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 $\overline{p}p$ annihilation
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Consequences of a Strong π - π Interaction in the Statistical Theory of p - $\bar{p}$ Annihilation.

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Summary. — The Fermi statistical model for multiple particle production is applied to the p - $\bar{p}$ annihilation process; calculations are performed for three assumptions on the π - π interaction, *viz.* *a*) a strong resonance in the state $l=1$, $T=1$ with total energy $4m_\pi$; *b*) the same with total energy $3m_\pi$; *c*) no strong interactions. Multiplicities and spectra for 2, 4 and 6-prong stars are computed. The π - π isobar model *a*) agrees well with experiment, taking an interaction volume $\Omega_0 = (\hbar/m_\pi c)^3 \cdot 4\pi/3$. The 2-prong spectrum shows a hump around 760 MeV kinetic energy.

1. — Introduction.

Most of the experiments on p - $\bar{p}$ annihilation ⁽¹⁻⁴⁾ have been compared with the Fermi theory of multiple production, and it has been found that all measured quantities were in fair agreement with this theory, provided the interaction volume was taken to be $10 \Omega_0$, where

$$\Omega_0 = \frac{4\pi}{3} \left(\frac{\hbar}{m_\pi c} \right)^3.$$

⁽¹⁾ ANTIPROTON COLLABORATION EXPERIMENT: *Phys. Rev.*, **105**, 1037 (1957).

⁽²⁾ O. CHAMBERLAIN G. GOLDHABER, L. JANNEAU, T. KALOGEROPOULOS, E. SEGRÉ, R. SILBERBERG: *Phys. Rev.*, **113**, 1615 (1959).

⁽³⁾ E. KALOGEROPOULOS: UCRL 8677.

⁽⁴⁾ N. HORWITZ, D. MILLER, J. MURRAY and R. TRIPP: UCRL 8591.

This is, of course, very unsatisfactory, all the more so because the Fermi-theory gives quite good results in the meson production from p-p collisions, at various energies ^(5,7), with an interaction volume $\Omega = \Omega_0$.

It was therefore proposed ^(8,9) to incorporate into the statistical theory of annihilation a presumed pion-pion resonant state. This improved the result.

We shall show that if one calculates correctly all phase-space integrals, and takes all contributions, to the final state of n_π pions and n_K K-mesons into account, the agreement with experiment is excellent; an important difference with the usual Fermi-theory appears, which seems to be in favour of the isobar-model and could be settled with improved statistics for the 2-prong events.

2. - Outline of calculations.

We take the probability for a certain end state, labelled b , of a p- $\bar{p}$ annihilation to be proportional to

$$S_b = \Omega_0^{n_\pi + n_\sigma} \Omega_K^{n_K} \frac{(2l+1)^{n_\sigma}}{n_\pi! n_\sigma!} (P_b^{(1)} + P_b^{(0)}) e_{n_\pi, n_\sigma, n_K}(E, 0),$$

where:

Ω_0 = interaction volume with radius of pion Compton wavelength,

Ω_K = interaction volume with a smaller radius, to be discussed hereafter,

n_π, n_K : numbers of pions and K-mesons in the state b ,

n_σ : number of π - π isobars, l their angular momentum,

$P_b^{(1)}$ and $P_b^{(2)}$ give the probability of different charge states in total isospin states 0 and 1. These coefficients are normalized so that the sum over the charge states of the n_π pions, the n_σ π - π isobars and the n_K K-mesons gives the usual statistical factor $W_{\alpha, \beta}$, as defined by YEIVIN-DE SHALIT ⁽¹⁰⁾ and BARAŠENKOV-BARBAŠEV ⁽¹¹⁾,

α : number of isospin $\frac{1}{2}$ particles,

β : number of isospin 1 particles,

⁽⁵⁾ F. CERULUS and R. HAGEDORN: CERN Report 59-3.

⁽⁶⁾ F. CERULUS and R. HAGEDORN: *Meson production at 6.2 GeV*, to be published.

⁽⁷⁾ S. Z. BELENKI *et al.*: *Usp. Fiz. Nauk*, **62** (2), 1 (1957).

⁽⁸⁾ E. EBERLE: *Nuovo Cimento*, **8**, 610 (1958).

⁽⁹⁾ T. GOTÔ: *Nuovo Cimento*, **8**, 625 (1958).

⁽¹⁰⁾ Y. YEIVIN and A. DE-SHALIT: *Nuovo Cimento*, **1**, 1147 (1955).

⁽¹¹⁾ V. S. BARAŠENKOV and B. M. BARBAŠEV: *Suppl. Nuovo Cimento*, **7**, 19 (1957).

$\varrho_{n_\pi, n_\sigma, n_K}(E; 0)$ is the integral over momentum space:

$$\varrho_{n_\pi, n_\sigma, n_K}(E, p) = \int \dots \int d\mathbf{q} \dots d\mathbf{q}_N \delta(E - \sum_{i=1}^N \sqrt{m_i^2 + q_i^2}) \delta(\mathbf{p} - \sum_{i=1}^N \mathbf{q}_i),$$

with

$$m_1, \dots, m_{n_\pi} = \mu \quad E = \text{total energy,}$$

$$m_{n_\pi+1}, \dots, m_{n_\pi+n_\sigma} = m_\sigma, \quad p = \text{total momentum,}$$

$$m_{n_\pi+n_\sigma+1}, \dots, m_{n_\pi+n_\sigma+n_K} = m_K \quad N = n_\pi + n_\sigma + n_K.$$

In the following investigation we took the π - π isobar to have $T=1$, $l=1$, in accordance with recent speculations on the electromagnetic form factor of the nucleon ⁽¹²⁾. The calculations were done for two masses of the isobar: $m_\sigma = 3\mu$ and $m_\sigma = 4\mu$. According to ref. ⁽¹²⁾ the 4μ isobar should give the best fit.

The $\varrho_{n_\pi, n_\sigma, n_K}(E; 0)$ were computed on the CERN Ferranti «Mercury» computer, by a method described previously ⁽⁵⁾ using a programme written by R. HAGEDORN.

The $P_b^{(0)}$, $P_b^{(1)}$ have been calculated for a large number of end states, and will shortly be published.

The method used ⁽¹³⁾ allows not only the calculation of the multiplicities, but also the spectra of each kind of particle, for each reaction considered.

We shall call then

$$S_{b,j}(\varepsilon_j) d\varepsilon_j = \text{probability that in reaction } b, \text{ particle } j \text{ has a kinetic energy between } \varepsilon_j \text{ and } \varepsilon_j + d\varepsilon_j.$$

The normalization is such that

$$\int_0^{\varepsilon_j \max} S_{b,j}(\varepsilon_j) d\varepsilon_j = S_b.$$

The multiplicities of pions, e.g. are computed from the S_b by summing over all reactions yielding the same number of pions, and dividing by the sum over all S_b .

Soon after the annihilation, the isobar decays into two pions, and these pions have quite a different spectrum from directly produced ones. The com-

⁽¹²⁾ W. FRAZER and J. FULCO: *Phys. Rev. Lett.*, **2**, 365 (1959).

⁽¹³⁾ F. CERULUS and R. HAGEDORN: *Suppl. Nuovo Cimento*, **9**, 646, 659 (1958).

putation of the decay from the isobar spectrum is part of the programme of the computer.

To compare with experiment, it is useful to compute the spectra of pions produced in two, four and six-prong events. This can be done by selecting those b with 2, 4 or 6 charged pions, and adding the spectra for each reaction. This adding, together with the computation of mean energies and mean particle numbers can again be performed on the computer, following another programme of R. HAGEDORN (*).

3. - Results.

We considered all reactions as taking place from $p\bar{p}$ annihilation at rest; *i.e.*, we took

$$E = 2m_N c^2 \quad \text{and} \quad p = 0.$$

We compared in each case with the results of the Fermi model without isobar, computed by taking $\Omega_\pi = 10 \Omega$.

For Ω_K we took various values, from Ω_0 to $(1/30) \Omega_0$. Variation of Ω_K leaves the form of all spectra unchanged, it has only a slight influence on the pion number (except in the no-isobar hypothesis).

The reactions considered were

$$\begin{aligned} p + \bar{p} &\rightarrow 2, 3, \dots, 7 \text{ pions,} \\ &\rightarrow \sigma + \pi, \sigma + 2\pi, \dots, \sigma + 5\pi, \\ &\rightarrow 2\sigma, 2\sigma + \pi, \dots, 2\sigma + 4\pi, \\ &\rightarrow 3\sigma, 3\sigma + \pi, \\ &\rightarrow K + \bar{K}, K + \bar{K} + \pi, \dots, K + \bar{K} + 3\pi, \\ &\rightarrow K + \bar{K} + \sigma, K + \bar{K} + \sigma + \pi. \end{aligned}$$

It is easy to see from a plot of the weights of these reactions that all the others, with higher particle numbers, would contribute almost nothing, because of their vanishing probability.

(*) An extensive description of the two programmes used is available from CERN upon request.

a) *Multiplicities.* — Table I is computed by counting only the prongs in events without K-mesons.

TABLE I. — Probability in % of events with different charged pion numbers.

Ω_π :	Ω_0		$2\Omega_0$		$\frac{1}{2}\Omega_0$		no isobar $\Omega_\pi = 10\Omega_0$		
m_σ :	3 μ	4 μ	3 μ	4 μ	3 μ	4 μ	$\Omega_K = \Omega_\pi$	$\Omega_K = \frac{1}{10}\Omega_\pi$	$\Omega_K = \frac{1}{100}\Omega_\pi$
2 prongs	26.7	45.35	21.0	37.9	34.9	54.0	27.5	35.0	35.0
4 prongs	65.0	51.5	69.0	58.2	59.0	42.75	43.7	55.5	56.0
6 prongs	7.5	2.1	9.4	3.1	5.6	1.4	5.7	7.3	7.3

TABLE II. — Average number of pions.

Ω_π :	Ω_0		$2\Omega_0$		$\frac{1}{2}\Omega_0$		no isobar $\Omega_\pi = 10\Omega_0$		
m_σ :	3 μ	4 μ	3 μ	4 μ	3 μ	4 μ	$\Omega_K = \Omega_\pi$	$\Omega_K = \frac{1}{10}\Omega_\pi$	$\Omega_K = \frac{1}{100}\Omega_\pi$
$\bar{n}_{\pi^\pm}$	3.6	3.1	3.75	3.3	3.4	2.9	3.1	3.4	3.4
$\bar{n}_{\pi^0}$	1.7	1.55	1.8	1.6	1.6	1.5	1.35	1.7	1.7
$\bar{n}_\pi$	5.3	4.65	5.6	4.9	5.0	4.4	4.4	5.1	5.1

TABLE III. — Average number of K-mesons per p- $\bar{p}$ annihilation.

isobar; $\Omega_\pi = \Omega_0$			no isobar; $\Omega_\pi = 10\Omega_0$	
Ω_K/Ω_0	$\bar{n}_K$	m_σ	Ω_K/Ω_0	$\bar{n}_K$
1	(not com- puted)	0.30	10	0.43
0.4	0.0019	0.051	4	0.084
0.1	—	0.0034	1	0.0055
0.04	—	0.00052	0.4	0.00088
			0.1	$5.5 \cdot 10^{-5}$

TABLE IV. — Average energy of prongs (rest energy of 140 MeV included) (in MeV).

m_σ	isobar case; $\Omega_\pi = \Omega_0$		no isobar; $\Omega_\pi = 10\Omega_0$
	3 μ	4 μ	
0 prongs	640	640	435
2 prongs	430	472	419
4 prongs	335	372	366
6 prongs	294	310	299
average $\pi^\pm$	345	399	371
average π^0	355	410	371

b) *Spectra.* — The spectra of pions from 2, 4 and 6 prong events are displayed in Figs. 1, 2, 3. All the curves are normalized to unit area.

4. - Discussion of results.

Comparison with the published experimental data ⁽³⁾ shows very good agreement with the isobar model on the whole. Let us consider this in detail:

1) *Multiplicities of pions.* The multiplicities are more sensitive to a variation of the isobar mass than to a variation of the interaction volume. If the isobar idea is right, the mass m_σ should be closer to 4μ than to 3μ . This would be in agreement with the average number observed ⁽¹⁴⁾, and will probably yield the right percentage for the two and four prong events. One should watch out, with better statistics, for the probability of 6 prongs events, which are rather different for the various hypotheses.

The no-isobar model, with small Ω_K , comes fairly close to the experimental numbers too, which is well-known, although with very large statistics the differences might become significant.

2) *Multiplicities of K-mesons.* - It has been recognized by various authors that the most straightforward way to correct the too large proportion of K-mesons which statistical theories of meson production tend to give, is to take for K-mesons a smaller interaction volume. There is, in our opinion, no compelling reason to take for it a sphere with the K-Compton wavelength as radius. The smallness of Ω_K is probably only a crude way to express the relative weakness of K-meson interactions, which do not allow statistical equilibrium to be reached during the collision time. If one considers the interaction volume as a substitute for the transition matrix element between the initial $p\bar{p}$ state, and the final state b , and if one takes from other theories the conclusion that the K-meson interaction strength is about 1/10 of pion-nucleon interaction strength, then it seems reasonable to look for an Ω_K in the neighbourhood of 1/10 Ω_π .

Applying this idea to meson production in p - p collisions at Bevatron energies, it was found ⁽⁵⁾ that this was indeed the magnitude of Ω_K that gave a.o. a ratio $K/\bar{K}$ compatible with experiment.

We want to emphasize, however, that the details of K-meson production are quite irrelevant for the problem of the validity of a statistical theory of annihilation. It may be that such a theory is unjustified for K-mesons, whereas it is entirely right for the strongly interacting pions, which experimentally make up the bulk of the annihilation products.

⁽¹⁴⁾ E. AMALDI: private communication.

3) *Spectra and average energies of charged pions in 2, 4 and 6 prong events.* The form of the spectra, and the average energy, are quite insensitive against reasonable variations of Ω_π , and only moderately sensitive against variations of the isobar mass.

Again there is only a small difference on the whole between the spectra from the 4μ isobar model and those from the no-isobar model. Both seem to fit experiments within the limits of statistics.

There is, however, an important feature in the 2 prong spectrum for the isobar model which is entirely absent in the «no-isobar» spectrum *viz.* the peak at the end of the spectrum, around 800 Mc^2 kinetic energy. This is due to the two-body reaction

$$p + \bar{p} \rightarrow \sigma + \pi,$$

where the directly produced pion has a sharp energy. The width of the bump is—for $p\text{-}\bar{p}$ collisions truly from rest—given by the width of the $\pi\text{-}\pi$ resonance. There is a second, lower bump at a somewhat higher energy, due to

$$p + \bar{p} \rightarrow \pi + \pi.$$

The two together should contain about 5% of all the pions from 2 prong events.

In the bubble chamber data ⁽³⁾ there seems indeed to be a bump in the energy histogram at the right place. This might still be a freak of statistics, and further information on this point is very desirable.

There is no way of getting something similar with the no-isobar model, all the more as the two pion annihilation, in that model, has a probability less than 10^{-3} .

If such a bump is found experimentally, it is conclusive evidence for a strong $\pi\text{-}\pi$ interaction, such as is described by an isobar in a statistical theory.

5. - Conclusion.

As conclusion, we can state that a Fermi-type theory of annihilation, with the assumptions of a $\pi\text{-}\pi$ interaction and a normal interaction volume can account in detail for all observed facts, using values of the parameters ($\Omega_\pi = \Omega_0$, $\Omega_K \approx \frac{1}{10} \Omega_0$, $m_\sigma = 4\mu$) which are taken from other applications of the Fermi-theory, or from the theory of the electromagnetic form factor of the nucleon. In the 2 prong spectrum the theory predicts a small bump around 780 Mc^2 (kinetic pion energy). Other tests of the isobar model, based on cor-

relations between annihilation products might be devised. This is being investigated.

* * *

The author wishes to thank R. HAGEDORN for the use of his programmes and for numerous discussions and S. FUBINI for discussions on the π - π interaction.

RIASSUNTO (*)

Il modello statistico di Fermi per la produzione di particelle multiple viene applicato al processo di annichilazione $p\bar{p}$; i calcoli sono effettuati partendo da tre supposizioni circa la interazione π - π , cioè: a) forte risonanza nello stato $l=1$, $T=1$ con energia totale $4m_\pi$; b) lo stesso con energia totale $3m_\pi$; c) nessuna interazione forte. Si computano le molteplicità e gli spettri di stelle a 2, 4 e 6 rami. Il modello π - π isobaro a) si accorda bene agli esperimenti, assunto un volume di interazione $\Omega_0 = (\hbar/m_\pi c)^3 \cdot 4\pi/3$. Lo spettro delle stelle a 2 rami presenta una gobba verso 760 MeV di energia cinetica.

(*) Traduzione a cura della Redazione.