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Work done under auspices of the U. S. Atomic Energy Commission.

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μ -MESON DECAY

C. Fronsdal

CERN, Geneva, Switzerland

and

S. L. Glashow*

Institute for Theoretical Physics, Copenhagen, Denmark, and CERN, Geneva, Switzerland

(Received November 18, 1959)

It was originally suggested by Yukawa¹ that the muon decay may be mediated by an unstable boson with weak couplings to lepton fields. More recently,^{2,3} a charged, spin-one boson has been proposed in order to generate the $V-A$ form of the weak decay interactions. It has already been pointed out⁴ that a charged scalar intermediary can give rise to the desired couplings, providing an alternative but generally equivalent theory of the weak interactions. This applies when the first-order Kemmer formalism for spinless bosons is employed, but not with the use of the more conventional Klein-Gordon second-order theory.

The field equations in the Kemmer theory may be written

$$\varphi_{\lambda} = m^{-1} \partial_{\lambda} \varphi + m^{-2} (g_1^{(\mu)} J_{\lambda}^{(\mu)} + g_1^{(e)} J_{\lambda}^{(e)}),$$

$$\varphi = m^{-1} \partial_{\lambda} \varphi_{\lambda} + m^{-2} (g_2^{(\mu)} J^{(\mu)} + g_2^{(e)} J^{(e)}), \quad (1)$$

when both scalar and vector interactions are present, where

$$J_{\lambda}^{(e)} = \bar{\psi}_{(e)} \gamma_{\lambda} \frac{1-\gamma_5}{2} \psi_{(\nu)} + \text{c.c.},$$

$$J^{(e)} = \bar{\psi}_{(e)} \frac{1-\gamma_5}{2} \psi_{(\nu)} + \text{c.c.}$$

Only in the absence of any vector interaction is φ_{μ} simply proportional to $\partial_{\mu} \varphi$. The induced normal muon decay interaction, to lowest order in $g_1^{(e)}$ and $g_2^{(e)}$, is

$$\begin{aligned} & m^{-2} g_1^{(\mu)} g_1^{(e)} \int d^4 x J_{\lambda}^{(e)}(x) J_{\lambda}^{(\mu)}(x) \\ & + \left(g_2^{(\mu)} + g_1^{(\mu)} \frac{m^{(\mu)}}{m} \right) \left(g_2^{(e)} + g_1^{(e)} \frac{m^{(e)}}{m} \right) \\ & \times \int d^4 x d^4 x' J^{(e)}(x) D_F(x-x', m) J^{(\mu)}(x'). \end{aligned} \quad (2)$$

The first term is a purely local $V-A$ interaction, the second a nonlocal $S-P$ interaction. If $g_2^{(e)} = 0$, the scalar admixture for muon decay becomes $m^{(\mu)} m^{(e)} / m^2$ —less than 10^{-4} for nucleonic mass of the intermediary.⁵ Alternatively, if either or both of the factors $[g_2^{(\mu)} + g_1^{(\mu)} (m^{(\mu)} / m)]$ and $[g_2^{(e)} + g_1^{(e)} (m^{(e)} / m)]$ are set equal to zero, the normal μ -decay interaction (to this order in g) becomes a purely local one.

Using the Klein-Gordon formalism, we obtain

a muon decay interaction identical to Eq. (2) except for the absence of the direct vector coupling. The two versions of the spin-zero theory are identical for $g_1^{(e)} = 0$, and in general differ only by a direct self-interaction of $J_\lambda^{(e)}$. The choice between these formalisms depends upon whether one regards first-order or second-order field

equations as the more basic. [It is only the Kemmer theory that may be developed according to the canonical formulation of Schwinger.⁶]

For the discussion of $\mu \rightarrow e + \gamma$, we shall assume $g_2^{(e)} = 0$ and neglect finite contributions proportional to the electron mass. The matrix element for this mode becomes (q = photon momentum)

$$\frac{e}{96\pi^2} \frac{g_1^{(\mu)} g_1^{(e)}}{m^2} \bar{\psi}_{(e)} \frac{1-\gamma_5}{2} \left\{ \frac{1}{2} m^{(\mu)} \sigma_{\alpha\beta} F^{\alpha\beta} - \gamma_\mu A_\mu q^2 \left[\ln \left(\frac{K^2}{m^2} \right) + c + O \left(\frac{q^2}{m^2} \right) \right] \right\} \psi_{(\mu)},$$

where K is a cutoff to the divergent momentum integration, and c is a finite constant which depends upon the manner in which the cutoff is introduced. The decay interaction contains a finite, point-like, "moment" interaction, and a divergent "charge" interaction which does not contribute to the free-particle decay. The branching ratio for the decay of a free muon into the $e + \gamma$ channel is

$$\rho = \alpha/24\pi \approx 10^{-4},$$

which is ~ 40 times greater than the current experimental upper limit.⁷ In contrast to the result with a vector intermediary,⁸⁻¹⁰ the result is both convergent and unambiguous.

We should like to thank Professor C. J. Bakker

and Professor M. Fierz for the hospitality extended to us at CERN.

*National Science Foundation Post-Doctoral Fellow.

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SOME GRAVITATIONAL WAVES*

Asher Peres

Department of Physics, Israel Institute of Technology, Haifa, Israel

(Received October 19, 1959)

The few hitherto known wave-like exact solutions of the Einstein gravitational equations represented either plane^{1,2} or cylindrical³ waves. We here intend to derive a new class of solutions, displaying a lesser degree of symmetry, and thus more generality.

Let us consider the metric

$$ds^2 = -dx^2 - dy^2 - dz^2 + dt^2 - 2f(x, y, z+t)(dz + dt)^2.$$

One can easily show that $R_{zz} = R_{zt} = R_{tt} = f_{xx} + f_{yy}$, and other components vanish. If f is a harmonic function of x and y , this metric thus satisfies Einstein's equations in vacuo, whatever may be its dependence on $(z+t)$.

A convenient tetrad of orthonormal vectors is

$$h_{(1)}^k = (\cos a, \sin a, 0, 0),$$

$$h_{(2)}^k = (-\sin a, \cos a, 0, 0),$$

$$h_{(3)}^k = (0, 0, 1-f, f),$$

$$h_{(4)}^k = (0, 0, -f, 1+f),$$

where $\tan(2a) = (f_{xy}/f_{xx})$. The only nonvanishing independent physical components⁴ of the curvature tensor are

$$\sigma = R_{(m1n1)} = -R_{(m2n2)} = (f_{xx}^2 + f_{yy}^2)^{1/2},$$

where m and n take the values 3 and 4 only. Our