional form, where the corresponding weight functions are directly related to the absorptive parts of the amplitudes in the various channels. As mentioned in the previous section, one may neglect the absorptive parts in the photoproduction channels. As shown by equations (11) furthermore, the absorptive parts of the amplitudes in the $\gamma \pi \rightarrow NN$ channel are simply functions of t in the approximation where only the $I = J = 1$ two-pion intermediate state is retained. These considerations lead us to write the H^0 amplitude (5) using (11) as follows:

$$\begin{aligned} H^0 = H_{\text{BORN}}^0 + \frac{1}{4\sqrt{2}} \frac{\Lambda}{\mu^3} \left[M_A \frac{1}{\pi} \int_{(2\mu)^2}^{\infty} \frac{t' \phi(t') \text{Im } F_2^V(t') dt'}{t'-t} - \right. \\ \left. - M_B \frac{1}{\pi} \int_{(2\mu)^2}^{\infty} \frac{\phi(t') \text{Im } F_2^V(t') dt'}{t'-t} - M_D \frac{1}{\pi} \int_{(2\mu)^2}^{\infty} \frac{\phi(t') \text{Im } F_1^V(t') dt'}{t'-t} \right] \end{aligned} \quad (13)$$

where possible subtractions have been omitted. The form of the first integral in equation (13) would seem to suggest that a subtraction might be effected so as to cast this integral into the same form as the second one. If one does this equation (13) becomes

$$H^0 = H_{\text{BORN}}^0 + \frac{1}{4\sqrt{2}} \frac{\Lambda}{\mu^3} \left[A_0 M_A + (t M_A - M_B) \frac{1}{\pi} \int_{(2\mu)^2}^{\infty} \frac{\phi(t') \text{Im } F_2^V(t') dt'}{t' - t} \right. \\ \left. - M_D \frac{1}{\pi} \int_{(2\mu)^2}^{\infty} \frac{\phi(t') \text{Im } F_1^V(t') dt'}{t' - t} \right] \quad (14)$$

At this point we make the assumption of a sharp pion-pion resonance. Writing the $J = 1$ $I = 1$ pion-pion scattering amplitude as

$$e^{i\delta} \sin \delta = \frac{\gamma q^3}{t_R - t - i \gamma q^3} \quad (15)$$

where t_R is the square of the total c.m. resonance energy of the two pions and where γ is proportional to the resonance width, we shall make the approximation of replacing $\sin^2 \delta$ by $\pi \gamma q_R^3 \delta(t_R - t)$. With this approximation it has been shown in reference ⁷⁾ that the imaginary parts of the isovector nucleon form factors are of the form

$$\text{Im } F_1^V(t) = -\frac{e}{2} \pi a t_R \delta(t - t_R) \quad (16)$$

$$\text{Im } F_2^V(t) = -\frac{eg}{2M} \pi b t_R \delta(t - t_R)$$

where g is the isovector part of the anomalous gyromagnetic ratio.

The constants a and b are related to γ , t_R , and two new constants C_1 and C_2 describing in the same approximation the $J = 1$ $I = 1$ $\pi\pi \rightarrow \pi\pi$ reaction amplitude:

$$a = \frac{2C_1}{\gamma} \frac{t_R - \delta}{(t_R)^{3/2}} \quad (17)$$

$$b = \frac{M}{g} \frac{2C_2}{\gamma} \frac{t_R - \delta}{(t_R)^{3/2}}$$

Insertion of (16) into (14) yields the simple form

$$H^0 = H_{\text{BORN}}^0 + \frac{1}{8\sqrt{2}} \frac{e\Lambda}{\mu^3} t_R \phi(t_R) \left[A'_0 M_A - \frac{g^b}{M} \frac{t_M A - M_B}{t_R - t} + a \frac{M_D}{t_R - t} \right] \quad (18)$$

We have thus obtained an expression for the isoscalar amplitude in terms of the following parameters:

- a , b and t_R which are linked to the nucleon form factors or, equivalently, to the $\pi + \pi \rightarrow N + \bar{N}$ reaction amplitudes⁷⁾. A possible set of values for these three parameters has been given in reference⁷⁾ by comparison of the experimental data with the nucleon form factors.

- Λ which occurs in the expression for the $\gamma + \pi \rightarrow \pi + \pi$ reaction amplitude¹⁰⁾. One might hope to estimate this constant by comparison with experimental data for photomeson production on nucleons or eventually Compton scattering on nucleons. Alternatively, this constant could be measured directly, at least in principle, by the procedures discussed in reference¹⁰⁾.

- An adjustable parameter A'_0 . The use of further subtractions would, of course, introduce additional parameters of this type.

The simplicity of the present treatment is essentially due to our neglect of the absorptive parts of the isoscalar multipole amplitudes in channel I. Had we taken these absorptive parts into account we would, of course, have been led to a system of coupled Omnès-type integral equations in the multipole amplitudes for H^0 .

VI. "Bipion" model

It is of interest to note that the results of the preceding section in the case of a narrow resonance may be obtained quite simply by means of an elementary first order perturbation calculation. We can indeed introduce a new intermediate particle B which we will take to simulate the effect of a narrow $J = 1 \quad I = 1$ pion pion resonance in the intermediate state. This particle whose mass will be taken to be $\sqrt{t_R}$ must, of course, be a vector boson as well as a vector in isotopic spin space.

More specifically, we shall compute the graph drawn in Figure 2.

Fig. 2

The $\gamma \pi B$ vertex may be written as:

$$\lambda \delta_{\alpha\beta} \epsilon_{\mu\nu\rho\sigma} e_{\mu}^k v_{\nu} q_{\rho} \quad (19)$$

by Lorentz and gauge invariance.

The $\overline{N}NB$ vertex can be shown to contain two types of couplings:

$$\overline{u}(p_2) \left[c_1 \gamma_\tau + \frac{i}{2} c_2 [\gamma_\tau, \gamma_\lambda] (p_2 - p_1)_\lambda \right] \tau_\beta u(p_1) \quad (20)$$

a third coupling proportional to the divergence of the B field is a priori possible but may be shown to give no contribution.

The boson propagator¹⁶⁾ is proportional to:

$$\frac{\int d^4x \frac{(q-k)_\mu (q-k)_\nu}{t_R}}{t_R - t} \quad (21)$$

(we recall that $t = -(q-k)^2$); by combination of (19) with (21) one immediately sees that the second term in the boson propagator does not contribute to the matrix element under consideration. After some algebraic manipulations the final result can be expressed in terms of the fundamental forms appearing in (3):

$$H_{\text{bipion}}^0 = \lambda \left[\frac{c_2}{t-t_R} (t M_A - M_B) - \frac{c_1}{t-t_R} M_D \right] \quad (22)$$

Inspection of equations (17), (18) and (22) shows that they are of the same form if one takes $A'_0 = 0$ and

$$\frac{c_2}{c_1} = \frac{c_2}{c_1} . \quad (23)$$

It is of some interest to note that by using the "bipion" model we are able to reproduce the main features of the dispersion treatment given in the previous section under the assumption of a narrow pion pion resonance.

VII. Isovector amplitudes; tripion model

If one now considers the Mandelstam representation for the isovector amplitudes $A^\pm B^\pm C^\pm D^\pm$ one finds that owing to G invariance one cannot directly carry through the Cini-Fubini reduction. Indeed, one sees that

for the graph drawn in Figure 3(a) the variables s_1 and t may both reach their lower limits of integration in the Mandelstam integrals. The

Fig. 3

same situation holds for the variable s_1 and s_2 if one considers the graph drawn in Fig. 3(b). On the other hand, nothing is as yet known regarding the $\gamma + \pi \rightarrow \pi + \pi + \pi$ and $\pi + \pi + \pi \rightarrow \bar{N} + N$ production amplitudes.

Under these circumstances, we shall consider a very naïve model in which the three pion intermediate state is replaced by the tripion particle considered in section III, with quantum numbers $J = 1, I = 0$. As we are again dealing with a vector boson, we may write, in complete analogy with section VI

$$H_{\text{tripion}}^+ = \lambda' \left[\frac{e_2}{t - t_R'} (t M_A - M_B) - \frac{e_1}{t - t_R'} M_D \right] \quad (24)$$

where λ' is the coupling constant corresponding to the $\gamma \pi$ tripion vertex and where e_1 and e_2 are the coupling constants of the tripion to the nucleon; t_R' is the square of the tripion mass.

In order to get some information about the parameters appearing in formula (24), we will apply a similar model to the calculation of the isoscalar nucleon form factors, and assume that these may be written as a sum of two terms, the first coming from the graph $\gamma \rightarrow \text{tripion} \rightarrow N + \bar{N}$, and the second from graphs comprising higher mass intermediate states; we

will consider the latter contributions to be adequately represented by additive constants:

$$\begin{aligned} F_1^S(t) &= \frac{e}{2} \left[1 - \frac{a't}{t_R' - t} \right] \\ F_2^S(t) &= \frac{g'e}{2m} \left[1 - \frac{b't}{t_R' - t} \right] \end{aligned} \quad (25)$$

where g' is the isoscalar part of the anomalous gyromagnetic ratio, and where

$$\frac{b'}{a'} = \frac{m}{g'} \frac{e_2'}{e_1'} \quad (26)$$

Experimentally $F_1^S(t)$ and $F_1^V(t)$ are rather close in value owing to the smallness of the neutron charge form factor. We shall therefore take

$$a' \simeq a ; \quad t_R' \simeq t_R \quad (27)$$

where the strict equality would correspond to $F_1^N(t) \equiv 0$. On the other hand, for values of t between $-25\mu^2$ and 0 we have¹⁷⁾:

$$0.7 < \frac{F_2^N(t)/F_2^N(0)}{F_2^P(t)/F_2^P(0)} < 1$$

These experimental values lead to the following limits on the value of the ratio e_2'/e_1' :

$$0.5 < \frac{m}{g'} \frac{e_2'}{e_1'} < 5 \quad (28)$$

Returning now to the photoproduction amplitude, it is clear that one cannot simply add on the tripion contribution to the solution obtained by CGLN for H^+ owing to the presence of the $J = 3/2$ $I = 3/2$ pion nucleon final state interactions in the isovector amplitudes.

If we now make the tripion approximation, the cut in the t plane of the amplitudes is replaced by a pole at $t = t_R$, so that only the cuts in s_1 and s_2 remain. The amplitude H^+ may then be written:

$$H^+ = H_{\text{Born}}^+ + H_{\text{tripion}}^+ + \sum_{ABCD} N_A \frac{1}{\pi^2} \int_{(m+\mu)^2}^{\infty} \int_{(m+\mu)^2}^{\infty} \frac{\rho_A(s_1, s_2) ds_1 ds_2}{(s_1' - s_1)(s_2' - s_2)} \quad (29)$$

Taking a fixed value of t we can reduce the double integral to a sum of two one-dimensional integrals and write for A^+ , for example:

$$A^+(s_1, t) = A_{\text{Born}}^+(s_1, t) + A_{\text{tripion}}^+(t) + \frac{1}{\pi} \int_{(m+\mu)^2}^{\infty} ds' \text{Im } A^+(s', t) \left\{ \frac{1}{s' - s_1 - i\epsilon} + \frac{1}{s' - 2m^2 - \mu^2 + t + s_1} \right\} \quad (30)$$

Similar representations satisfying crossing symmetry hold for B^+ , C^+ and D^+ . Note that as a result of inequality (28) and of equation (24), the tripion contribution will mainly affect the D^+ amplitude.

It may easily be seen that the amplitudes A^- , B^- , C^- , D^- satisfy representations of the same form with the exception that the tripion terms will be absent; these are simply the equations written down by CGLN.

When unitarity is taken into account, this representation leads to a system of coupled Omnès type integral equations in the multipoles, involving as a result of crossing both the $(-)$ and $(+)$ amplitudes. By well known techniques¹⁸⁾, this system may be transformed into a system of Fredholm type integral equations. These considerations will be deferred to a succeeding paper.

VIII. Programme for a comparison with experiment and conclusions

The amplitudes for photoproduction of mesons with definite charge are linear combinations of T^+ , T^- and T^0 :

$$\begin{aligned}
 \gamma + P &\rightarrow N + \pi^+ & \sqrt{2} (T^- + T^0) \\
 \gamma + N &\rightarrow P + \pi^- & -\sqrt{2} (T^- - T^0) \\
 \gamma + P &\rightarrow P + \pi^0 & T^+ + T^0 \\
 \gamma + N &\rightarrow N + \pi^0 & T^+ - T^0
 \end{aligned} \tag{31}$$

We see that the amplitudes for production of charged mesons are mainly affected by the bipion term and the CGLN terms as the tripion contribution to T^- coming from crossing will be less important. The photoproduction of π_0 's on the other hand will be affected by both the bipion and the tripion as well as by the CGLN terms.

The energy dependence of the experimentally measured ratio $d\sigma^-/d\sigma^+$ at 90° is in disagreement with the theoretical prediction based on the CGLN calculation. Whereas the theoretical curve is relatively flat, the experimental curve decreases steadily with energy in the 170 to 250 MeV region¹⁷⁾. Inspection of equation (31) shows that this ratio is extremely sensitive to the pion pion resonance correction to T^0 . It is hoped, therefore, that the comparison of our theory with the experimental data for this ratio measured at different angles may provide a severe test of the existence of a $J = 1$, $I = 1$ pion pion resonance^(*). It should be pointed out that since the experimental values of $d\sigma^-/d\sigma^+$ at 90° are of the order of 1 for the energies under consideration, and since T^- is affected by the $3/2 \ 3/2$ resonance, whereas T^0 is not, the ratio $|T^- - T^0|^2 / |T^- + T^0|^2$ will be relatively insensitive to the small tripion correction to T^- . As this

(*) According to Munczek $d\sigma^-/d\sigma^+$ is indeed improved by the inclusion of the bipion term but there are still some difficulties with the π^+ photoproduction cross-section itself.

latter correction can only be evaluated by means of a very simplified model, this circumstance is rather fortunate insofar as the comparison of this aspect of our theory with experiment is concerned.

For the comparison of our theory with the experimental results on π^0 production, one could in principle hope to fit the existing experimental curves for the differential cross-sections by means of expressions of the type:

$$a + b \cos \theta + c \cos^2 \theta + \frac{d}{t-t_R} + \frac{e}{(t-t_R)^2}$$

assuming $t_R = t_R'$, and where e is expressed in terms of the bipion and tripion parameters. An analogous method was used by Moravczik to isolate the retardation term in the case of charged meson photoproduction¹⁹⁾. In our case, however, this method does not seem too hopeful at first sight owing to the large distance of the pole $t = t_R$ from the physical region at low energies.

Another method of comparison would involve solving the integral equations for T^+ and T^- and comparing directly with the experimental total and differential cross-sections.

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