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On the pion-hyperon resonances and on the possible uses thereof(Determination of the $\Sigma - \Lambda$ relative parity and of the $\alpha_\Lambda / \alpha_0$ ratio)Ph. Meyer ^{*)}, J. Prentki, Yoshio Yamaguchi ^{**)}

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ABSTRACT

The existence of a resonance in the hyperon-pion system leads to a possibility of determining the $\Sigma - \Lambda$ relative parity and the ratio $\alpha_\Lambda / \alpha_0$ of asymmetry coefficients α_Λ (in $\Lambda \rightarrow p + \pi^-$) and α_0 (in $\Sigma^+ \rightarrow p + \pi^0$). In this paper a proposal is made which allows the determination of these quantities by measuring only the up/down asymmetries of pions coming from Λ^- and Σ^- decays. Various aspects of this proposal are discussed.

The possible existence of other resonances has also been emphasized. In particular it is shown that the existence of a resonance in the isotopic spin state $I=1$ leads to some predictions concerning the existence of another resonance, probably in the $I=2$ state.

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Introduction

Evidence for the existence of a resonance in the pion-hyperon system has been recently reported ¹⁾. In the same way that the (33) resonance has played an essential role in the development of pion physics it may happen (if the experimental results are confirmed) that the pion-hyperon resonance(s) will help us in understanding certain aspects of strange particle physics.

One of the most immediate applications that has been proposed ²⁾ is to use this resonance in order to determine the relative $\Sigma - \Lambda$ parity and the ratio of the asymmetry coefficients α_{Λ} (in $\Lambda \rightarrow p + \pi^-$) to α_0 (in $\Sigma^+ \rightarrow p + \pi^0$).

The main content of this paper is to describe in some detail a series of experiments which might allow the determination of the above mentioned quantities by measuring only up/down asymmetries.

In order to fulfil the proposed programme a number of conditions must be met which can be checked only through experiment. These conditions will be specified, and a set of possible reactions by which the resonance in question can be excited will be suggested.

Strictly speaking, the method which is proposed and the relations which are derived are rigorously valid only in the case where the resonance is so narrow and pronounced that the excited hyperon formed can be considered to have a "long" lifetime and that all channels except the resonating one can be neglected. The validity of such a model and the corrections to be expected will be briefly discussed.

Quite independently of the possible measurement of the $\Sigma - \Lambda$ parity and of the ratio $\alpha_{\Lambda} / \alpha_0$ the properties of the resonance(s) of the pion-hyperon system should be studied both theoretically and experimentally. This is particularly true of the isotopic spin properties since a number of models which shall be discussed predict the existence of other resonances for which it would be interesting to have experimental evidence.

In chapter 1 the experimental situation concerning the $\pi - \Lambda$ resonance is briefly recalled. In chapter 2 the method for the determination of the $\Sigma - \Lambda$ relative parity from the existence of the H^* isobar is described in some detail. In chapter 3 we show how to determine $\pi(\Sigma)/\pi(\Lambda)$ and α_Λ/α_0 from the measurement of up/down asymmetries only. Chapter 4 is devoted to isotopic spin properties of the isobars; it is shown, by studying various models, that a new isobar of $I=2$ should very probably exist. The general discussion is given in chapter 5.

1. The $\pi - \Lambda$ resonance : Production and decay of the hyperon isobar.

The study of the reaction

with a separated K^- beam of 1.15 BeV/c shows that there exists a peak around 400 MeV/c in the π^+ and π^- spectra. The most obvious interpretation ¹⁾ is that reaction (1) proceeds in 2 steps; first an excited hyperon H^* is formed

followed by the immediate decay

The position of the peak in the π -spectrum shows that H^* has a mass of 1370 MeV which gives a Q value of ~ 115 MeV in the decay (3). According to preliminary results, the width of the resonance seems to be about 25 MeV, approximately 75 % of the π 's in (1) going through the resonant channel.

The Q value in (3) shows that the decay

must also exist with a Q value of ~ 40 MeV. Therefore the study of the reaction

$$K^- + p \rightarrow \Sigma + \pi + \pi \quad (5)$$

should also show a peak in the π -spectrum corresponding to the resonant channel (2), if the branching ratio Σ/Λ is not too small. In the case of an incident K^- beam the set of reactions involving H^* (with a "threshold" of 400 MeV/c) would be

$$K^- + p \rightarrow \begin{cases} H^{*+} + \pi^- \\ H^{*0} + \pi^0 \\ H^{*-} + \pi^+ \end{cases} \quad (6)$$

$$K^- + p \rightarrow \begin{cases} H^{*+} + \pi^- \\ H^{*0} + \pi^0 \\ H^{*-} + \pi^+ \end{cases} \quad (7)$$

$$K^- + p \rightarrow \begin{cases} H^{*+} + \pi^- \\ H^{*0} + \pi^0 \\ H^{*-} + \pi^+ \end{cases} \quad (8)$$

with subsequent decay (the lifetime of H^* being $\sim 10^{-23}$ sec)

$$H^{*+} \rightarrow \begin{cases} \Sigma^+ + \pi^0 \\ \Sigma^0 + \pi^+ \\ \Lambda + \pi^+ \end{cases} \quad (9)$$

$$H^{*+} \rightarrow \begin{cases} \Sigma^+ + \pi^0 \\ \Sigma^0 + \pi^+ \\ \Lambda + \pi^+ \end{cases} \quad (10)$$

$$H^{*+} \rightarrow \begin{cases} \Sigma^+ + \pi^0 \\ \Sigma^0 + \pi^+ \\ \Lambda + \pi^+ \end{cases} \quad (11)$$

$$H^{*0} \rightarrow \begin{cases} \Sigma^+ + \pi^- \\ \Sigma^0 + \pi^0 \\ \Lambda + \pi^0 \end{cases} \quad (12)$$

$$H^{*0} \rightarrow \begin{cases} \Sigma^+ + \pi^- \\ \Sigma^0 + \pi^0 \\ \Lambda + \pi^0 \end{cases} \quad (13)$$

$$H^{*0} \rightarrow \begin{cases} \Sigma^+ + \pi^- \\ \Sigma^0 + \pi^0 \\ \Lambda + \pi^0 \end{cases} \quad (14)$$

$$H^{*-} \rightarrow \begin{cases} \Sigma^- + \pi^0 \\ \Sigma^0 + \pi^- \\ \Lambda + \pi^- \end{cases} \quad (15)$$

$$H^{*-} \rightarrow \begin{cases} \Sigma^- + \pi^0 \\ \Sigma^0 + \pi^- \\ \Lambda + \pi^- \end{cases} \quad (16)$$

$$H^{*-} \rightarrow \begin{cases} \Sigma^- + \pi^0 \\ \Sigma^0 + \pi^- \\ \Lambda + \pi^- \end{cases} \quad (17)$$

The existence of the decay (3) demonstrates that H^* must be in a pure isotopic spin state $I=1$. This, of course, forbids the decay $K^- + p \rightarrow \Sigma^0 + \pi^0 + \pi^0$ going through the $I=1$ resonant channel.

Needless to say that other reactions than (2) can be used in principle to excite the H^* resonance. As examples we list here the following ^{*)}

$$\pi + p \rightarrow H^* + K \quad (1.24 \text{ GeV}) \quad (18)$$

$$K^- + d \rightarrow H^* + N \quad (\text{rest}) \quad (19)$$

$$\bar{p} + p \rightarrow H^* + \bar{\Lambda} \quad (1.41 \text{ GeV}) \quad (20)$$

$$\bar{p} + p \rightarrow H^* + \bar{H}^* \quad (2.11 \text{ GeV}) \quad (20')$$

$$\gamma + p \rightarrow H^* + K \quad (1.39 \text{ GeV}), \text{ etc.} \quad (21)$$

The thresholds for these reactions are put in brackets.

It is, of course, difficult to say a priori which reaction is most favourable for the production of the resonating state and at what energy one should work. Furthermore, in the considerations to follow it will be essential for H^* to have a large polarization and again the most favourable situation can only be discovered through experiment.

2. Determination of the $\Sigma - \Lambda$ relative parity

A method will now be proposed to measure the $\Sigma - \Lambda$ parity taking advantage of the existence of the H^* resonance. This method will be successful provided the two following conditions are satisfied :

- (i) The isobar H^* must be a resonance which is sufficiently narrow and pronounced so that one can choose experimental conditions under which the resonant process dominates. This means that the reaction which excites the isobar H^*

^{*)} The H^* resonance cannot be excited in K-N elastic scattering. The corresponding energy being in the unphysical region.

can be considered as a quasi-two-body reaction. Knowing the "mass" of H^* one selects the events for which the energy of reaction product c is in a narrow band around the resonance peak. If H^* is a "good" resonance this choice of events will ensure that the observed reaction is passing through the formation of the isobar H^* and that under these experimental conditions the background for the non-resonant channel $a+b \rightarrow c+d+e$ is minimised. We shall come back later to this point.

- (ii) The method proposed for determining the $\Sigma - \Lambda$ relative parity and the ratio $\alpha_\Lambda / \alpha_0$ rests upon the measure of the polarization of the Σ 's and Λ 's in the decay of H^{*+} for instance. Therefore a second condition which must be met is that the branching ratio of reactions $(9)/(11) = (10)/(11)$ be not too small, and that the polarization of H^* be large. Large polarizations having been observed in strange particle creation it is expected that one can find an energy for which this will also be true of the isobar.

To discuss the method proposed let us take as an example the excitation of the isobar through the reaction

Knowing the position of the resonance H^* for a given energy of the incident π^+ one selects the K^+ associated with the resonant channel. In what follows, the analysis will always refer to such processes. If condition (i) is satisfied, then reaction (22) can be thought of [see Fig.1] as a two step process :

followed by the immediate decay

Furthermore, the direction of the outgoing K^+ together with that of the incident π^+ defines a plane which shall be referred to as the production plane S in what follows.

Now consider P_Λ and P_Σ defined as the "average" polarization of the Λ 's and Σ 's coming via resonant reaction (23) through the decay (a), (b) and (c). By "average" we mean an average over all directions of hyperons keeping the energy and direction of the associated K^+ fixed. From conservation of parity it is clear that the polarization of H^* must be normal to S . In the decay of H^* , for a given direction of Λ (Σ) the polarization of Λ (Σ) will in general not be perpendicular to the plane S . But if an average is made over all directions of the hyperons this average polarization $\vec{P}_\Lambda$ and $\vec{P}_\Sigma$ will be necessarily normal to the plane S .

Let us call $\pi(\Lambda)$, $\pi(\Sigma)$ and $\pi(H^*)$ the parity of Λ , Σ , and H^* respectively (only relative parities are relevant) and J the spin of H^* . If M is the component of J along the normal to S which we shall take as the z axis, the polarization of H^* is given by

$$P_{H^*} = \frac{1}{J} \frac{\sum_M M |W_J^M|^2}{\sum_M |W_J^M|^2} \quad (24)$$

where W_J^M is the amplitude for the production of H^* in the spin state (J, M) .

The final state wave function of the $\pi - \Lambda$ ($\pi - \Sigma$) system coming from the decay of H^* is given, in the rest system of H^* , by

$$\psi_J = \sum_M W_J^M a_J \left\{ \sqrt{\frac{J+M}{2J}} \cdot \alpha \cdot Y_{J-\frac{1}{2}}^{M-\frac{1}{2}}(\theta, \varphi) + \sqrt{\frac{J-M}{2J}} \cdot \beta \cdot Y_{J-\frac{1}{2}}^{M+\frac{1}{2}}(\theta, \varphi) \right\} \quad (25)$$

for $\ell = J - \frac{1}{2}$ and

$$\psi_J = \sum_M W_J^M b_J \left\{ \sqrt{\frac{J-M+1}{2J+2}} \alpha Y_{J+\frac{1}{2}}^{M-\frac{1}{2}}(\theta, \varphi) - \sqrt{\frac{J+M+1}{2J+2}} \beta Y_{J+\frac{1}{2}}^{M+\frac{1}{2}}(\theta, \varphi) \right\} \quad (26)$$

for $\ell = J + \frac{1}{2}$; where ℓ is the orbital angular momentum of the decay products of H^* , the a_J , b_J are the decay amplitudes for the given ℓ and given Λ (Σ) channels and where α and β are the 2 component spin functions of the hyperon for spin up and down respectively. The final state wave function is given by either (25) or (26) depending on the value of ℓ , i.e. on the relative parity of Λ (Σ) and H^* .

After averaging over θ and φ it is easily seen that only the z component of $\vec{\sigma}$ has a non vanishing expectation value in the Λ (Σ)- π final state, and one obtains in the usual way

$$P_{\Lambda(\Sigma)} = \langle \sigma_z \rangle = \frac{\sum_M |a_J|^2 |W_J^M|^2 \left(\frac{J+M}{2J} - \frac{J-M}{2J} \right)}{\sum_M |a_J|^2 |W_J^M|^2} = \sum_M \frac{M}{J} \frac{|W_J^M|^2}{\sum_{M'} |W_J^{M'}|^2} \quad (27)$$

for $\ell = J - \frac{1}{2}$

$$P_{\Lambda(\Sigma)} = \langle \sigma_z \rangle = \frac{\sum_M |b_J|^2 |W_J^M|^2 \left(\frac{J-M+1}{2J+2} - \frac{J+M+1}{2J+2} \right)}{\sum_M |b_J|^2 |W_J^M|^2} = \sum_M \left(-\frac{M}{J+1} \right) \frac{|W_J^M|^2}{\sum_{M'} |W_J^{M'}|^2} \quad (28)$$

for $\ell = J + \frac{1}{2}$.

However, in the decay of H^* , ℓ is related to the parity by

$$\pi(H^*) = \pi(\Lambda \text{ or } \Sigma) (-1)^{\ell+1} \quad (29)$$

Using equations (24) and (29), the relations (27) and (28) can be written in the compact form

$$P_{\Lambda} [P_{\Sigma}] = \begin{Bmatrix} 1 \\ -\frac{J}{J+1} \end{Bmatrix} \times P_{H^*} \quad \text{for} \quad \pi(\Lambda) [\pi(\Sigma)] = \begin{Bmatrix} (-1)^{J+\frac{1}{2}} \\ (-1)^{J-\frac{1}{2}} \end{Bmatrix} \cdot \pi(H^*) \quad (30)$$

As a consequence one can specialize relation (30) to the cases of interest and make the following statements :

(i) If $\pi(\Lambda) = \pi(\Sigma)$ then

$$P_{\Lambda} / P_{\Sigma} = +1 \quad (31)$$

for arbitrary spin J of H^*

(ii) If $\pi(\Lambda) = -\pi(\Sigma)$ then

$$P_{\Lambda} / P_{\Sigma} = \begin{cases} -1/3 \quad (-5/3) & \pi(\Lambda) = \pi(H^*) \\ -3 \quad (-3/5) & \pi(\Lambda) = -\pi(H^*) \end{cases} \quad \text{for } J=1/2(3/2) \text{ and} \quad (32)$$

It is important to notice that for any J or $\pi(H^*)$, the sign of P_{Λ} / P_{Σ} is either positive or negative according to whether the $\Sigma - \Lambda$ relative parity is even or odd. A measure of the sign of P_{Λ} / P_{Σ} is therefore a measure of the $\Sigma - \Lambda$ parity.

However, P_{Λ} and P_{Σ} are not easily accessible to experiment. What one can actually measure most simply is the up/down asymmetries with respect to the production plane S , of the pions coming from the hyperon decays (a') and

(c') for instance. Both reactions are good analysers of polarization since α'_Λ and α_0 are known to be large in magnitude. Such measurements will provide us with the ratio ^{*)}

$$\frac{\alpha'_\Lambda P_\Lambda}{\alpha_0 P_{\Sigma^+}} \equiv n \quad (33)$$

Note that from our definition of $P_\Lambda (P_{\Sigma^+})$ the up/down asymmetry to be measured concerns the pions coming from the decay of all $\Lambda (\Sigma^+)$ in reactions (a) and (c') whatever their direction as long as they are associated with the K^+ 's which have been selected as explained above. The decay of the Λ 's coming from (b) can be distinguished from those coming from (c') by kinematics ^{**)}.

If $\alpha'_\Lambda / \alpha_0$ is known from other experiments, e.g. such as would be given by a measurement of the longitudinal polarization of the proton in the decay of unpolarized $\Lambda (\Sigma^+)$ ²⁾. Then, by measuring n one obtains P_Λ / P_{Σ^+} and hence the $\Sigma - \Lambda$ parity.

However, it is important to realise that the existence of the resonant channel H^{*+} makes it possible to obtain separately P_Λ / P_{Σ^+} and $\alpha'_\Lambda / \alpha_0$ by considering the whole set of reactions (a), (b) and (c) and by measuring only up/down asymmetries in the subsequent decay of the hyperons. This we discuss in the next chapter.

^{*)} The connection between the measured up/down asymmetries and the quantity (33) will be discussed in Appendix II.

^{**)} For this reason among many others, the use of hydrogen bubble chamber seems more suitable.

3. Determination of $\pi(\Lambda)/\pi(\Sigma)$ and $\alpha_\Lambda/\alpha_\Sigma$ from the measurement of up/down asymmetries.

A) Determination of $\pi(\Lambda)/\pi(\Sigma)$

Consider the Λ 's coming from reactions (b') and (c). Let us recall first that in the decay $\Sigma^0 \rightarrow \Lambda + \gamma$ one has the well-known relation ⁴⁾

$$P_\Lambda = -1/3 P_{\Sigma^0} \quad (34)$$

if an average is made over all directions of Λ 's. This statement is independent of the $\Sigma-\Lambda$ relative parity.

Remembering the definition of P_{Σ^0} (without an arrow) it can be shown (see Appendix II) that relation (34) will still hold to a very good approximation for the polarization (normal to S) when an average is made over all directions of Σ^0 coming from (b'). Calling $P_{\Lambda \text{ ind}}$ the polarization of the Λ 's produced from all Σ^0 's, in the indirect reaction (b') one has

$$P_{\Lambda \text{ ind}} = -\frac{1}{3} P_{\Sigma^0} \quad (35)$$

Now by measuring the up/down asymmetry with respect to S in the decay of all the Λ 's coming from direct reaction (c) and indirect reaction (b') one can get the ratio

$$\frac{\alpha_\Lambda P_\Lambda}{\alpha_\Lambda P_{\Lambda \text{ ind}}} = \frac{\alpha_\Lambda P_\Lambda}{\alpha_\Lambda (-\frac{1}{3} P_{\Sigma^0})} = -3 \frac{P_\Lambda}{P_{\Sigma^0}} \quad (36)$$

Hence we can deduce the $\Sigma-\Lambda$ relation parity as described in section 2, without knowing the signs of the α 's.

B) Determination of the sign of $\alpha_{\Lambda}/\alpha_0$:

The H^* has an isotopic spin $I=1$; the $\pi-\Sigma$ systems are therefore in a pure $I=1$ state, from which it follows that :

$$P_{\Sigma^0} \text{ in (b)} = P_{\Sigma^+} \text{ in (a)}.$$

Consequently a measurement of (36) is also a measurement of P_{Λ}/P_{Σ^+} .

Furthermore if one measures the ratio of the up/down asymmetries with respect to S in the decays (c') and (a') one gets

$$\frac{\alpha_{\Lambda} P_{\Lambda}}{\alpha_0 P_{\Sigma^+}} = \frac{\alpha_{\Lambda} P_{\Lambda}}{\alpha_0 P_{\Sigma^0}} = \frac{\alpha_{\Lambda}}{\alpha_0} \times \left(-\frac{P_{\Lambda}}{3P_{\Lambda \text{ ind}}} \right) \quad (37)$$

where equation (35) has been used. $P_{\Lambda}/P_{\Lambda \text{ ind}}$ being known from the previous experiment in A), one obtains in this way a direct measurement of $\alpha_{\Lambda}/\alpha_0$ *).

We want to conclude this section with the following remarks.

) The following remark seems to be appropriate here. If H^{+} isobar can be produced in a reaction then it is possible to measure both P_{Λ}/P_{Σ} and $\alpha_{\Lambda}/\alpha_0$ as it was explained above. That is not the case for reactions giving H^{*-} and H^{*0} . Indeed

$$H^{*-} \longrightarrow \begin{cases} \Lambda + \pi^- \\ \Sigma^0 + \pi^- \\ \Sigma^- + \pi^0 \end{cases} \quad \text{i) ;} \quad H^{*0} \longrightarrow \begin{cases} \Lambda + \pi^0 \\ \Sigma^+ + \pi^- \\ \Sigma^- + \pi^+ \end{cases} \quad \text{ii)}$$

As it is well known Σ^- is not a "good analyser", i.e. $\alpha_- = 0$.

From i) we see that only $P_{\Lambda}/P_{\Lambda \text{ ind}} = -3 P_{\Lambda}/P_{\Sigma^0}$ can be measured (which gives $\Sigma - \Lambda$ relative parity). From ii) only $\alpha_{\Lambda} P_{\Lambda}/\alpha_0 P_{\Sigma^+}$ can be obtained, because P_{Σ^+} in ii) and P_{Σ^0} in i) are in general not correlated.

All proposals discussed here are based on measurements of up/down asymmetries which are manifestly connected with polarization effects. Formula (30) shows that P_Λ and P_Σ are expressed in terms of P_{H^*} . It is necessary to choose such experimental conditions where P_{H^*} is as large as possible. Large P_{H^*} can be expected at least if the incident energy is not very far from the " H^* production threshold" (cf. the high degree of polarization of hyperons produced at moderate energies). Obviously all steps which lead to a loss of polarization make our proposal more difficult to perform (see Eq. (35) connected with $\Sigma^0 \rightarrow \Lambda + \gamma$, or also some cases of Eq. (30) connected with different possible combinations of spins, parities of H^* , Λ and Σ).

It is very important to stress that too much significance should not be attached to the actual numbers given in (31) and (32) nor to the numerical values of the α_Λ/α_0 ratio which can be deduced from the described proposal. Two reasons can be given for this : a) the interference effects caused by channels other than the resonating one. This will be discussed further in chapter 5. b) the fact that decaying hyperons are moving in the rest system of H^* with, however, a small velocity. This leads to minor corrections described in Appendix II.

Nevertheless, it is very likely that the signs of P_Λ/P_Σ and α_Λ/α_0 still survive after all these corrections and the only information which is needed for the determination of the $\Sigma-\Lambda$ relative parity is precisely the sign of P_Λ/P_Σ . Concerning the α_Λ/α_0 ratio, its absolute value is known experimentally to be close to 1⁵⁾. The sign of this ratio which can be trusted in these experiments, in spite of the above mentioned corrections, is particularly important to know, because of its bearing on the proposed theories of weak interactions⁶⁾.

4. Isotopic properties of the isobars and the probable existence of other resonances.

We have seen that the isotopic spin of H^* must be equal to 1. The resonant mechanism that has been discussed leads then to relations between the different cross-sections. For instance, consider the reactions

The branching ratios are hardly predictable in general but if the K^+ events are chosen as explained in chapter 2 in such a way that the reaction goes through the resonant channel (23) then one has the branching ratios $(39)/(40)=1$, which provide a valuable test of the isobar model.

In the resonant reactions (6) - (8) followed by the decay (9) - (17) the fact that H^* has $I=1$ implies again that one has the branching ratios $(9)/(10) = (12)/(13) = (15)/(16) = 1$. But now only triangular relations can be established between reactions involving Σ 's and π 's passing through different charge states of H^* since K^+p is not a pure isotopic spin state. Similar considerations can be applied to $\pi^- + p \rightarrow \Sigma + \pi + K$ if one of the π 's is on the resonance.

It is clear that in general nothing can be said on the branching ratios Σ/Λ , unless specific models are introduced. A number of models for the strong interactions have been proposed. It will be seen that they lead to certain important predictions concerning the existence of other resonances in the pion-hyperon system for which it would be interesting to have experimental evidence. For this reason these problems will be briefly discussed.

A) Global ⁷⁾ and restricted ⁸⁾ symmetry.

When global or restricted symmetries are valid it is possible to describe the Λ - and Σ -hyperons which have ordinary isotopic spins of 0 and 1 respectively by the Y and Z doublets :

$$Y = \begin{pmatrix} -\Sigma^+ \\ Y^0 \end{pmatrix} \quad Z = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix} \quad (41)$$

where

$$Y^0 = \frac{\Lambda - \Sigma^0}{\sqrt{2}} \quad \text{and} \quad Z^0 = \frac{\Lambda - \Sigma^0}{\sqrt{2}}$$

In the global symmetry description Y and Z have "isotopic spins of global symmetry" $i=1/2$. The $Y\pi$ and $Z\pi$ systems can be in states with $i=1/2$ and $3/2$. It is possible to express the wave functions of the $Y\pi$ and $Z\pi$ systems corresponding to $i=1/2$ and $3/2$ in terms of the wave functions of the $\pi\Lambda$ and $\pi\Sigma$ systems ⁹⁾ of ordinary isotopic spin $I=0,1,2$.

The relations are given in Appendix I (5.A). It is seen that the $I=0$ state is expressed in terms of the $i=1/2$ state only, $I=2$ in terms of the $i=3/2$ only, whereas for $I=1$ the following formulae are valid :

$$|1/2; 1 I_3\rangle = \sqrt{\frac{2}{3}} |1 I_3\rangle_{\Lambda} + \sqrt{\frac{1}{3}} |1 I_3\rangle_{\Sigma} \quad (42)$$

$$|3/2; 1 I_3\rangle = \sqrt{\frac{1}{3}} |1 I_3\rangle_{\Lambda} - \sqrt{\frac{2}{3}} |1 I_3\rangle_{\Sigma} \quad (43)$$

$|1 I_3\rangle_{\Lambda}$ and $|1 I_3\rangle_{\Sigma}$ are the normalised charge wave functions for $I=1$ and $|1/2; 1 I_3\rangle$ and $|3/2; 1 I_3\rangle$ are the normalised wave functions for $i=1/2$ and $i=3/2$ respectively corresponding to $I=1$.

As a consequence when one discusses the resonating state which we have considered and which has $I=1$, in the frame of global or restricted symmetry, the appropriate description is in terms of the functions $|1/2; 1 I_3\rangle$ and $|3/2; 1 I_3\rangle$.

In other words, the wave functions of $\pi-\Lambda$ and $\pi-\Sigma$ which correspond to the decay of the isobar H^* must appear either in the combination $|1/2 \ 1 \ I_3\rangle$ or in the combination $|3/2 \ 1 \ I_3\rangle$ of the $|1 \ I_3\rangle_\Lambda$ and $|1 \ I_3\rangle_\Sigma$. Evidently this determines the branching ratios $(\Lambda+\pi)$ to $(\Sigma+\pi)$. Two cases must be distinguished.

i) Global symmetry.

In global symmetry (where all π interactions are identical $g_1=g_2=g_3=g_4$)*) one knows that the resonant $(Y\pi)$ and $(Z\pi)$ systems must be in the state $i=3/2, J=3/2$ which is the analogue of the $(3,3)$ resonance of the $\pi-N$ system. It is therefore the function $|3/2 \ 1 \ I_3\rangle$ which appears in our considerations. One then obtains for the Σ/Λ branching ratios for instance

$$H^{*+} \longrightarrow \begin{cases} \Sigma^+ + \pi^0 & : & 1 \\ \Sigma^0 + \pi^+ & : & 1 \\ \Lambda + \pi^+ & : & 4 \end{cases} \quad (44)$$

where kinematical corrections **) due to the $\Sigma-\Lambda$ mass difference have not been included.

*) $g_1 \dots g_4$ are the coupling constants for $NN\pi$, $\Sigma\Lambda\pi$, $\Sigma\Sigma\pi$ and $\Xi\Xi\pi$, respectively.

**) This factor (~ 4) would predict a ratio $\Lambda/\Sigma^0 \simeq 16$.

In the case of global symmetry the fact that the resonating state has $i=3/2$ leads to an important prediction. Indeed from the formulae (6.A) in Appendix I, one sees that the state $I=2$ which corresponds to a $i=3/2$ state must also give rise to a resonance. If global symmetry were strictly valid these two resonances would coincide. However, the interactions which are responsible for the violation of global symmetry will tend to separate the position of the $I=1$ and $I=2$ resonances. Roughly speaking one would expect that this separation will be of the order of magnitude of the $\Sigma - \Lambda$ mass difference. It seems therefore that if global symmetry has any validity one must find a second resonance with $I=2$ corresponding to an isobar H^{**} of mass ~ 1500 MeV. It would be very interesting to detect it experimentally for instance in the reaction

through the appearance of a second resonance in the K spectrum. Obviously the H^{**} isobar can decay in $\Sigma + \pi$ and not in $\Lambda + \pi$. The branching ratios corresponding to the decay of H^{**} can be calculated in the standard manner. For instance the reaction

has the branching ratios indicated in the brackets.

ii) Restricted symmetry ($g_2 = g_3$).

The case of restricted symmetry is similar to the one just discussed. The only difference is that now the $\pi - \Sigma\Sigma$ and $\pi - \Sigma\Lambda$ coupling constants need not be equal to those of the $\pi - NN$. It is therefore impossible to assert that the resonance appears in the $i=3/2$ state. Two cases can occur :

a) The $I=1$ resonance corresponds to $i=3/2$. One then gets the same conclusions as in the case of global symmetry, i.e. the branching ratio $\Lambda/\Sigma^{0(+)}=4$ and a resonance with $I=2$ must exist.

b) The $I=1$ resonance corresponds to the state with $i=1/2$. In this case the appropriate wave functions are the $|1/2 \ 1 \ I_3\rangle$ given by (42). One gets for the branching ratio (ignoring again the $\Sigma - \Lambda$ mass difference)

$$H^{*+} \rightarrow \begin{cases} \Sigma^+ + \pi^0 & : 1 \\ \Sigma^0 + \pi^+ & : 1 \\ \Lambda + \pi^+ & : 1 \end{cases} \quad (47)$$

If this were the case, the situation would be more favourable for the set of experiments proposed in chapters 2 and 3.

Should the resonance occur in the $i=1/2$ state then one can predict the existence of another resonance in the isotopic state $I=0$, as can be seen from expression (5.A) of Appendix I. As to the position of this resonance analogous remarks can be made in this case as in the one discussed in i). The $I=0$ resonance H^{***} could be observed for instance in reactions $\pi^- + p \rightarrow H^{***} + K^0$ or $K^- + p \rightarrow H^{***} + \pi^0$ with the branching ratios *)

$$H^{O***} \rightarrow \begin{cases} \Sigma^- + \pi^+ & : 1 \\ \Sigma^+ + \pi^- & : 1 \\ \Sigma^0 + \pi^0 & : 1 \end{cases} \quad (48)$$

*) The isobars H^{**} and H^{***} with $I=2$ and $I=0$ respectively cannot decay in $\Lambda + \pi$. However, they can give $\Lambda + \pi + \pi$ or $\Sigma + \pi + \pi$ if energetically possible. The isobar H^{***} could, in principle, be seen in the $K^- - p$ scattering if its mass is of the order, or heavier, than 1.4 BeV.

To summarise : if restricted asymmetry holds the existence of a $I=1$ resonance implies :

- a) either the existence of another resonance with $I=2$ if the $I=1$ resonance corresponds to $i=3/2$. In this case one has a branching ratio $\Sigma^0/\Lambda^0=1/4$ ^{*)}.
- b) or the existence of an other resonance with $I=0$ if the $I=1$ resonance corresponds to $i=1/2$. The branching ratio Σ^0/Λ^0 is then equal to 1 ^{*)}.

Of course it might happen that there are resonances in all three isotopic spin states.

B) A composite theory.

We would like to add a remark on another possibility of a high symmetry property possessed by strong interactions, which is suggested from the Sakata model. Namely, a complete symmetry (valid of course only approximately) among (bare) proton, neutron and Λ -hyperon has been proposed ¹⁰⁾. It is then possible to find a correspondence between the 33 -resonance in π -N system and possible pion-hyperon resonance(s) in the $J=3/2$ state. Up to the "3 body configurations" one finds three possibilities :

case (1)	2 resonances with $I=0$ and 1
case (2)	1 resonance with $I=1$
case (3)	1 resonance with $I=0$

pion-hyperon resonances

*) Ignoring the $\Sigma - \Lambda$ mass difference.

Under this condition there are no isobars with $I=2$ ^{*)}. It is clear that experimental studies of the isospin nature of pion-hyperon resonances are very important for eliminating some models of strong interactions. E.g. we know already that case (3) is excluded.

C) Static model.

It is well known that the static model in the one meson approximation describes quite well the π -N scattering. The same methods can be applied in the case of π -hyperon scattering. This has been done by Amati, Stanghellini and Vitale ¹¹⁾, and we shall discuss here certain of their results.

The position and shape of the resonances in the $\Lambda(\Sigma)$ - π system depend on three parameters: a quantity related to the cut-off λ , the mass difference $m_\Sigma - m_\Lambda \equiv \Delta$, and a parameter $\delta = \frac{g_2^2 - g_3^2}{g_2^2 + g_3^2}$, where g_2 and g_3 were defined above.

The position of the resonance $I=1, J=3/2$ fixes the value of λ . If one uses the same λ as in the π -N scattering and coupling constants g_2 and g_3 of the same order of magnitude as g_1 , the calculated position of H^* agrees with experiment. Once λ is fixed to fit the $I=1$ resonance one can calculate the position of the other resonances. If $g_2 \simeq g_3$ a resonance H^{**} with $I=2$ is predicted. The energy difference between the resonances H^* and H^{**} depends only on Δ , the contribution due to δ being negligible in this case, (only some percent of the contribution is due to δ). The difference in the position of the 2 resonances is given by $M_{H^{**}} - M_{H^*} \simeq 2\Delta$ where M_H is the "mass" of the corresponding isobar. With $g_2 \simeq g_3$ there is no resonance in the isotopic spin state $I=0$. Also for reasonable coupling constants the resonances appear in the spin state $J=3/2$ and not $J=1/2$.

*) The isobar with $S=-1, I=2$ can be formed if one uses 5 (or more) body configurations.

Of course by varying g_2 and g_3 in an arbitrary manner other situations could arise. These will not be discussed here ¹²⁾. We wish only to point out that in the static model one expects the appearance of a resonance with $I=2$. The predictions of the static model briefly summarised here are very similar to the ones of the global symmetry discussed in i). This is not surprising since the conditions on the cut-off and the coupling constants introduce in the static model assumptions which are in a certain way connected with a globally symmetric picture of strong interactions.

We have seen that different models predict the existence of other resonances in particular in the state $I=2$. Experimental confirmation of these predictions would be of great interest. This could be most easily done for instance by observing the K^+ spectrum in the reaction

$$\pi^- + p \rightarrow \begin{cases} H^{*-} + K^+ \longrightarrow \Sigma(\Lambda) + \pi + K^+ \\ H^{** -} + K^+ \longrightarrow \Sigma + \pi + K^+ \end{cases} \quad (49)$$

Such an experiment could be performed either with counters or by a time of flight measurement. One expects to find 2 peaks in the K^+ spectrum, a peak corresponding to $I=1$ resonance and another to $I=2$. Knowing the energy of the incident π^- and the masses M_{H^*} and $M_{H^{**}}$ to be expected, it is easy to calculate the position of the peaks in the K spectrum.

Furthermore, from the observed energies and isotopic spins of the resonances it will be possible to get some indication on the strength of the coupling constants g_2 and g_3 .

In any case a counter experiment such as the one proposed here will give indications on the cross-sections for the resonant process. This information is necessary for the planning of the bubble chamber experiments proposed in chapters 2 and 3, to measure the $\Sigma - \Lambda$ parity and the ratio $\alpha_\Lambda / \alpha_0$.

To conclude, let us remark that all the considerations of chapter 4 have been made under the assumption that the $\Sigma - \Lambda$ parity is even. Calculations are in progress at CERN to study the predictions of the static model in the one meson approximation for the case of opposite $\Sigma - \Lambda$ parity.

5. Discussion and conclusions

Our proposal for determining the $\Sigma-\Lambda$ parity and the ratio α_Λ/α_0 rests upon the measure of up/down asymmetries with respect to the production plane S , in the decay (a'), (b'') and (c') of all the Σ^+ and of all the direct and indirect Λ 's respectively. By fixing the K^+ direction and energy, in the manner explained in chapter 2, one expects to minimize the background of non resonant channels (1) and (5) where the isobar H^* is not formed in the intermediate stage. If the lifetime of H^* were long one would be in a more favourable situation for the neglect of interference effects due to non resonating channels. The fact that the H^* resonance has an observed width of ~ 25 MeV (which means a lifetime $\sim 10^{-23}$ sec) implies that even with our choice of events around the resonance peak the total cross-section for the resonating process will be spread out thus reducing the ratio of resonating to non resonating cross-section in the K^+ energy band considered. In the present situation of the theory of strange particles it is extremely difficult to estimate this interference effect, and one will have to rely on future experimental investigation of the resonance to appraise the validity of the isobar model. Along this line several tests can be suggested. For instance in the resonating channel $I=1$ one must have $\sigma(\pi^+p \rightarrow K^+\Sigma^0\pi^+) = \sigma(\pi^+p \rightarrow K^+\Sigma^+\pi^0)$. The deviation from this expected branching ratio would give an indication on the relative importance of the non resonating interference terms mentioned above. Furthermore, one can investigate by the usual methods (Adair test etc.) the resonances and check if the angular distribution of the pions associated with the decay of the conjectured isobar is compatible with a $\pi-\Lambda$ ($\pi-\Sigma$) system in a pure total angular momentum state, etc.

Another possible test of the validity of the isobar model will be to check whether the $|\alpha_\Lambda/\alpha_0|$ obtained from our proposed experiment is close to 1 or not.

An indication which is favourable to the isobar model is that $\sim 3/4$ of the $K^+p \rightarrow \Lambda + \pi^+ + \pi^-$ events in the experiment ¹⁾ mentioned in chapter 1 seems to pass through the resonance H^* . This lends support to our assumption (1) of chapter 2 that H^* is a "good" resonance.

Let us also remark that in the case where the isobar is excited through the $K^- + p$ reactions (6) - (8) there is an additional interference due to the presence of 2 pions in the final state. Even if one considers only resonating channels the transition amplitude for the process describing a π^- associated with the production of H^{*+} will interfere coherently with the corresponding one for the π^- coming from the decay of H^{*-} produced in association with π^+ . This effect can be appreciable if H^* has a short lifetime. A similar situation arises in the pion production,

The influence of the short lifetime of the pion-nucleon isobar on the branching ratios and on the pion spectrum in reaction (50) has been previously considered ¹³⁾. In analogy to the problems discussed here it would be also interesting to study the validity of the pion-nucleon isobar model by measuring the branching ratios in reaction (50) when the energy of one of the emitted pion is on the resonance ^{*)}.

For the reasons just mentioned it seems therefore that reactions (18) with production of a K and a π in the final state might have smaller interference effects at the resonance ^{**) [than e.g. reactions (1) and (5)] and might be better suited for our purpose ^{**) .}}

Apart from the condition that H^* be a good resonance we have mentioned in chapter 2 that the proposed measurements of the $\Sigma - \Lambda$ parity and of $\alpha_\Lambda / \alpha_0$ is feasible if the Σ / Λ branching ratio is not disastrous. As was discussed in chapter 4, the cases of the global symmetry and of the static model (with however some assumptions on the coupling constants g_2 and g_3) lead to a very

) If, for example, the H^ isobar is the perfect analogue of the 3,3 isobar of the π -N system (see, e.g. the global symmetry case) then the experimental information observed by studying reaction (50) can be used for an estimation of the interference effects connected with $K^- + p \rightarrow \Lambda(\Sigma) + \pi + \pi$.

**) A systematic study of reactions of the type (22) can give eventually also some indications on the existence (or non-existence) of a ΛK and a πK resonances. If a ΛK resonance would exist, some interesting conclusions could be deduced from it.

unfavourable branching ratio Σ/Λ of the order of $1/20$. There are, however, no strong reasons to believe such predictions. In particular, it was assumed that the $\Sigma-\Lambda$ relative parity is even and this is just what we are looking for. It should be remarked that if the resonance happens to be in a state of $i=1/2$ or $J=1/2$ (or both) the Σ/Λ branching ratio would be more favourable for our proposal. In all other cases it is difficult to predict the Σ/Λ branching ratio and an experimental study of it is urgently needed.

In any case, and this will be our conclusion, it seems that a systematic investigation of reactions like (18) leading to resonant processes of the systems $\pi\Lambda$ and $\pi\Sigma$ can be very rich in consequences. If the resonances are sufficiently pronounced and if the polarizations are large enough, a quite ambitious programme giving the $\Sigma-\Lambda$ relative parity, α_0/α_Λ etc. can be attempted. Even if some of these conditions are not satisfied, such an investigation would certainly provide a check of the theoretical predictions concerning other resonances. The properties of various resonances or even their non-existence will give a clearer idea on the fundamental nature of the interactions of strange particles.

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A P P E N D I X I

In the global (and restricted) symmetry, the "doublet" description for Λ and Σ is used. The "i" spin is introduced; $i=1/2$ for all baryons

$$\begin{pmatrix} p \\ n \end{pmatrix}, \begin{pmatrix} -\Sigma^+ \\ Y^0 \end{pmatrix}, \begin{pmatrix} Z_0 \\ \Sigma^- \end{pmatrix}, \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$$

and $i=1$ for pions. The i spin is the same as the ordinary isotopic spin I for pions, nucleons and cascade.

The systems πY and πZ can be described in terms of charge wave functions

$$|1/2 \ i_3\rangle_Y, |1/2 \ i_3\rangle_Z \quad \text{and} \quad |3/2 \ i_3\rangle_Y, |3/2 \ i_3\rangle_Z \quad (1.A)$$

corresponding to the total $i=1/2$ and $i=3/2$ respectively. These functions are easily obtained. They are analogous to the wave functions of the πN system with $i=I=1/2$ or $3/2$.

Since the global (restricted) symmetry is a particular case of the charge independent theory the wave functions (1.A) can be expressed in terms of the (normalised) eigenfunctions

$$|I \ I_3\rangle_{\Lambda}, |I \ I_3\rangle_{\Sigma} \quad (I=0,1,2) \quad (2.A)$$

of the usual total isotopic spin I of the $\pi\Lambda$ and $\pi\Sigma$ systems. For our purpose it is necessary to have wave functions

$$|i; I \ I_3\rangle \quad (3.A)$$

which are simultaneously eigenfunctions of i and I . Such wave functions can be formed by linear combinations of wave functions (1.A). Some examples are given below :

$$|1/2; 0 0\rangle = \frac{1}{\sqrt{2}} (|1/2, 1/2\rangle_Y + |1/2, 1/2\rangle_Z)$$

$$|1/2; 1 1\rangle = |1/2 1/2\rangle_Y$$

$$|3/2; 1 1\rangle = 1/2 (|3/2, 1/2\rangle_Y + \sqrt{3} |3/2, 3/2\rangle_Z)$$

$$|3/2; 2 2\rangle = |3/2 3/2\rangle_Y$$

It is not necessary to write explicitly all these combinations. We want to express functions $|i; I I_3\rangle$ in terms of functions $|I I_3\rangle_\Lambda$ and $|I I_3\rangle_\Sigma$. From the charge independence, these relations are obviously independent of I_3 . They must be of the form

$$|i I I_3\rangle = \alpha_{Ii} |I I_3\rangle_\Sigma + \beta_{Ii} |I I_3\rangle_\Lambda \quad (4.A)$$

For $I=0$ and $I=2$ the second term of right hand side of (4.A) is evidently zero. One has then

$$|1/2; 0 0\rangle = |0 0\rangle_\Sigma \quad (5'.A)$$

and

$$|3/2; 2 I_3\rangle = |2 I_3\rangle_\Sigma \quad (5''.A)$$

because $|3/2 3/2\rangle_Y = |\pi^+ \Sigma^+\rangle \equiv |2 2\rangle_\Sigma$ and it is sufficient to find such a relation for a particular value of I_3 . For $I=1$ the situation is more complicated. Both α_{1i} and β_{1i} are different from zero. An explicit calculation shows

$$\left. \begin{aligned} |1/2; 1 I_3\rangle &= \sqrt{\frac{1}{3}} |1 I_3\rangle_\Sigma + \sqrt{\frac{2}{3}} |1 I_3\rangle_\Lambda \\ |3/2; 1 I_3\rangle &= -\sqrt{\frac{2}{3}} |1 I_3\rangle_\Sigma + \sqrt{\frac{1}{3}} |1 I_3\rangle_\Lambda \end{aligned} \right\} \quad (5'''.A)$$

From these formulae it is clear that wave functions with $i=1/2$ contain $I=1$ and $I=0$ only, whereas those with $i=3/2$ contain $I=1$ and $I=2$.

When the π -hyperon scattering is considered in the approximation of the global (or restricted) symmetry, one can express the S matrix elements of the I description by those of the i description. This is easily done by the use of formulae (5.A). With obvious notations one has :

$$\left. \begin{aligned} \langle \pi \Sigma | S | \pi \Sigma \rangle_{I=2} &= S_{3/2} \\ \langle \pi \Sigma | S | \pi \Sigma \rangle_{I=1} &= \frac{1}{3} S_{3/2} + \frac{2}{3} S_{1/2} \\ \langle \pi \Sigma | S | \pi \Lambda \rangle_{I=1} &= -\frac{\sqrt{2}}{3} S_{3/2} + \frac{\sqrt{2}}{3} S_{1/2} \\ \langle \pi \Lambda | S | \pi \Lambda \rangle_{I=1} &= \frac{2}{3} S_{3/2} + \frac{1}{3} S_{1/2} \\ \langle \pi \Sigma | S | \pi \Sigma \rangle_{I=0} &= S_{1/2} \end{aligned} \right\} \quad (6.A)$$

where

$$S_i = \langle i I I_3 | S | i I I_3 \rangle = \langle i i_3 | S | i i_3 \rangle_Y$$

do not depend on I_3 by charge independence and also not on I , because

$$_Y \langle i i_3 | S | i i_3 \rangle_Y = _Z \langle i i_3 | S | i i_3 \rangle_Z \quad \text{and} \quad _Z \langle i i_3 | S | i i_3 \rangle_Y = 0$$

If a resonance occurs in a given state of i , formulae (5.A) give various branching ratios. From (6.A) it is seen that if a resonance occurs in $i=3/2$ (or $1/2$) then there must exist two resonances with $I=1$ and 2 (or $I=1$ and 0).

A P P E N D I X II

We want to discuss briefly the effect of the motion of the Λ 's (or Σ 's) on the up/down asymmetries. Corrections connected with this problem enter when the average described in chapter 3 is performed. Obviously, the discussion can be done in the rest system of H^* (referred to as the H^* system). It is possible to treat this problem exactly but for our purpose a simple, approximate treatment is quite sufficient because, as was said, we do not think that the actual numbers given in this paper have more than a 20 % significance. A non-relativistic approximation for baryons may be sufficient for our purpose (pions are treated relativistically).

Let us introduce the following notations :

$\vec{v}_\Lambda$ velocity of Λ in H^* system ; $v_\Lambda \equiv |\vec{v}_\Lambda| = \text{const} \simeq \frac{1}{4} c$
 $\vec{v}_\pi = \frac{\vec{q}_\pi}{\omega}$ " " π^- coming from Λ decay in H^* system
 $\vec{v}_\pi^* = \frac{\vec{q}_\pi^*}{\omega^*}$ " " π^- " " " in the rest system of the Λ

$$v_\pi^* = |\vec{v}_\pi^*| = \text{const} = 0.58 c$$

θ^* angle between Z axis (normal to the plane S which is the direction of $\vec{P}_{H^*}$) and $\vec{v}_\pi^*$

ξ angle between Z axis and $\vec{v}_\Lambda$

Now $\vec{v}_\pi = \vec{v}_\pi^* + \vec{v}_\Lambda$, so for the z component

$$(\vec{v}_\pi)_z = v_\pi^* \cos \theta^* + v_\Lambda \cos \xi \tag{1.B}$$

The π^- emitted upward (downward) with respect to the plane S have $(\vec{v}_\pi)_z > 0$ ($(\vec{v}_\pi)_z < 0$). From (1.B) it is seen that the division into "up" and "down" corresponds to a critical angle θ_c^* given by :

$$\cos \theta_c^* = -\frac{v_\Lambda}{v_\pi^*} \cos \xi \quad (2.B)$$

Defining $\vec{P}_\Lambda(\vec{v}_\Lambda)$ as the polarization of the Λ 's in the rest system of Λ emitted in the direction $\vec{v}_\Lambda^*$ by a polarized H^* the up-down asymmetry, can be found from :

$$\frac{\int_{-1}^1 f(\xi) d \cos \xi \int_{\cos \theta_c^*}^1 d \cos \theta^* \int_0^{2\pi} d\varphi^* \left[1 + \alpha_\Lambda \vec{P}_\Lambda(\vec{v}_\Lambda) \cdot \frac{\vec{q}^*}{|\vec{q}^*|} \right]}{\int_{-1}^1 f(\xi) d \cos \xi \int_{-1}^1 d \cos \theta^* \int_0^{2\pi} d\varphi^* \left[1 + \alpha_\Lambda \vec{P}_\Lambda(\vec{v}_\Lambda) \cdot \frac{\vec{q}^*}{|\vec{q}^*|} \right]} \quad (3.B)$$

where $f(\xi)$ is the angular distribution of Λ 's in the H^* system (an average over the azimuthal angle is already done).

Now

$$\begin{aligned} & \int_{-1}^1 f(\xi) d \cos \xi \int_{\cos \theta_c^*}^1 d \cos \theta^* \int_0^{2\pi} d\varphi^* \left(1 + \alpha_\Lambda \vec{P}_\Lambda(\vec{v}_\Lambda) \cdot \frac{\vec{q}^*}{|\vec{q}^*|} \right) \\ &= 2\pi \int_{-1}^1 f(\xi) d \cos \xi \cdot \int_{\cos \theta_c^*}^1 d \cos \theta^* (1 + \alpha_\Lambda P_{H^*} g(\xi) \cos \theta^*) \quad (4.B) \\ &= 2\pi \int_{-1}^1 f(\xi) d \cos \xi \left[1 - \frac{v_\Lambda}{v_\pi^*} \cos \xi + \frac{1}{2} \left\{ \alpha_\Lambda P_{H^*} g(\xi) \left(1 - \frac{v_\Lambda^2}{v_\pi^{*2}} \cos^2 \xi \right) \right\} \right] \end{aligned}$$

where (3.B) has been used. $g(\xi)$ is a known function of ξ since the explicit form of $\vec{P}_\Lambda(\vec{v}_\Lambda)$ can be found. The linear term in $\cos \xi$ in the last integrand does not contribute to the final result because $f(\xi)$ is an even function of $\cos \xi$ due to the parity conservation. This shows that the corrections due to the finite velocity of Λ 's are quadratic in $\frac{v_\Lambda}{v_\pi^*}$.

) For given J and $\pi(H^)/\pi(\Lambda)$, $\vec{P}_\Lambda(\vec{v})$ can be explicitly written down in terms of P_{H^*} and $\vec{v}_\Lambda$.

Further we notice that

$$\frac{\int_{-1}^1 f(\xi) g(\xi) P_H^* d \cos \xi}{\int_{-1}^1 f(\xi) d \cos \xi} = P_{\Lambda} \quad (5.B)$$

is precisely what we have called the "average polarization" of the Λ 's in the text [see chapter 2]. From (3.B), (4.B) and (5.B) one has

$$\frac{(\text{up})}{(\text{down})} = \frac{1 + \frac{1}{2} \alpha_{\Lambda} P_{\Lambda} \left\{ 1 - \varepsilon \left(\frac{v_{\Lambda}}{v_{\pi}^*} \right)^2 \right\}}{1 - \frac{1}{2} \alpha_{\Lambda} P_{\Lambda} \left\{ 1 - \varepsilon \left(\frac{v_{\Lambda}}{v_{\pi}^*} \right)^2 \right\}} \quad (6.B)$$

where

$$\varepsilon = \frac{\int_{-1}^1 f(\xi) g(\xi) \cos^2 \xi d \cos \xi}{\int_{-1}^1 f(\xi) g(\xi) d \cos \xi} \quad (7.B)$$

we have calculated ε for the very simple case $J=\frac{1}{2}$, $\pi(H^*)/\pi(\Lambda) = -1$ with the result $\varepsilon = 1/3$. In general $|\varepsilon|$ must be a smaller number. From this property of ε together with $(v_{\Lambda}/v_{\pi}^*)^2 \simeq 1/5$ it is seen that the corrections on the up/down asymmetry induced by the motion of the Λ 's are of the order of 5 - 10 % at most, so they are of no importance for the considerations of chapters 3 and 4. Similar estimations can be made for the case of decays of H^* via Σ . The corrections induced by the motion of Σ 's are even smaller because $v_{\Sigma} < v_{\Lambda}$.

Furthermore, similar calculation applied to the chain decay process $\Sigma^0 \rightarrow \Lambda + \gamma \rightarrow p + \pi^-$ shows that the effect of finite velocity of the Λ 's to the up/down asymmetry of π^- with respect to the plane perpendicular to the Σ^0 -polarization is negligibly small ($< 1\%$).

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