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A Note on Production Dispersion Relation *)

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In the deduction of the dispersion relations for the production processes

$$A+B \rightarrow C+D$$

a feature is met, which has no counterpart in the elastic scattering dispersion relation, connected with the fact that the thresholds of the two processes $A+B \rightarrow A+D$ and $C+D \rightarrow C+D$ are in general different. This problem has been discussed by R.F. Streater and the author ¹⁾, but although correct that discussion on this subject is unnecessary. We review first the discussion of ref. ¹⁾ and at the end we show how the point is settled by using the rotational invariance.

The extension of what is done in the elastic scattering dispersion relation amounts to write first a dispersion relation for some negative values of

$$\xi = \frac{m_A^2 + m_C^2}{2}$$

with $\frac{m_A^2 - m_C^2}{2}$, m_B^2 and m_D^2 fixed at their physical values and the momentum transfer between the B and D particles also fixed, and after, to perform the analytical continuation up to the physical ξ value. At first, it seems that the absorptive part of the amplitude is not analytic in a neighbourhood S of the real ξ axis

$$S = \left[\xi = \xi_1 + i\xi_2 : \xi_1 \leq \left(\frac{m_A^2 + m_C^2}{2} \right)_{\text{phys.}} + \delta ; \xi_2 \leq \epsilon \right]$$

as is the case in the elastic scattering dispersion relation even if both the dispersion relation $A+B \rightarrow A+B$ $C+D \rightarrow C+D$ can be proved up to suitable large momentum transfer. In fact, from the application of the Dyson representation because of the absorptive part of the amplitude one has:

$$A = \int \frac{\mathcal{L} d\alpha d u_i d u_{i0} d x_i}{x_1 x_2 - \sqrt{x_1^2 - k_1^2} \sqrt{x_2^2 - k_2^2} - k_1 k_2 \cos(\theta - \alpha)} \quad i=1,2 \quad (1)$$

$$x_1 = \frac{k_1^2 + |u_{11}|^2 + x_1^2 - \left(u_{10} + \frac{m_B^2 - \alpha - \mathfrak{F}}{2W}\right)^2}{2|u_{11}|}$$

$$x_2 = \frac{k_2^2 + |u_{21}|^2 + x_2^2 - \left(u_{20} + \frac{m_D^2 + \alpha - \mathfrak{F}}{2W}\right)^2}{2|u_{21}|}$$

and

$$k_1 = \frac{1}{2W} \sqrt{(W + m_B)^2 - \mathfrak{F} - \alpha} \sqrt{(W - m_B)^2 - \mathfrak{F} - \alpha}$$

$$k_2 = \frac{1}{2W} \sqrt{(W + m_C)^2 - \mathfrak{F} + \alpha} \sqrt{(W - m_C)^2 - \mathfrak{F} + \alpha}$$

$$W^2 = (p_A + p_B)^2$$

The presence of the product $k_1 k_2$ in the denominator (instead of k^2 in the elastic scattering case) seems to imply that in the unphysical range the region of analyticity of A in $\mathfrak{F}$ is the strip S around the real $\mathfrak{F}$ axis minus a neighbourhood around the branch lines of $k_1 k_2$ because one expects in general that there are W values in which only one of the k_1 and k_2 becomes "imaginary". (This is not the case in the photo-production amplitude owing to the neglect of the higher order electromagnetic correction). These cuts are harmless if they can be chosen such that a continuous path in S exists linking the unphysical $\mathfrak{F}$ mass value to its physical value, this path being also contained in the forward tube. In order that A is the absorptive part of the amplitude, it is also required that in the physical range the arguments of k_1 and k_2 be both zero. By letting run the branch lines from the four branch points

$$(W \pm m_B)^2 + \alpha, (W \pm m_C)^2 - \alpha$$

up to infinity in the direction of, say, the positive imaginary ξ axis allows to fulfil all the requirements for a dispersion relation to hold, provided that the elastic scattering $A+B \rightarrow A+B$, $C+D \rightarrow C+D$ dispersion relation holds for suitable large momentum transfer.

We want to show that no such cuts exist. From Lorentz invariance the weight function depends only on $\cos \alpha$ such that (2) can be cast in the form:

$$A = \int \frac{\rho \cdot (x_1 x_2 - \sqrt{x_1^2 - k_1^2} \sqrt{x_2^2 - k_2^2} - k_1 k_2 \cos \theta \cos \alpha) dx d\mu_1 d\mu_2 dx_2}{(x_1 x_2 - \sqrt{x_1^2 - k_1^2} \sqrt{x_2^2 - k_2^2} - k_1 k_2 \cos \theta \cos \alpha)^2 - k_1^2 k_2^2 \sin^2 \theta \sin^2 \alpha}$$

and because

$$k_1 k_2 \cos \theta = \left(\frac{W^2 + m_B^2 - \alpha - \beta}{2W} \right) \left(\frac{W^2 + m_D^2 + \alpha - \beta}{2W} \right) - \frac{m_B^2 + m_D^2}{2} - 2 \Delta^2$$

$$\Delta^2 = -\frac{1}{4} (p_B - p_D)^2$$

it follows that the singularities of A can arise only from the zeros of the denominator, exactly as it is the case in the proof of elastic scattering dispersion relation.

REFERENCE

- 1) A. Minguzzi, R.F. Streater, Nuovo Cimento 17, 946 (1960)