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Green's functions in some covariant soluble problems.⁺)

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ABSTRACT

Examples of soluble covariant field theory are given, where there is no scattering but where the Green's functions are non-trivial. Exact expressions for the Green's functions are obtained, and in each case they show a peculiar behaviour on the light cone. Some case exhibit the ghost-like trouble. In one case, it is shown that Lehmann's proof for the spectral representation of the two-body Green's function fails. All examples given here contradict Haag's theorem and the significance of this contradiction is pointed out.

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ERRATA

Page 5 : Eq. (16) $-S_i S_j (y_i - y_j)$ should be $-S_i S_j \Delta_F(y_i - y_j)$

Page 12 : Eq. (25) $G(x,y) = \dots S_F(x-y)$: $S_F(x-y)$ is missing

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1.

It is well-known that the vector coupling theory of the neutral scalar meson with a fermion field can be solved exactly. Although this theory gives no scattering at all, the Green's functions are not equivalent to those of the free field ¹⁾. The purpose of this note is to show that there is an infinite number of soluble theories belonging to a certain class, which are relativistically invariant and give no scattering at all but with non-trivial Green's functions. Exact expressions for these Green's functions have been obtained and they show peculiar singularities on the light cone. Because of this, they in general cannot satisfy Lehmann's spectral representation ²⁾ (at least his original proof fails). Another interesting point is the appearance of the ghost-like trouble in one case. This example may give some insight into the ghost problem of the covariant field theory. Finally, all examples considered here violate the so-called Haag theorem and an explanation for this phenomenon is discussed.

2.

We start from the following Lagrangian ^{*)}

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_1, \\ \mathcal{L}_0 &= -\frac{1}{4} \left[\bar{\psi} \left(\gamma \frac{\partial}{\partial x} + m \right) \psi + \psi \left(\gamma^T \frac{\partial}{\partial x} - m \right) \bar{\psi} + c.c. \right] \\ &\quad - \frac{1}{2} \left[\left(\frac{\partial}{\partial x_\mu} \phi \right)^2 + \mu^2 \phi^2 \right] \\ \mathcal{L}_1 &= \frac{i}{4} \left(\bar{\psi} \gamma_\mu \psi - \psi \gamma_\mu^T \bar{\psi} \right) \frac{\partial}{\partial x_\mu} F(\phi) \\ &\quad + \frac{i}{4} \frac{\partial}{\partial x_\mu} F(\phi) \cdot \left(\bar{\psi} \gamma_\mu \psi - \psi \gamma_\mu^T \bar{\psi} \right)\end{aligned}\tag{1}$$

*)

If we use the Hamiltonian equation, as we shall shortly, we can

prove that $\frac{\partial}{\partial x_\mu} F(\phi) = \frac{1}{2} \left[\frac{\delta F}{\delta \phi} \frac{\partial \phi}{\partial x_\mu} + \frac{\partial \phi}{\partial x_\mu} \frac{\delta F}{\delta \phi} \right]$

This remark will be useful in what follows.

where $F(\phi)$ is an arbitrary local hermitian function of $\phi(x)$.
When we put $F(\phi) \equiv g\phi$, g being a real constant, then Eq. (1) reduces to the vector coupling theory of the neutral scalar meson, which had been treated previously¹⁾.

The equations of motion are given by

$$\begin{aligned} (\gamma \frac{\partial}{\partial x} + m) \psi &= \frac{i}{2} \gamma_{\mu} \left[\psi \frac{\partial F(\phi)}{\partial x_{\mu}} + \frac{\partial F(\phi)}{\partial x_{\mu}} \psi \right] \\ (\square - \mu^2) \phi &= 0 \end{aligned} \quad (2)$$

The Hamiltonian of our theory is easily computed by the conventional method.

$$\begin{aligned} H &= \int d^3x [H_0(x) + H_1(x)] \\ H_0(x) &= \psi^* (\alpha \cdot p + \beta m) \psi + \frac{1}{2} [(\nabla \phi)^2 + \mu^2 \phi^2 + \pi^2] \\ H_1(x) &= -\frac{i}{2} \sum_{k=1}^3 \frac{\partial}{\partial x_k} F(\phi) [\bar{\psi} \gamma_k \psi - \psi \gamma_k^T \bar{\psi}] \\ &\quad - \frac{1}{4} \left(\frac{\delta F}{\delta \phi} \pi + \pi \frac{\delta F}{\delta \phi} \right) (\psi^* \psi - \psi \psi^*) \\ &\quad + \frac{1}{8} \left(\frac{\delta F}{\delta \phi} \right)^2 (\psi^* \psi - \psi \psi^*)^2 \end{aligned} \quad (3)$$

where $\pi(x)$ is the canonical conjugate of $\phi(x)$, so that

$$\pi(x) = \frac{\partial \phi}{\partial t} + \frac{1}{2} \frac{\delta F}{\delta \phi} (\psi^* \psi - \psi \psi^*)$$

For an arbitrary operator Q , we must have the Hamiltonian equation

$$\frac{dQ}{dt} = i [H, Q] \quad (4)$$

The quantization can be achieved by the canonical commutation relations:

$$\begin{aligned} [\pi(x), \phi(x')] &= -i \delta^{(3)}(x-x') \\ \{\psi(x), \psi^*(x')\}_+ &= \delta^{(3)}(x-x') \\ [\pi(x), \psi(x')] &= [\phi(x), \psi(x')] = 0 \\ [\pi(x), \pi(x')] &= [\phi(x), \phi(x')] = 0 \end{aligned} \quad (5)$$

It is easy to check that our quantization is consistent, i.e. the Hamiltonian equation (4) reproduces exactly the Lagrangian equ. (2) on the basis of the commutation laws Eq. (5)

Now let us consider the following unitary operator U .

$$U = \exp \left[\frac{i}{2} \int d^3x (\psi^*(x) \psi(x) - \psi(x) \psi^*(x)) F(\phi(x)) \right] \quad (6)$$

Then, after some algebra, we can easily show that

$$H = U H_0 U^{-1} \quad (7)$$

and also we have

$$\begin{aligned} U^{-1} \phi(x) U &= \phi(x) \\ U^{-1} \psi(x) U &= \exp[i F(\phi(x))] \cdot \psi(x) \end{aligned} \quad (8)$$

Because of Eq. (7), the eigen-functions Ψ_n of H and Φ_n of H_0 are related to each other by the following relation

$$\Psi_n = U \Phi_n \quad (9)$$

Next, we define two time-dependent operators, the Heisenberg operator $Q(t)$ and the free-field operator $Q_0(t)$ (of the interaction representation) respectively, from the time-independent Q as follows:

$$\begin{aligned} Q(t) &= \exp(iHt) Q \exp(-iHt) \\ Q_0(t) &= \exp(iH_0t) Q \exp(-iH_0t) \end{aligned} \quad (10)$$

Then, using Eqs. (7) and (8), the Heisenberg operators ψ and ϕ are related to the free-field operator ψ_0 and ϕ_0 by

$$\begin{aligned} U^{-1} \phi(x) U &= \phi_0(x) \\ U^{-1} \psi(x) U &= \exp[iF(\phi_0(x))] \cdot \psi_0(x) \end{aligned} \quad (11)$$

where x denotes a point in the four-dimensional space with the co-ordinate $(\underline{x}, t)$.

By means of Eqs. (9) and (11), we can compute the Green's function exactly. For example, let us consider the two-body case.

$$G(x, y) = - (\bar{\Psi}_0^* T(\psi(x) \bar{\psi}(y)) \Psi_0) \quad (12)$$

Inserting Eq. (9) into Eq. (12) and using Eq. (11), we obtain

$$G(x, y) = S_F(x-y) (\bar{\Phi}_0^* P(\exp[iF(\phi_0(x))], \exp[-iF(\phi_0(y))]) \Phi_0) \quad (12')$$

where

$$S_F(x-y) = - (\bar{\Phi}_0^* T(\psi_0(x) \bar{\psi}_0(y)) \Phi_0)$$

is the free-field propagator.

In order to evaluate the matrix element appearing in Eq. (12'), we use the following trick ³⁾; we express $\exp (i \int F(z))$ as a Fourier transform.

$$\exp (i \int F(z)) = \int_{-\infty}^{\infty} d\tau h(\tau) \exp (i \tau z) \quad (13)$$

and use the equation ⁴⁾

$$\begin{aligned} & (\Phi_0^* P(\exp [i \tau \phi_0(x)], \exp [-i \sigma \phi_0(y)]) \Phi_0) \\ &= \exp [\tau \sigma \Delta_F(x-y) - \frac{1}{2} (\tau^2 + \sigma^2) \Delta_F(0)] \end{aligned} \quad (14)$$

where Δ_F is the free propagator for the meson field, defined by

$$\Delta_F(x-y) = (\Phi_0^* P(\phi_0(x) \phi_0(y)) \Phi_0)$$

Thus, finally, we have

$$\begin{aligned} G(x, y) &= S_F(x-y) \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\sigma h(\tau) h^*(\sigma) \\ &\times \exp [\tau \sigma \Delta_F(x-y) - \frac{1}{2} (\tau^2 + \sigma^2) \Delta_F(0)] \end{aligned} \quad (15)$$

This result can be generalized for all Green's functions. For example, the n-body fermion Green's function is given by

$$\begin{aligned} G(x_1, \dots, x_n; y_1, \dots, y_n) &= G_0(x_1, \dots, x_n; y_1, \dots, y_n) \\ &\times \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} d\tau_1 \dots d\tau_n d\sigma_1 \dots d\sigma_n h(\tau_1) \dots h(\tau_n) h^*(\sigma_1) \dots h^*(\sigma_n) \\ &\times \exp \left(\frac{1}{2} \sum_{i,j=1}^n [2\tau_i \sigma_j \Delta_F(x_i - y_j) - \tau_i \tau_j \Delta_F(x_i - x_j) - \sigma_i \sigma_j \Delta_F(y_i - y_j)] \right) \end{aligned} \quad (16)$$

where G_0 is the corresponding Green's function of the free field.

For the vector coupling of the scalar meson theory, $F = g \phi$, g being a real constant. In this case, $h(t) = \delta(g-t)$ and then Eq. (16) reduces to the formula which has been derived previously ⁵⁾ by using a different method. Also, from Eq. (16) we can conclude that there is no scattering at all among fermions. We can prove this by using the same method described previously ⁵⁾. This conclusion can be easily generalized to include mesons. In this connection, it may be interesting to mention the existence of other theories of a similar nature, i.e. the ones having non-trivial Green's functions but which give no scattering at all. One such example has been treated by Bardakci and Sudarshan ⁶⁾, whose theory is the one obtained when we keep only finite terms in the so-called Haag expansion of the Heisenberg operator into the infinite series of the free operators. Furthermore, Araki et al. ⁷⁾ gave a theorem of a similar nature.

Finally, we would like to point out the meaning of Eq. (11). This can be interpreted as follows. In the original Lagrangian Eq. (1), we have chosen ψ and ϕ as the fundamental canonical variables. However, if we choose as the canonical variables ψ' and ϕ' such that $\phi' = \phi$, $\psi' = \exp(i F(\phi)) \psi$ from the beginning, then we can see that L reduces to L_0 in terms of the new variables ψ' and ϕ' . This is the situation contained in Eq. (11), if we identify ψ' and ϕ' with ψ_0 and ϕ_0 , respectively. In the conventional Lagrangian theory, there always exist such ambiguities with regard to the variables which are to be chosen as the canonical ones. In our case, two different choices of the canonical variables give two different Green's functions, although the physical content is the same, at least, on the surface. However, in some cases a particular choice like the present case will cause some trouble as will be seen in the following section.

3.

In this section, we first investigate two special cases:

(i) $F = g \phi$ and (ii) $F = g \phi^2$, corresponding to each case, the Fourier transform of Eq. (13) is given by

$$(i) \quad h(t) = \delta(x-g)$$

$$(ii) \quad h(t) = (4\pi|g|)^{-\frac{1}{2}} \exp\left[\frac{\pi}{4} \frac{g}{|g|} i - \frac{x^2}{4g} i\right]$$

Accordingly the Green's function can be computed from Eq. (15).

$$(i) \quad G(x, y) = S_F(x-y) \exp[g^2 \Delta_F(x-y) - g^2 \Delta_F(0)] \quad (17a)$$

$$(ii) \quad G(x, y) = S_F(x-y) [1 + 4g^2 \Delta_F^2(0) - 4g^2 \Delta_F^2(x-y)]^{-\frac{1}{2}} \quad (17b)$$

Therefore, the renormalization constant Z_2 and the renormalized charge g_r^2 are given respectively by

$$(i) \quad Z_2 = \exp[-g^2 \Delta_F(0)] \quad (18a)$$

$$(ii) \quad Z_2 = [1 + 4g^2 \Delta_F^2(0)]^{-\frac{1}{2}} \quad (18b)$$

$$(i) \quad g_r^2 = g^2 \quad (19a)$$

$$(ii) \quad g_r^2 = g^2 [1 + 4g^2 \Delta_F^2(0)]^{-1} \quad (19b)$$

The reason why $g_r^2 = g^2$ in the first case can be explained as due to the existence of the Ward identity. Finally, the renormalized Green's functions are given by

$$(i) \quad S_F'(x-y) = S_F(x-y) \exp [g_r^2 \Delta_F(x-y)] \quad (20a)$$

$$(ii) \quad S_F'(x-y) = S_F(x-y) [1 - 4 g_r^2 \Delta_F^2(x-y)]^{-\frac{1}{2}} \quad (20b)$$

where we have written $G_V(x, y) = S_F'(x-y)$.

We note that in the second case $F = g \phi^2$, Eq. (19b) has exactly the same form as in the Lee model⁸⁾ or the Landau model⁹⁾, and so generally we have a ghost-like situation. This can be best seen from Eq. (20b), since $S_F'(x-y) \cdot \gamma_4$ would become imaginary for small $(x-y)^2$, thus contradicting the positive definite character of $-G(x, x) \gamma_4$. In view of this difficulty, we are forced to introduce the indefinite metric in our theory. It is quite interesting to observe that Eq. (20b) is less singular on the light cone than the free field propagator $S_F(x-y)$, as we can see from Eq. (20b). The same situation does not apply for the first case Eq. (20a), which is exponentially singular on the light cone. In this case, if we make a formal analytic continuation of g_r^2 into $i g_r^2$, then Eq. (20a) exhibits an infinite oscillation near the light cone, which has some similarity to the suggestion given by Heisenberg¹⁰⁾. Such a situation has been already utilized for the renormalizability proof of a certain type of theories¹⁾ otherwise un-renormalizable in the usual sense.

Now let us investigate the validity of Lehmann's spectral representation²⁾ in this connection. For the second case, Eq. (20b) it does not hold in the sense that our theory does not admit a positive definite quantization as has been remarked already. For the first case, Eq. (20a) has an exponential singularity on the light cone, since

$$\Delta_F(x) \sim \frac{1}{4\pi^2} \frac{1}{x^2 + i\epsilon} \quad (x^2 \sim 0)$$

Thus Eq. (20a) has a singularity on the light cone, which increases exponentially inside the space-like region. So we may guess that Lehmann's spectral representation cannot hold. Actually, in this case, we will show that indeed, Lehmann's original proof fails. Lehmann²⁾ has proved that any two-body Green's function $S^{(1)}$ satisfies the following representations with the same ρ_1 and ρ_2 for all propagators:

$$S^{(1)}(x) = S_F^{(1)}(x, m) + \int_{(m+\mu)}^{\infty} dt [\rho_1(t)(\gamma \frac{\partial}{\partial x}) - \rho_2(t)] \Delta^{(1)}(x, t) \quad (21)$$

provided that the definitions of ρ_1 and ρ_2 by means of the following relation have meaning.

$$\begin{aligned} (i \partial k) \rho_1(\sqrt{-k^2}) - \rho_2(\sqrt{-k^2}) &= -(2\pi)^3 \sum_n (\Psi_0^* \psi_r(0) \Psi_n) (\Psi_n^* \bar{\psi}_r(0) \Psi_0) \delta^{(4)}(k - k_n) \\ &= -\frac{1}{2\pi} \int d^4k \, e^{-ik(x-y)} (\Psi_0^* \psi_r(x) \bar{\psi}_r(y) \Psi_0) \end{aligned}$$

$$t \rho_1(t) \geq \rho_2(t) \geq -t \rho_1(t) \quad (t > 0) \quad (22)$$

In Eq. (22) $\psi_r(x)$ is the renormalized operator of $\psi(x)$. Now the point is R.H.S. (right-hand side) of Eq. (22) does not exist at all in our case and hence ρ_1 and ρ_2 are meaningless. Thus, Lehmann's original proof fails at this point. The above mentioned statement can be proved by exactly calculating the integrand on the R.H.S. of Eq. (22). By using the method of the previous section, we can easily show that^{*)}

$$\begin{aligned} S^{(+)}(x-y) &= i (\Psi_0^* \psi_r(x) \bar{\psi}_r(y) \Psi_0) \\ &= S^{(+)}(x, m) \exp [i g^2 \Delta^{(+)}(x, \mu)] \end{aligned} \quad (23)$$

*) All propagators in this paper are the same as in reference 2) except for a factor 2 for the Δ_F and S_F functions.

where we have written the S and Δ functions explicitly as functions of m and μ , the nucleon and meson masses respectively. Now Eq. (23) has an exponentially divergent singularity on the light cone in the space-like region $x^2 > 0$, since

$$\Delta^{(+)}(x) \sim -\frac{i}{4\pi^2} \frac{1}{x^2 - (x_0 - i\epsilon)^2} \quad (x^2 \rightarrow 0)$$

This shows that the R.H.S. of Eq. (22) is meaningless, and so the Lehmann's argument fails. Of course, this does not exclude a possibility that S'' function still could satisfy the relations like Eq. (21). However, if this were the case, ρ_1 and ρ_2 in Eq. (21) have to be different for different propagators. This can be shown as follows: Consider S' function defined by

$$S'(x-y) = i(\Psi_0^* (\Psi_\gamma(x) \bar{\Psi}_\gamma(y) + \bar{\Psi}_\gamma(y) \Psi_\gamma(x)) \Psi_0)$$

By using the technique of the previous section, we find

$$\begin{aligned} S'(x) = & S^{(+)}(x, m) \exp [i g^2 \Delta^{(+)}(x, \mu)] \\ & + S^{(-)}(x, m) \exp [-i g^2 \Delta^{(-)}(x, \mu)] \end{aligned} \quad (24)$$

S' function behaves nicely near the light cone, in contrast to the S_F' function. This is due to the fact that S' function is zero in the space-like region because of the causality. Notice that for the time-like region $x^2 < 0$, $\Delta(x, t)$ can be written as

$$\Delta(x, t) = \frac{t}{4\pi\sqrt{-x^2}} \varepsilon(x_0) J_1(t\sqrt{-x^2})$$

Then, Eq. (21) for S' function is equivalent to the Fourier-Bessel transform ¹¹⁾ of the $S'(x)$ function, so that the weight function ρ_1 and ρ_2 can be computed by the inverse Fourier-Bessel formula ¹¹⁾. In this way, we get

$$x \rho_1(x) = \frac{m^2}{2} \int_0^\infty ds s J_2(sx) \left\{ H_2^{(2)}(ms) \left[\exp\left(\frac{ig^2}{8\pi} \left(\frac{\mu}{s}\right) H_1^{(2)}(\mu s)\right) - 1 \right] + H_2^{(1)}(ms) \left[\exp\left(\frac{ig^2}{8\pi} \left(\frac{\mu}{s}\right) H_1^{(1)}(\mu s)\right) - 1 \right] \right\}$$

$$\rho_2(x) = \frac{m^2}{2} \int_0^\infty ds s J_1(sx) \left\{ H_1^{(2)}(ms) \left[\exp\left(\frac{ig^2}{8\pi} \frac{\mu}{s} H_1^{(2)}(\mu s)\right) - 1 \right] + H_1^{(1)}(ms) \left[\exp\left(\frac{ig^2}{8\pi} \frac{\mu}{s} H_1^{(1)}(\mu s)\right) - 1 \right] \right\}$$

From this we find that for $t \rightarrow \infty$

$$\rho_1(t) \sim o(t^{-\frac{3}{2}}), \quad \rho_2(t) \sim o(t^{-\frac{1}{2}})$$

Then we can compute the right hand side of Eq. (21) in case of S'_F function. It is easy to show that then it must have a singularity of at most $(x^2)^{-2}$ near $x^2 \rightarrow +0$, whereas Eq. (20a) behaves like $\exp\left[\frac{g^2}{4\pi}(x^2)^{-1}\right]$. This is a contradiction and therefore, if Eq. (21) is true, then ρ_1 and ρ_2 must be different for different propagators - a situation which is different from the Lehmann's case. Actually, it is quite unlikely that S'_F function will ever be written in the form of Eq. (21). By using a similar argument, we can show that if $\int_0^\infty dx |\rho_{1,2}(x)| x^{-n}$ exists for some positive integer n , then the R.H.S. of Eq. (21) have to have a singularity of at most $(x^2)^{-n}$ at $x^2 \rightarrow +0$, which is again a contradiction with Eq. (20a)

Finally, we consider one more singular case.

$$(iii) \quad F(\phi) = g \log |\phi| = \frac{1}{2} g \log (\phi^2)$$

In this case, we can still calculate Green's function from Eq. (15) to get

$$G(x, y) = \frac{1}{\cosh(\frac{\pi}{2}g)} \cosh(g \sin^{-1} \frac{\Delta_F(x-y)}{\Delta_F(0)}) \quad (25)$$

It is now no longer possible to separate the divergent constants into a multiplicative renormalization factor, but the renormalization constant z_2 can still be defined by

$$G(x, y) \rightarrow z_2 S_F(x-y) \quad (x-y)^2 \rightarrow \infty$$

Then we have

$$z_2 = \frac{1}{\cosh(\frac{\pi}{2}g)}$$

It is remarkable that z_2 is finite now and satisfies $0 < z_2 < 1$ as the previous cases.

4.

In this last section we discuss briefly the connection of our theory with the so-called Haag theorem¹²⁾. Since we have introduced the Heisenberg and interaction representation at $t = 0$ the canonical variables $\psi(x)$, $\phi(x)$, $\pi(x)$ of the former agrees with $\psi_0(x)$, $\phi_0(x)$, $\pi_0(x)$ of the latter at the time $t = 0$. Or more generally, at an arbitrary time t , we can find an unitary operator V_t such that

$$V_t^{-1} \psi(x, t) V_t = \psi_0(x, t)$$

$$V_t^{-1} \phi(x, t) V_t = \phi_0(x, t)$$

$$V_t^{-1} \pi(x, t) V_t = \pi_0(x, t)$$

where V_t is explicitly given by

$$V_t = U \cdot \exp \left[-\frac{i}{2} \int d^3x [\psi_0^*(x, t) \psi_0(x, t) - \psi_0(x, t) \psi_0^*(x, t)] F(\phi_0(x, t)) \right]$$

Therefore, according to the first part of the Haag theorem¹²⁾, the two vacuum states Ψ_0 and Φ_0 must agree (notice that at $t = 0$, $V_t = 1$). Therefore, according to Eq. (9), we must have $U = 1$, i.e. $F = 0$. Then the second part of the Haag theorem^{12), 13)} is automatic, i.e. our Green's functions must be equivalent to those of the free fields in contrast to Eqs. (15) and (16). This situation suggests that the operator U is not a proper unitary operator in a given Hilbert space, but a transformation which carries one Hilbert space into another inequivalent one^{14), 15)}. This can be illustrated also as follows. Define

$$U(\sigma) = \exp \left[\frac{i}{2} \int_{\sigma} d\sigma_{\mu} (\bar{\psi} \gamma_{\mu} \psi - \psi \gamma_{\mu}^{\dagger} \bar{\psi}) F(\phi) \right] \quad (25)$$

where σ is a space-like surface and $U(\sigma)$ is obviously covariant. Then Eq. (9) can be re-written as

$$\Psi_n = U(\sigma) \Phi_n \quad (27)$$

Now, since our theory is relativistic invariant, we can choose any reference system for the quantization from the beginning. This means that for a Lorentz transformation including translations, Eq. (21) must change as (note that Ψ_n and Φ_n have the same energy and momentum)

$$\Psi_{\tilde{n}} = U(\tilde{\sigma}) \Phi_{\tilde{n}} \quad (28)$$

where $\tilde{n}$ and $\tilde{\sigma}$ are the quantities after Lorentz transformation of the state n and the space-like surface σ , respectively. However, the states n cover all possible states and in particular the states $\tilde{n}$. Therefore, Eq. (27) must still hold after substituting $n \rightarrow \tilde{n}$. Thus we have

$$\Psi_{\tilde{n}} = U(\sigma) \Phi_{\tilde{n}}$$

comparing this with Eq. (28), we conclude that $U(\tilde{\sigma}) = U(\sigma)$

This means that $U(\sigma)$ is independent of the space-like surface σ . Then note that for any two space-like surface σ_1 and σ_2 we have

$$i \int_{\sigma_2}^{\sigma_1} d^4x (\bar{\Psi} \gamma_\mu \Psi - 4 \gamma_\mu^T \bar{\Psi}) \frac{\partial F(\Phi)}{\partial x_\mu} = \left(\int_{\sigma_1} - \int_{\sigma_2} \right) d\sigma_\mu (\bar{\Psi} \gamma_\mu \Psi - 4 \gamma_\mu^T \bar{\Psi}) F(\Phi) \equiv 0$$

where we have used the fact that $U(\sigma_1) = U(\sigma_2)$. This result is reconcilable only if $F = 0$. (since $F = \text{const.}$ is trivial) i.e. there is no interaction again. This means that by Lorentz transformation Ψ_n does not go to $\Psi_{\tilde{n}}$ of the same Hilbert space when $F \neq 0$, provided that Φ_n does go to $\Phi_{\tilde{n}}$. Thus in a sense, Ψ_0 is not invariant under a Lorentz transformation, and the Lorentz transformation in general (always except for the pure space rotation and translations) brings one Hilbert space into another inequivalent Hilbert space. This is the reason why we can have Green's functions differing from the free field ones against the Haag theorem.

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