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Dispersion Relations for some Photon Reactions.

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(ricevuto il 1° Agosto 1960)

Summary. — By using Dyson's integral representation directly for the absorptive part of the T matrix, certain dispersion relations involving photons and nuclei are proved more simply than by the method of BOGOLIUBOV *et al.* The range of validity is always less than that obtained by the best possible techniques, but fewer physical assumptions are needed. In particular the existence of an interpolating field associated with light nuclei is not needed. In the usual e^2 approximation, we can treat photoproduction, Compton scattering and bremsstrahlung from nuclei.

Introduction.

We study the matrix element

$$(1) \quad T(p_1, p_2, k_1 + k_2) = \int \langle p_1 | \theta(x) \left[j\left(\frac{x}{2}\right), j\left(-\frac{x}{2}\right) \right] | p_2 \rangle \exp \left[i \frac{k_1 + k_2}{2} x \right] d^4x,$$

which, owing to its retarded nature, is a regular function of $q = (k_1 + k_2)/2$ in the forward tube $\mathcal{S}(q)$, *i.e.* if $\text{Im } q$ is a forward timelike vector. This information leads to analyticity in the cut energy plane (and hence to a dispersion relation) only for suitable negative values of k^2 . Instead of following the continuation procedure of BOGOLIUBOV ⁽¹⁾, here we add the mass-spectral conditions directly to the forward tube. This problem was solved by the

⁽¹⁾ N. N. BOGOLIUBOV and D. V. SHIRKOV: *Introduction to the Theory of Quantized Fields* (New York, 1959); H. LEHMANN: *Suppl. Nuovo Cimento*, **14**, 153 (1959); H. J. BREMERMAN, R. OEHME and J. G. TAYLOR: *Phys. Rev.*, **109**, 2178 (1958).

Jost-Lehmann-Dyson representation⁽²⁻⁵⁾. We find that the domain of regularity can immediately be shown to be the cut energy plane only in cases where the photon is involved, and in the usual approximation in which all intermediate states containing photons are ignored. The method proves dispersion relations for the process $\gamma + n \rightarrow \gamma + n$ up to $\Delta^2 = .8\mu^2$, for the process $\gamma + n \rightarrow \pi + n$ for Δ^2 at the threshold, for the process $\gamma + n \rightarrow \gamma + \gamma + n$ for a suitable momentum transfer between the nucleons, and a suitable configuration of the final boson system, and for γ -nucleus elastic scattering for light nuclei (i.e. provided $(Z/\sqrt{137})^2 \ll 1$), for momentum transfers of the order of the binding energy of the least tightly bound nucleon. This can be shown without the necessity of ascribing an interpolating field⁽⁶⁾ to the nucleus, which remains in the state vector throughout the proof.

In order to prove dispersion relations, we must eliminate the contributions from the one-particle intermediate states, which give rise to poles at, say, $(\pm q + (p_1 + p_2)/2)^2 = M^2$. This is done by considering, instead of $T(p_1 p_2 q)$, the function

$$(2) \quad \left[\left(q + \frac{p_1 + p_2}{2} \right)^2 - M^2 \right] \left[\left(-q + \frac{p_1 + p_2}{2} \right)^2 - M^2 \right] T(p_1, p_2, q) \equiv \tilde{T},$$

which does not contain this contribution, but otherwise has the same support and spectrum. Then $T(\omega)$ has the same domain as $\tilde{T}(\omega)$ except for poles at $(\pm \omega + E_A)^2 - \omega^2 + \Delta^2 = M^2$.

1. - Photon-nucleon elastic scattering⁽²⁾.

We introduce $\tilde{T}$, given by (1) and (2) where M is the nucleon mass and $J(\omega)$ the photon current, and apply the Jost-Lehmann⁽³⁾ representation with $m_1 = m_2 = M + \mu$, (μ = meson mass) in the Breit system $\mathbf{p} + \mathbf{p}' = 0$. We get

$$(3) \quad \tilde{T} = \frac{1}{2\pi} \int d^3u \, dk^2 \frac{\Phi_1(\mathbf{u}, k) + \frac{k_{10} + k_{20}}{2} \Phi_2(\mathbf{u}, k)}{k^2 + \left(\frac{\mathbf{k}_1 + \mathbf{k}_2}{2} - \mathbf{u} \right)^2 - \left(\frac{k_{10} + k_{20}}{2} \right)^2}.$$

(2) R. OEHME and J. G. TAYLOR: *Phys. Rev.*, **113**, 371 (1959).

(3) R. JOST and H. LEHMANN: *Nuovo Cimento*, **5**, 1598 (1957).

(4) F. J. DYSON: *Phys. Rev.*, **110**, 1460 (1951).

(5) R. F. STREATER: *Garding's proof of the Jost-Lehmann-Dyson representation*, CERN Report 8483/TH. 89.

(6) W. ZIMMERMANN: *Nuovo Cimento*, **10**, 597 (1958); K. NISHIJIMA: *Phys. Rev.*, **111**, 995 (1958).

By defining

$$(4) \quad \omega = \frac{(k_1 + k_2)(p_1 + p_2)}{2\sqrt{(p_1 + p_2)^2}},$$

we have

$$\frac{K_{10} + K_{20}}{2} = \omega, \quad p_1 = -p_2, \quad p_{1,0} = p_{2,0} = \sqrt{\Delta^2 + M^2} \equiv E_A,$$

and the support of $\varphi_{1,2}$ is

$$(5) \quad k \geq \max \{0, M + \mu - \sqrt{E_A^2 - u^2}\}, \quad 0 \leq u^2 \leq E_A.$$

By choosing a polar axis in the scattering plane perpendicular to $\mathbf{p}$ we get

$$(6) \quad \tilde{T} = \frac{1}{2\pi} \int_{-1}^1 u^2 du \int_0^{2\pi} d(\cos \theta) \int_0^{2\pi} d\varphi dk^2 \cdot \frac{\Phi_1(\sin \theta, \cos \varphi | \mathbf{u} | k) + \omega \Phi_2(\sin \theta, \cos \varphi | \mathbf{u} | k)}{k^2 - \Delta^2 + u^2 - 2|u| \cos \theta \sqrt{\omega^2 - \Delta^2}},$$

which can be written

$$(7) \quad \tilde{T} = \frac{1}{2\pi} \int_{-1}^1 u^2 du \int_0^{2\pi} d(\cos \theta) \int_0^{2\pi} d\varphi \frac{[\Phi_1 + \omega \Phi_2] 2[k^2 - \Delta^2 - u^2]}{(k^2 - \Delta^2 + u^2)^2 - 4u^2 \cos^2 \theta (\omega^2 - \Delta^2)}.$$

If the smallest value of $k^2 - \Delta^2 + u^2$, (as k, u vary over the support of $\Phi_{1,2}$) is positive, we can prove dispersion relations. Otherwise, no point in the ω -plane is a regular point. The latter case holds for all processes not involving photons (*). For $\gamma + n \rightarrow \gamma + n$, however, $\min(k^2 - \Delta^2 + u^2)$ is positive, provided that $\Delta^2 < \sim 0.8 \mu^2$.

On introducing the two weight functions

$$(8) \quad \tilde{\Phi}_{1,2}(\omega') = \int u^2 du dk^2 \int_0^{2\pi} d(\cos \theta) \cdot \int_0^{2\pi} d\varphi \Phi_{1,2}(2k^2 - 2\Delta^2 + 2u^2) \delta[(k^2 - \Delta^2 + u^2)^2 - 4u^2 \cos^2 \theta (\omega'^2 - \Delta^2)],$$

(*) This remark was stressed by J. G. TAYLOR: *Lectures on dispersion relations* University of Maryland, ASTIA no. 202640.

we get

$$(9) \quad \tilde{T} = \frac{1}{2\pi} \int_{\omega_0}^{\infty} \frac{2\tilde{\Phi}_1(\omega')\omega' d\omega'}{\omega'^2 - \omega^2} + \frac{\omega}{2\pi} \int_{\omega_0}^{\infty} \frac{2\tilde{\Phi}_2(\omega')\omega' d\omega'}{\omega'^2 - \omega^2},$$

where

$$\omega_0 = \left[\min \left(\frac{k^2 - \Delta^2 + u^2}{2|u|\cos\theta} \right)^2 + \Delta^2 \right]^{\frac{1}{2}} = \frac{-2E_A + m^2 + m_1^2}{2E_A},$$

the «correct» threshold corresponding to $(p+k)^2 = (M+\mu)^2$. Then

$$(10) \quad \tilde{T} = \frac{1}{2\pi} \left(\int_{-\infty}^{-\omega_0} + \int_{\omega_0}^{\infty} \right) \frac{\varepsilon(\omega') [\tilde{\Phi}_1(\omega') + \omega' \tilde{\Phi}_2(\omega')]}{\omega' - \omega} d\omega'.$$

The numerator is exactly $\tilde{A}(\omega')$, the absorptive part, defined as

$$\tilde{A} = \int dx^4 \exp[iqx] \left[\left(\pm i \frac{\partial}{\partial x} + \frac{p_1 + p_2}{2} \right)^2 - M^2 \right] \langle p_1 | \left[j\left(\frac{x}{2}\right), j\left(-\frac{x}{2}\right) \right] | p_2 \rangle.$$

For a theory invariant under TP , $\tilde{A}(\omega')$ is real and equal to the imaginary part of $\tilde{T}(\omega')$ (⁷). For elastic scattering, this follows already from momentum conservation, without the use of TP .

In the same way, neglecting intermediate states with more than one photon, we can show dispersion relations for the process

$$\gamma + \text{nucleus} \rightarrow \gamma + \text{nucleus}$$

for $\Delta^2 \leq \Delta_0^2 \simeq$ binding energy of the most loosely bound nucleons.

2. - Photo-production from nucleons.

It turns out that dispersion relations can be shown for the largest momentum transfer if we apply the reduction formula to the nucleon field, *i.e.* we consider

$$(11) \quad \tilde{T} = \left[\left(\frac{k_1 + k_2}{2} \pm q \right)^2 - M^2 \right] \int \langle \gamma | \theta(x) \left[f\left(\frac{x}{2}\right), f^*\left(-\frac{x}{2}\right) \right] | \pi \rangle \exp[iqx] d^4x,$$

$$q = \frac{p_1 + p_2}{2}, \quad k_1^2 = 0, \quad k_2^2 = \mu^2.$$

(⁷) R. OEHME: *Phys. Rev.*, **100**, 1503 (1955), Appendix.

Define

$$(12) \quad \omega = \frac{(p_1 + p_2)(k_1 + k_2)}{2\sqrt{(k_1 + k_2)^2}},$$

and in the Breit system of the bosons ($k_1 + k_2 = 0$)

$$\begin{aligned} q &= \left\{ \omega, 0, \frac{2\omega\mu^2}{4kw}, \sqrt{\omega^2 \frac{\Delta^2}{k^2} - m^2 - \Delta^2} \right\}, \\ k_1 &= \left\{ \frac{\bar{w}^2 - \mu^2}{2\bar{w}}, 0, k, 0 \right\}, \\ k_2 &= \left\{ \frac{\bar{w}^2 + \mu^2}{2\bar{w}}, 0, -k, 0 \right\}, \end{aligned}$$

where

$$\bar{w}^2 = 2\mu^2 + 4\Delta^2, \quad \bar{k}^2 = \frac{(\bar{w}^2 - \mu^2)^2}{4\bar{w}^2}.$$

Then as above

$$(13) \quad \tilde{T} = \frac{1}{2\pi} \int d^3u dk^2.$$

$$\frac{\Phi_1 + q_0 \Phi_2}{k^2 + u^2 - \Delta^2 - m^2 + 2|u| \cos \theta \sqrt{\omega^2 (\Delta^2 / k^2) - m^2 - \Delta^2} + 2|u| \sin \theta \cos \varphi (\omega \mu^2 / 2\bar{k}\bar{w})},$$

where $\Phi_{1,2}$ have support

$$0 \leq u^2 \leq \frac{\mu^2}{2} + \Delta^2; \quad k \geq \max \left\{ 0, m + \mu - \sqrt{\frac{\mu^2}{2} + \Delta^2 - u^2} \right\}.$$

As before one can eliminate the square root in (13) by using rotational invariance. The domain of regularity is the cut plane, provided

$$(14) \quad \left[\min \left(\frac{k^2 + u^2 - m^2 - \Delta^2}{2|u|} \right) \right]^2 \geq \frac{\frac{1}{4}(\mu^4 / \bar{k}^2 \bar{w}^2)(\Delta^2 + m^2)}{1 - \frac{1}{4}(\mu^4 / \bar{k}^2 \bar{w}^2)},$$

and one can show that with the actual values of the meson and nucleon masses, (14) is satisfied with $\Delta^2 = \Delta_{\text{thresh}}^2$.

3. - Bremsstrahlung from nucleons.

We can prove regularity in the cut plane also for the process

$$\gamma + n \rightarrow \gamma' + \gamma'' + n'.$$

Consider

$$\tilde{T} \propto \int \langle \gamma | \theta(x) \left[f\left(\frac{x}{2}\right), f^+\left(\frac{x}{2}\right) \right] | \gamma' \gamma''_{\text{out}} \rangle \exp \left[i \frac{n' + n}{2} x \right].$$

Define the Breit system by

$$\gamma + \gamma' + \gamma'' = 0$$

and fix the «mass» of the $\gamma' + \gamma''$ system as $\xi^2 = (\gamma' + \gamma'')^2$, and define

$$\omega = \frac{(\gamma + \gamma' + \gamma'')(n + n')}{2\sqrt{(\gamma + \gamma' + \gamma'')^2}},$$

then we get exactly the same expression as in (13) with ξ^2 instead of μ^2 , Δ^2 being always the momentum transfer between the nucleons. It follows that ω is expressed in terms of $W^2 = (\gamma + n)^2$, $\Delta^2 = -\frac{1}{4}(n - n')^2$, $\xi^2 = (\gamma' + \gamma'')^2$ only. As the two other invariants we take the polar co-ordinates θ' , φ'' of the vector $\gamma' - \gamma''$, one axis being along n' and the polar axis perpendicular to $\gamma' + \gamma''$. Then the kinematics of the problem is specified by the five parameters W^2 , Δ^2 , ξ^2 , φ' , θ' ⁽⁸⁾. One can follow the procedure in (13), proving dispersion relations in ω under condition (14), with μ^2 replaced by ξ^2 . There are two modifications. In the first place, the square root in the denominator of eq. (13) cannot be removed in this case, since the weight-function is no longer invariant under rotations around the γ -axis. Secondly the absorptive part is no longer the imaginary part.

One can localise the branch point at the value $W = \pm m + \mu$, $\pm m + 2\mu \dots$ by considering, instead of T , the related functions

$$T^+ = T(\theta', \varphi') + T(\pi - \theta', \varphi'),$$

$$T^- = \frac{T(\theta', \varphi') - T(\pi - \theta', \varphi')}{\sqrt{\omega^2(\Delta^2/k^2) - M^2 - \Delta^2}}.$$

4. - Conclusion.

Although, by this method, we cannot show dispersion relations for as large a value of Δ^2 as by the standard method, this method is so much simpler that we can hope to incorporate some of the consequences of unitarity without the work becoming too complicated. The possibility of showing photon disper-

⁽⁸⁾ R. ASCOLI and A. MINGUZZI: *Phys. Rev.*, **118**, 1435 (1960).

sion relations by Dyson's representation is closely related to the possibility ⁽⁹⁾ of justifying « polology » for some photon processes. As remarked in ref. ⁽⁹⁾ it is essential that we can treat the electromagnetic forces to order e^2 .

⁽⁹⁾ J. GUNSON and J. G. TAYLOR: *Nuovo Cimento*, **15**, 806 (1960).

RIASSUNTO

In questa nota si dimostrano relazioni di dispersione in cui intervengono fotoni senza usare la tecnica di Bogoliubov, partendo dalla rappresentazione integrale di Dyson del commutatore non ritardato. I valori del « momentum transfer » che si ottengono sono minori di quelli ottenuti con la tecnica migliore; ma il nostro procedimento richiede un minor numero di ipotesi; in particolare non è necessario introdurre « campi interpolanti » per i nuclei. Nell'approssimazione e^2 , si provano relazioni di dispersione per fotoproduzione, scattering Compton e bremsstrahlung su nuclei.