Dispersion Analysis of Possible Parity Nonconservation in Low-Energy Pion-Nucleon Scattering

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An attempt is made to analyze the possible parity nonconservation in low-energy pion-nucleon scattering. The use of relativistic dispersion relations enables us to relate such effects to the possible parity nonconservation in strange-particle production from pion-nucleon collisions. We show that large violations of parity conservation in strange-particle production are indeed compatible with small effects in low-energy pion-nucleon scattering. It is suggested that our result might be useful in order to understand the very good evidence for parity conservation in nuclear physics.

I. INTRODUCTION

THE question of parity conservation in strong interactions is still an open one both from the theoretical and experimental¹ point of view. We have no longer any fundamental theoretical reason for believing in parity conservation in any interaction. On the other hand there is a strong experimental evidence² in low-energy nuclear physics where parity conservation has been tested within a factor 10^{-3} in amplitude. A way of understanding such a situation without explicitly assuming parity conservation was recently found. It was shown³⁻⁵ that for pion and electromagnetic interactions, charge symmetry and gauge invariance are enough to insure parity conservation. Those arguments do not apply to interactions involving strange particles and it is in effects involving such particles that the most significant violations of parity conservation are to be expected. So the problem arises whether such parity-nonconserving strange-particle interactions could be reconciled with the actual experimental evidence in low-energy nuclear physics and pion physics.

In this paper we shall try to study the possible violation of parity conservation in low-energy pion-nucleon scattering using the method of dispersion relations. The central point in our approach is that, under the assumption of time-reversal invariance and charge symmetry, there is no pion-nucleon renormalized coupling constant for parity violation. This means that if the strange-particle contributions are neglected, our dispersion relations yield a solution where there is no violation of parity conservation in pion-nucleon scattering. The possible parity nonconservation in the pion-nucleon interaction is therefore related to the amplitudes for strange-particle production.

Our dispersion relations will relate the parity noncon-

servation in low-energy pion-nucleon scattering to that of the process

$$\pi + N \rightarrow K + Y + \text{pions}.$$

Since no parity nonconservation in this reaction has yet been directly observed, we shall only be able to give an upper bound to such effects by using an optical theorem.

We shall show that in any case, the parity violation in pion-nucleon scattering must be expected to be very small. Our results confirm the view that large violation of parity conservation in strange-particle production is indeed compatible with small effects in low-energy pion physics and therefore, probably also in low-energy nuclear physics.

II. KINEMATICS

We use the same approach and notation as that of Chew *et al.*⁶ Many of the formulas of reference 6 are repeated for completeness.

1

The four-vector momenta of the incident and outgoing pion are q_1 and q_2 , respectively, while those of the nucleon are p_1 and p_2 . Conservation of energy-momentum gives

$$p_1 + q_1 = p_2 + q_2. \quad (2.1)$$

Using the mass-shell restrictions on these vectors we are left with two independent scalars which we choose as

$$\nu = -P \cdot Q / M, \quad K^2 = \frac{1}{4}(q_1 - q_2)^2, \quad (2.2)$$

where

$$P = \frac{1}{2}(p_1 + p_2), \quad Q = \frac{1}{2}(q_1 + q_2).$$

We also find the following relations holding:

$$K^2 = \frac{1}{2}q^2(1 - \cos\theta), \quad (2.3)$$

$$\nu = \nu_L - (K^2/M), \quad (2.4)$$

$$\nu_L = (W^2 - M^2 - \mu^2)/2M, \quad (2.5)$$

$$q^2 = M^2(\nu_L^2 - \mu^2)/(M^2 + \mu^2 + 2M\nu_L), \quad (2.6)$$

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¹ Blumenfeld, Chinowsky, and Lederman, *Nuovo cimento* **8**, 296 (1958).

² D. H. Wilkinson, *Phys. Rev.* **109**, 1603, 1610, 1614 (1958).

³ V. G. Soloviev, *Nuclear Phys.* **6**, 618 (1958).

⁴ Drell, Frautschi, and Lockett, Stanford University Physics Department, Technical Report No. 30, June, 1958 (unpublished).

⁵ G. Morpurgo and B. F. Tauschek (unpublished). The generalization to the physical vertex function given in the text is due to W. Thirring.

⁶ Chew, Goldberger, Low, and Nambu, *Phys. Rev.* **106**, 1337 and 1345 (1957).

where $\mathbf{q}^2$, $\cos\theta$, and W are the momentum, angle of scattering, and total energy in the center-of-mass system and ν_L is the incident meson energy in the laboratory system.

2

The T -matrix is defined by

$$S = \delta_{fi} - (2\pi)^4 i \delta(p_2 + q_2 - p_1 - q_1) \times \left(\frac{M^2}{4E_1 E_2 \omega_1 \omega_2} \right)^{\frac{1}{2}} \bar{u}_2 T u_1, \quad (2.7)$$

where E_1 and E_2 are the initial and final energies of the nucleon and ω_1 and ω_2 are the corresponding quantities for the meson. Our spinors are normalized to

$$\bar{u}_2 u_2 = \bar{u}_1 u_1 = 1,$$

T is the Lorentz-invariant function and may be written

$$T = -A(\nu, K^2) + i\gamma \cdot Q B(\nu, K^2) + i\gamma \cdot Q \gamma_5 C(\nu, K^2) - \gamma_5 D(\nu, K^2). \quad (2.8)$$

These are the only invariants involving γ matrices which remain as independent scalars since any other terms can be reduced with the help of the Dirac equation

$$(i\gamma \cdot p_1 + M)u_1 = 0, \quad \bar{u}_2(i\gamma \cdot p_2 + M) = 0. \quad (2.9)$$

The two terms with γ_5 are permitted since our theory no longer conserves parity.

3

In order to simplify our calculations we shall assume charge independence, which means we can write

$$A_{\beta\alpha} = A^+(\nu, K^2)\delta_{\beta\alpha} + A^-(\nu, K^2)\frac{1}{2}[\tau_\beta, \tau_\alpha], \quad (2.10)$$

and the same for B , C , and D . It is important to note, however, that our arguments in Sec. 10 about the nonappearance of a Born term in the dispersion relation for C depend only on the much weaker assumption of charge *symmetry* for our theory.

The amplitudes $(\pm)$ can easily be related to the amplitudes in terms of total isotopic spin by

$$A^+ = \frac{1}{3}(2A_{\frac{3}{2}} + A_{\frac{1}{2}}), \quad A^- = \frac{1}{3}(A_{\frac{3}{2}} - A_{\frac{1}{2}}), \quad (2.11)$$

etc.

4

In the center-of-mass system we can write

$$\frac{d\sigma}{d\Omega} = \sum \left| \left\langle f \left| f_1 + g_1 \frac{\boldsymbol{\sigma} \cdot \mathbf{q}_1}{q_1} + g_2 \frac{\boldsymbol{\sigma} \cdot \mathbf{q}_2}{q_2} + f_2 \frac{\boldsymbol{\sigma} \cdot \mathbf{q}_2 \boldsymbol{\sigma} \cdot \mathbf{q}_1}{q_2 q_1} \right| i \right\rangle \right|^2, \quad (2.12)$$

where $\sum$ is a sum over initial and final spin states. It is understood that the appropriate combination of am-

plitudes of different isotopic spin is to be taken. One finds

$$f_1^\pm = \left(\frac{E+M}{2W} \right) \left(\frac{A^\pm + (W-M)B^\pm}{4\pi} \right), \quad (2.13)$$

$$f_2^\pm + \left(\frac{E-M}{2W} \right) \left(\frac{-A^\pm + (W+M)B^\pm}{4\pi} \right), \quad (2.14)$$

$$g_1^\pm = \left(\frac{q}{2W} \right) \left(\frac{D^\pm + WC^\pm}{4\pi} \right), \quad (2.15)$$

$$g_2^\pm = \left(\frac{q}{2W} \right) \left(\frac{-D^\pm + WC^\pm}{4\pi} \right), \quad (2.16)$$

where E is the nucleon energy in the center-of-mass and $q = \sqrt{\mathbf{q}^2}$. One finds from the usual scattering analysis⁷

$$f_1 = \sum_{l=0}^{\infty} f_{l+} P_{l+1}'(X) - \sum_{l=2}^{\infty} f_{l-} P_{l-1}'(X), \quad (2.17)$$

$$f_2 = \sum_{l=1}^{\infty} (f_{l-} - f_{l+}) P_l'(X), \quad (2.18)$$

$$g_1 = \sum_{l=1}^{\infty} (g_{l+} - g_{l-}) P_l'(X), \quad (2.19)$$

$$g_2 = - \sum_{l=0}^{\infty} g_{l+} P_{l+1}'(X) + \sum_{l=2}^{\infty} g_{l-} P_{l-1}'(X), \quad (2.20)$$

where $X = \cos\theta$. If $g_1 = g_2 = 0$, then $f_{l\pm}$ is the scattering amplitude in the state of parity $-(-1)^l$ and $j = l \pm \frac{1}{2}$.

$$f_{l\pm} = \frac{e^{i\delta_{l\pm}} \sin\delta_{l\pm}}{q}, \quad (g_1 = g_2 = 0) \quad (2.21)$$

where the phase shifts are to be introduced for the amplitudes of given total isotopic spin.

$g_{l\pm}$ is, except for a factor $2iq$, the S -matrix element connecting the states l to the states $l \pm 1$ through the angular momentum $J = l \pm \frac{1}{2}$. (See Sec. 6.) Also

$$\bar{\sigma}_S = 4\pi \sum_{J=\frac{1}{2}}^{\infty} (J + \frac{1}{2}) (|f_{l+}|^2 + |f_{l+1-}|^2 + |g_{l+}|^2 + |g_{l+1-}|^2), \quad (2.22)$$

where $\bar{\sigma}_S$ is the elastic scattering cross section summed and averaged over spins.

5

Invariance of our theory under time reversal says⁸ that

$$D^\pm(\nu, K^2) = 0. \quad (2.23)$$

⁷ J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, New York, 1952), p. 426. Note, however, that for the proper conditions of unitarity and symmetry of the S -matrix one must have $S_{ll'} = i^{l-l'} \eta_{ll'}$ where $\eta_{ll'}$ is defined in the above. The extra $i^{l-l'}$ makes no difference in a parity-conserving theory.

⁸ We thank Dr. D. Geffen for discussions on this point.

This means

$$g_1 = g_2 \equiv g = (q/8\pi)C, \quad (2.24)$$

which in turn implies

$$g_{l+} = g_{l+1-} \equiv g_J, \quad (2.25)$$

and means our S -matrix for a given J is symmetric.

6

Unitarity of the S -matrix, combined with the symmetry property above means that we can relate the f 's and g 's for a given J and isotopic spin state T ($T = \frac{3}{2}, \frac{1}{2}$) through two characteristic phase shifts and a real mixing parameter. Thus,

$$S_{J,T} = \begin{pmatrix} S_{++} & S_{+-} \\ S_{-+} & S_{--} \end{pmatrix} = R^{-1} \begin{pmatrix} e^{2i\delta+} & 0 \\ 0 & e^{2i\delta-} \end{pmatrix} R, \quad (2.26)$$

where R is a real rotation,

$$R = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix}, \quad (2.27)$$

if S is symmetric, i.e., $S_{+-} = S_{-+}$. $\delta_{\pm}$ are the characteristic phase shifts and are real numbers (for elastic scattering) to ensure the unitarity of the matrix

$$\begin{pmatrix} e^{2i\delta+} & 0 \\ 0 & e^{2i\delta-} \end{pmatrix}.$$

Thus we find

$$S_{J,T} = \begin{pmatrix} e^{2i\delta+} \cos^2\epsilon + e^{2i\delta-} \sin^2\epsilon & \frac{1}{2}(e^{2i\delta+} - e^{2i\delta-}) \sin 2\epsilon \\ \frac{1}{2}(e^{2i\delta+} - e^{2i\delta-}) \sin 2\epsilon & e^{2i\delta-} \cos^2\epsilon + e^{2i\delta+} \sin^2\epsilon \end{pmatrix}, \quad (2.28)$$

where $\delta_{\pm}$ and ϵ all carry an index J and T and, from their method of introduction⁷

$$\begin{aligned} S_{++} &= 2iqf_{l+1-} + 1, & S_{--} &= 2iqf_{l+} + 1, \\ S_{+-} &= S_{-+} = 2iqg_{l+} = 2iqg_{l+1-} \equiv 2iqg_J. \end{aligned} \quad (2.29)$$

We note that for small ϵ (i.e., keeping only first order) we can take the characteristic phase shifts to be the measured phase shifts. The phase of the off-diagonal matrix elements is then known as in photoproduction.

7

We include the formula for calculating the longitudinal polarization of the recoil nucleon. If

$$\begin{aligned} d\sigma/d\Omega &= \sum |\langle f | a + \mathbf{b} \cdot \boldsymbol{\sigma} | i \rangle|^2, \\ a &= f_1 + f_2 \cos\theta, \end{aligned} \quad (2.30)$$

$$\mathbf{b} = g \left(\frac{\mathbf{q}_1 + \mathbf{q}_2}{q} \right) + if_2 \left(\frac{\mathbf{q}_2 \times \mathbf{q}_1}{q^2} \right),$$

we find

$$\begin{aligned} d\sigma/d\Omega &= \frac{1}{2} \text{Tr}(a^* + \mathbf{b}^* \cdot \boldsymbol{\sigma})(a + \mathbf{b} \cdot \boldsymbol{\sigma}) = |a|^2 + |\mathbf{b}|^2 \\ &= |f_1|^2 + |f_2|^2 + 2 \text{Re} f_1^* f_2 \cos\theta \\ &\quad + 2|g|^2(1 + \cos\theta). \end{aligned} \quad (2.31)$$

Introducing projection operators for the longitudinal polarization of the recoil nucleon,

$$\begin{aligned} p_+ &= (1 + \boldsymbol{\sigma} \cdot \hat{p}_2)/2, & p_- &= (1 - \boldsymbol{\sigma} \cdot \hat{p}_2)/2, \\ \hat{p}_2 &= -\hat{q}_2, \end{aligned} \quad (2.32)$$

where $\hat{p}_2$ and $\hat{q}_2$ are unit vectors in the directions of $\mathbf{p}_2$ and $\mathbf{q}_2$. The polarization is

$$\begin{aligned} P &= \frac{\frac{1}{2} \text{Tr}(a^* + \mathbf{b}^* \cdot \boldsymbol{\sigma})(p_+ - p_-)(a + \mathbf{b} \cdot \boldsymbol{\sigma})}{\frac{1}{2} \text{Tr}(a^* + \mathbf{b}^* \cdot \boldsymbol{\sigma})(p_+ + p_-)(a + \mathbf{b} \cdot \boldsymbol{\sigma})} \\ &= \frac{2 \text{Re} a^*(\mathbf{b} \cdot \hat{p}_2) + i \hat{p}_2 \cdot (\mathbf{b} \times \mathbf{b}^*)}{|a|^2 + |\mathbf{b}|^2}, \end{aligned} \quad (2.33)$$

$$P \frac{d\sigma}{d\Omega} = -2(1 + \cos\theta) \text{Re} g^*(f_1 + f_2). \quad (2.34)$$

8

It is also of interest to generalize the optical theorem for the case of parity nonconservation. We find

$$\sigma_t = \frac{4\pi}{q} \text{Im} f(0) = \frac{4\pi}{q} [\text{Im}(f_1 + f_2) + 2\sigma_z \text{Im} g]_{\theta=0}. \quad (2.35)$$

This is the total cross section for pions on aligned nucleons in a given state of isotopic spin T . σ_z is the value of the nucleon spin in the incident direction. One thus has

$$\sigma_{t\uparrow} = \frac{4\pi}{q} [\text{Im}(f_1 + f_2) + 2 \text{Im} g]_{\theta=0}, \quad (2.36)$$

$$\sigma_{t\downarrow} = \frac{4\pi}{q} [\text{Im}(f_1 + f_2) - 2 \text{Im} g]_{\theta=0},$$

so that $\text{Im} g|_{\theta=0}$ is a directly measurable quantity. By demanding that our cross sections be positive, one finds

$$|\text{Im} g|_{\theta=0} \leq (q/8\pi) \bar{\sigma}_t, \quad (2.37)$$

where $\bar{\sigma}_t$ is the total unpolarized cross section.

This analysis can be extended to each J state by writing

$$\sigma = \sum_J \sigma_J, \quad (2.38)$$

$$\sigma_J = \frac{4\pi}{q} (J + \frac{1}{2}) [\text{Im}(f_{l+} + f_{l+1-}) - 2\sigma_z \text{Im} g_J],$$

and also to any reaction channel. This gives another inequality

$$|\text{Im} g_J| \leq \frac{q}{8\pi(J + \frac{1}{2})} \bar{\sigma}_J. \quad (2.39)$$

III. THE DISPERSION RELATIONS

9

We now proceed to write the dispersion relations. We must first relate values of the T matrix on the negative ν axis to those on the positive axis. To do this we make use of the crossing symmetry of our theory, that is

$$T_{fi}^{\beta\alpha}(\nu, K^2) = [T_{if}^{\beta\alpha}(-\nu^*, K^2)]^* \quad (3.1)$$

for $\nu = \nu_{\text{real}} + i\epsilon$. f and i designate the final and initial nucleon states, and β and α the final and initial isotopic spin states of the meson. In terms of A , B , and C one has

$$A^\pm(\nu) = \pm [A^\pm(-\nu^*)]^*, \quad (3.2)$$

$$B^\pm(\nu) = \mp [B^\pm(-\nu^*)]^*, \quad (3.3)$$

$$C^\pm(\nu) = \mp [C^\pm(-\nu^*)]^*, \quad (3.4)$$

$$\nu = \nu_{\text{real}} + i\epsilon.$$

10

To calculate the contribution of the pole, we must evaluate

$$\langle b | j_\alpha(0) | a \rangle = ig_0 \bar{u}_b [G(\Delta p^2) + F(\Delta p^2) \gamma_5] \tau_\alpha u_a, \quad (3.5)$$

where $j_\alpha(0)$ is the meson current, a and b are one-nucleon states, g_0 is the unrenormalized coupling constant, and G and F are arbitrary functions of the square of the four-momentum transfer at the vertex which in the case of interest is

$$\Delta p^2 = -\mu^2. \quad (3.6)$$

The above form of the matrix element is written from invariance considerations. If we ask that our theory be invariant under time reversal we arrive at

$$\langle b | j_\alpha(0) | a \rangle = \eta_T ig_0 \bar{u}_b [G(-\mu^2) - F(-\mu^2) \gamma_5] \tau_\alpha u_a, \quad (3.7)$$

where η_T is the intrinsic parity of the pion under time reversal and is -1 . We thus conclude that

$$G(-\mu^2) = 0, \quad (3.8)$$

and define

$$g_0 F(-\mu^2) = g_r, \quad (3.9)$$

the renormalized coupling constant. Thus we see that there is no contribution of this term to the dispersion relation for C , that is, there is *no pole* in the dispersion relation for C . The above argument is just a restatement of a well-known theorem about π -meson interactions.³⁻⁵ For the sake of completeness, we shall show that this argument actually only depends on the assumption of charge symmetry. With charge symmetry, we must retain an invariance under rotations about the 3 axis in isotopic-spin space and reflections in the 1-2 plane. This means we can write

$$\langle b | j_3(0) | a \rangle = ig_0^N \bar{u}_b [G^N(-\mu^2) + F^N(-\mu^2) \gamma_5] \tau_3 u_a,$$

$$\langle b | j_\alpha(0) | a \rangle = ig_0^C \bar{u}_b [G^C(-\mu^2) + F^C(-\mu^2) \gamma_5] \tau_\alpha u_a, \quad (3.10)$$

$$\alpha = 1, 2.$$

Again we apply invariance under time reversal and find with $\eta_T = -1$ for the charged and neutral pions

$$\begin{aligned} G^C(-\mu^2) &= G^N(-\mu^2) = 0, \\ g_0^N F^N(-\mu^2) &= g_r^N, \quad g_0^C F^C(-\mu^2) = g_r^C. \end{aligned} \quad (3.11)$$

11

Our dispersion relations are thus

$$\text{Re} A^\pm(\nu, k^2)$$

$$= -\frac{1}{\pi} \int_{\nu_0}^{\infty} d\nu' \text{Im} A^\pm(\nu', k^2) \left[\frac{1}{\nu' - \nu} \pm \frac{1}{\nu' + \nu} \right], \quad (3.12)$$

$$\text{Re} B^\pm(\nu, k^2)$$

$$\begin{aligned} &= \frac{g_r^2}{2M} \left[\frac{1}{\nu_B - \nu} \mp \frac{1}{\nu_B + \nu} \right] + \frac{1}{\pi} \int_{\nu_0}^{\infty} d\nu' \\ &\quad \times \text{Im} B^\pm(\nu', k^2) \left[\frac{1}{\nu' - \nu} \mp \frac{1}{\nu' + \nu} \right], \end{aligned} \quad (3.13)$$

$$\text{Re} C^\pm(\nu, k^2)$$

$$= -\frac{1}{\pi} \int_{\nu_0}^{\infty} d\nu' \text{Im} C^\pm(\nu', k^2) \left[\frac{1}{\nu' - \nu} \mp \frac{1}{\nu' + \nu} \right], \quad (3.14)$$

where

$$\nu_0 = \mu - (K^2/M), \quad \nu_B = -(\mu^2/2M + K^2/M).$$

These dispersion relations are written using the no-subtraction philosophy, i.e., by requiring that the high-energy dispersion integrals are convergent. It will become apparent that our results depend only on the convergence of Eq. (3.14) for C . We feel this assumption is justified since from Eqs. (2.24) and (2.36), we have⁹

$$\text{Im} C(\nu, 0) = \frac{1}{2}(\sigma_{i\uparrow} - \sigma_{i\downarrow}). \quad (3.15)$$

We can write the dispersion relations in the laboratory system by using (2.4). We give the result for C .

$$\begin{aligned} \text{Re} C^\pm(\nu_L, K^2) &= \frac{P}{\pi} \int_{\mu}^{\infty} d\nu_L' \text{Im} C_{\text{el.}}^\pm(\nu_L', K^2) \\ &\quad \times \left[\frac{1}{\nu_L' - \nu_L} \mp \frac{1}{\nu_L' + \nu_L - 2K^2/M} \right] + I^\pm(\nu_L, K^2), \end{aligned} \quad (3.16)$$

$$\begin{aligned} I^\pm(\nu_L, K^2) &= -\frac{P}{\pi} \int_{T_i = \text{inelastic threshold}}^{\infty} d\nu_L' \text{Im} C_{\text{inel.}}^\pm(\nu_L', K^2) \\ &\quad \times \left[\frac{1}{\nu_L' - \nu_L} \mp \frac{1}{\nu_L' + \nu_L - 2K^2/M} \right]. \end{aligned} \quad (3.17)$$

⁹ We recall that by using the analogous relation involving $\sigma^{(+)} - \sigma^{(-)}$ one gets a very good determination of $\delta_1 - \delta_3$.

We have explicitly separated the contributions to the integral coming from the elastic and inelastic parts.

In order to utilize these dispersion relations, we follow closely the methods and approximations which have been used successfully in reference 6.

We make the usual one-meson approximation to A and B and neglect the inhomogeneous terms coming from inelastic processes since these contributions are negligible compared to the term in g_r^2 .

For C , however, the situation is different. The only inhomogeneous term is given by the inelastic contribution. The fact that the dispersion relation for C does not contain any pole is of the utmost importance for it leads to the conclusion that in the one-meson approximation the trivial solution $C=0$ satisfies all the requirements of causality and unitarity. This means that the possible parity violations in low-energy pion-nucleon scattering must be induced by the contributions of inelastic processes to $\text{Im}C$. We shall therefore keep the term $I^\pm(\nu_L, K^2)$ in our dispersion relations. We note that for low-energy scattering we can neglect ν_L with respect to ν_L' in the denominators of our integral for $I^\pm$ since the ν_L' start at the inelastic threshold and thus the $I^\pm$ become constants independent of ν_L in this limit (see Sec. 19).

12

(a) We are looking for a solution to our system of dispersion relations, in terms of the constants g_r^2 and I , which has small parity nonconservation at low energies. In order to do this we introduce in these equations a small ϵ and see if they are satisfied. For small ϵ the characteristic phase shifts and the usual phase shifts coincide and the analysis of reference 6 goes through as before.

(b) Under the integral for the elastic scattering part we keep only the contribution of¹⁰

$$g_{33} = (f_{33}^D - f_{33}^P) \epsilon_{33}, \quad (3.18)$$

since this is the only term which will have a large imaginary part because of the resonance. We therefore have

$$\begin{aligned} \text{Im}C_{\text{el.}}^\pm(\nu_L, K^2) &= \frac{8\pi}{q} \text{Im}g_{\text{el.}}^\pm \\ &= \frac{16\pi}{q} \text{Im}g_3^\pm \left(\frac{3K^2}{q^2} - 1 \right), \end{aligned} \quad (3.19)$$

as the integrand for the elastic scattering part of the dispersion integral. Eventually we use

$$g_3^+ = \frac{2}{3}g_{33}, \quad g_3^- = -\frac{1}{3}g_{33}.$$

¹⁰ $f_{ij} \Rightarrow \begin{matrix} J=2i. \\ T=2j. \end{matrix}$

13

We are now interested in low-energy pion-nucleon scattering so we keep only the contribution of $J=\frac{1}{2}$ and $\frac{3}{2}$ to g and project out dispersion relations for each J amplitude.

$$g = g_3 - g_1 - 3g_3 \left(1 - \frac{2K^2}{q^2} \right) + O(J=\frac{5}{2}). \quad (3.20)$$

Therefore

$$\frac{6}{q^2} g_3 = g'(0), \quad (3.21)$$

$$-g_3 = g(0) + \frac{1}{3}q^2 g'(0), \quad (3.22)$$

where the derivatives and the 0's refer to the variable K^2 .

$$\text{Reg}_3 = \frac{q^3}{48\pi} \text{Re}C'(\nu_L, 0), \quad (3.23)$$

$$-\text{Reg}_3 = \frac{q}{8\pi} [\text{Re}C(\nu_L, 0) + \frac{1}{3}g^2 \text{Re}C'(\nu_L, 0)]. \quad (3.24)$$

Inserting the dispersion relations, one has

$$\begin{aligned} \text{Reg}_3^\pm &= \frac{q^3}{48\pi} I^{\pm'} + \frac{P}{\pi} \int_\mu^\infty \frac{\text{Im}g_3^\pm d\nu_L'}{q'^3} \\ &\times \left[\frac{1}{\nu_L' - \nu_L} \mp \frac{1}{\nu_L' + \nu_L} \pm \frac{2q'^2}{3M(\nu_L' + \nu_L)^2} \right], \end{aligned} \quad (3.25)$$

$$\begin{aligned} -\text{Reg}_3^\pm &= \frac{q}{8\pi} [I^\pm + \frac{1}{3}q^2 I^{\pm'}] + 2q \frac{P}{\pi} \int_\mu^\infty \frac{\text{Im}g_3^\pm d\nu_L'}{q'} \\ &\times \left[\left(\frac{1}{\nu_L' - \nu_L} \mp \frac{1}{\nu_L' + \nu_L} \right) \left(\frac{q^2}{q'^2} - 1 \right) \right. \\ &\quad \left. \pm \frac{2q^2}{3M(\nu_L' + \nu_L)^2} \right]. \end{aligned} \quad (3.26)$$

To facilitate the handling of these equations we reduce to the static limit using (2.5) and $\omega \equiv W - M$, $\omega' \equiv W' - M$:

$$\begin{aligned} \text{Reg}_3^\pm &= \frac{q^3}{48\pi} I^{\pm'} + \frac{P}{\pi} \int_\mu^\infty \frac{\text{Im}g_3^\pm d\omega'}{q'^3} \\ &\times \left[\frac{1}{\omega' - \omega} \mp \frac{1}{\omega' + \omega} \right], \end{aligned} \quad (3.27)$$

$$\begin{aligned} -\text{Reg}_3^\pm &= \frac{q}{8\pi} [I^\pm + \frac{1}{3}q^2 I^{\pm'}] \\ &- 4q \frac{P}{\pi} \int_\mu^\infty \frac{\text{Im}g_3^\pm d\omega'}{q'^3} \left(\frac{\omega}{\omega'} \right). \end{aligned} \quad (3.28)$$

14

To use unitarity, we must find expressions for the characteristic amplitudes (in J and T). Using the notation

$$\begin{aligned}\lambda_3 &= \frac{-1}{48\pi} [I^{+'} - I^{-'}], & \gamma_3 &= \frac{-1}{8\pi} [I^{+'} - I^{-'}], \\ \lambda_1 &= \frac{-1}{48\pi} [I^{+'} + 2I^{-'}], & \gamma_1 &= \frac{-1}{8\pi} [I^{+'} + 2I^{-'}],\end{aligned}\quad (3.29)$$

one finds

$$\begin{aligned}\text{Reg}_{33} &= -q^3\lambda_3 + q^3 \frac{P}{\pi} \int_{\mu}^{\infty} \frac{d\omega'}{q'^3} \\ &\quad \times \text{Im}g_{33} \left[\frac{1}{\omega' - \omega} - \frac{1}{3} \frac{1}{\omega' + \omega} \right],\end{aligned}\quad (3.30)$$

$$\text{Reg}_{31} = -q^3\lambda_1 - \frac{4q^3}{3} \frac{1}{\pi} \int_{\mu}^{\infty} \frac{d\omega'}{q'^3} \text{Im}g_{33} \left[\frac{1}{\omega' + \omega} \right], \quad (3.31)$$

$$\text{Reg}_{13} = q\gamma_3 + 2q^3\lambda_3 + \frac{4q}{3\pi} \int_{\mu}^{\infty} \frac{(2\omega + \omega')}{q'^3} \text{Im}g_{33} d\omega', \quad (3.32)$$

$$\text{Reg}_{11} = q\gamma_1 + 2q^3\lambda_1 + \frac{8q}{3\pi} \int_{\mu}^{\infty} \frac{\omega - \omega'}{q'^3} \text{Im}g_{33} d\omega'. \quad (3.33)$$

Equations (3.30) to (3.33) can give one a determination of the g 's in terms of only the constants γ and λ . These constants will be estimated in the next section and turn out to be small. Therefore, we can satisfy those equations with a small ϵ justifying the ansatz 12(a).

15

Keeping only f_{33} , we must solve the equation for g_{33} which is

$$\begin{aligned}\epsilon_{33} \text{Ref}_{33} &= q^3\lambda_3 + q^3 \frac{P}{\pi} \int_{\mu}^{\infty} \frac{d\omega'}{q'^3} \epsilon_{33} \\ &\quad \times \text{Im}f_{33} \left[\frac{1}{\omega' - \omega} - \frac{1}{3} \frac{1}{\omega' + \omega} \right].\end{aligned}\quad (3.34)$$

Substituting

$$\epsilon_{33}(\omega) = q\omega\alpha(\omega), \quad (3.35)$$

$$\begin{aligned}\alpha \text{Ref}_{33} &= \frac{q^2}{\omega} \left[\lambda_3 + \frac{2}{3\pi} \int_{\mu}^{\infty} \frac{d\omega'}{q'^2} \alpha \text{Im}f_{33} \right] \\ &\quad + q^2 \frac{P}{\pi} \int_{\mu}^{\infty} \frac{d\omega'}{q'^2} \alpha \text{Im}f_{33} \left[\frac{1}{\omega' - \omega} + \frac{1}{3} \frac{1}{\omega' + \omega} \right],\end{aligned}\quad (3.36)$$

while from reference 6 we know

$$\begin{aligned}\text{Ref}_{33} &= \frac{q^2}{\omega} \left[\frac{4}{3} \left(\frac{f}{\mu} \right)^2 \right] \\ &\quad + q^2 \frac{P}{\pi} \int_{\mu}^{\infty} \frac{d\omega'}{q'^2} \text{Im}f_{33} \left[\frac{1}{\omega' - \omega} + \frac{1}{9} \frac{1}{\omega' + \omega} \right].\end{aligned}\quad (3.37)$$

We see that to the extent to which we can consider the effect of the crossed terms as small, we have a solution for $\alpha = \text{constant}$.

$$\frac{\lambda_3}{\alpha} + \frac{2}{3\pi} \int_{\mu}^{\infty} \frac{d\omega'}{q'^2} \text{Im}f_{33} = \frac{4}{3} \left(\frac{f}{\mu} \right)^2. \quad (3.38)$$

Using⁶

$$\text{Im}f_{33} = (4\pi/3)(f/\mu)^2 q'^2 \delta(\omega' - \omega_R), \quad (3.39)$$

we find

$$\epsilon_{33} = \frac{q\omega\lambda_3}{(4/9)(f/\mu)^2}. \quad (3.40)$$

16

Inserting this solution in the other equations, that is, using

$$\text{Im}g_{33} = -3\pi\omega q^3\lambda_3\delta(\omega - \omega_R), \quad (3.41)$$

and neglecting D waves, we find

$$\epsilon_{31} = \frac{q^3}{\text{Ref}_{31}} \left[\lambda_1 - 4\lambda_3 \frac{\omega_R}{\omega + \omega_R} \right], \quad (3.42)$$

$$\epsilon_{13} = \frac{q}{\text{Re}(f_{13} - f_3)} \{ \gamma_3 + 2\lambda_3 [q^2 - 2\omega_R(\omega_R + 2\omega)] \}, \quad (3.43)$$

$$\epsilon_{11} = \frac{q}{\text{Re}(f_{11} - f_1)} [\gamma_1 + 8\lambda_3\omega_R(\omega_R - \omega) + 2q^2\lambda_1]. \quad (3.44)$$

Equations (3.40) to (3.44) are the solutions to our problem. Since we are only able to give order of magnitude estimates to these quantities, we approximate the formula further in order to gain a simple orientation.

(a) We keep only the Born term for Ref_{31} ⁶

$$\text{Ref}_{31} = -\frac{2}{3} \left(\frac{f}{\mu} \right)^2 \frac{q^2}{\omega}. \quad (3.45)$$

(b) We keep only the S -wave amplitudes in the denominators of ϵ_{13} and ϵ_{11} ,⁶

$$\begin{aligned}\text{Ref}_3 &= f_3 = -0.11/\mu, \\ \text{Ref}_1 &= f_1 = +0.16/\mu.\end{aligned}\quad (3.46)$$

(c) For the constants I , we note that a prime puts an extra power of q^2 in the denominator of the integral and therefore the constants γ (or at least the limits we are able to put on them) are much larger than the λ . We therefore drop the constants λ in ϵ_{13} and ϵ_{31} .

These approximations give us the following simple expressions:

$$\epsilon_{33} = \frac{q\omega\lambda_3}{(4/9)(f/\mu)^2}, \quad (3.47)$$

$$\epsilon_{31} = \frac{q\omega}{-\frac{2}{3}(f/\mu)^2} \left[\lambda_1 - 4\lambda_3 \frac{\omega_R}{\omega + \omega_R} \right], \quad (3.48)$$

$$\epsilon_{13} = \frac{q}{0.11/\mu} \gamma_3, \quad (3.49)$$

$$\epsilon_{11} = \frac{q}{-0.16/\mu} \gamma_1. \quad (3.50)$$

IV. NUMERICAL RESULTS

17

The quantities γ and λ are connected with nonconservation of parity in high-energy interactions of pions and nucleons and in principle can be measured. Since no experimental information is yet available on those effects, we have to resort to some theoretical considerations in order to give an upper bound to these quantities. The most obvious upper limit is obtained by using the total pion-nucleon cross section (2.37). This gives a rigorous upper limit on $\text{Im}g$ but certainly not a very stringent one since, when the equality sign holds, either σ_{π^+} or σ_{π^-} , must be zero.

Since we are primarily interested in answering the question whether large parity nonconservation in strange-particle production is compatible with small effects in low-energy pion physics, we shall keep only the contributions to $\text{Im}g$ coming from this process. A justification of this can perhaps be given as follows. Parity nonconservation is induced in our dispersion relations for pion-nucleon processes through the inelastic strange-particle production.¹¹ We will also have contributions to the inelastic integrals coming from such processes as multiple pion production. There will, however, be further dispersion relations for these processes and the inhomogeneous terms in these relations will again be given by the strange-particle production amplitudes. When one inserts these results into our original expression, one will find two integrals over inelastic processes and one hopes that the extra denominators will make these terms small or at least the same order of magnitude as the direct production amplitude. We therefore use

$$|\text{Im}C(0)| < \bar{\sigma}_{YK} \quad (4.1)$$

in either state of T .

We also have to obtain a limit for $\text{Im}C'(0)$. This can only be done by introducing a phase-shift analysis and using the inequality (2.39) for each partial wave.

¹¹ We remark that our equations are analogous to those of photoproduction⁶ where the strange-particle production amplitude plays the role of the Born terms involving the charge and magnetic moment.

Because of extra convergence factors in the denominators coming from the differentiation with respect to K^2 , the expressions for $C'(0)$ are dominated by the low-energy behavior of $\pi + N \rightarrow Y + K$. Therefore we shall keep only the contributions of the low-angular-momentum states $J = \frac{1}{2}$, $J = \frac{3}{2}$. We find again

$$g = g_3 - g_1 - 3g_3 \left(1 - \frac{2K^2}{q^2} \right) + O(J = \frac{5}{2}), \quad (4.2)$$

$$g'(0) = \frac{6}{q^2} g_3. \quad (4.3)$$

We thus have from (2.39)

$$|\text{Im}C'(0)| < \frac{3}{q^2} \bar{\sigma}_{YK}(J = \frac{3}{2}) < \frac{3}{q^2} \bar{\sigma}_{YK}. \quad (4.4)$$

18

We have

$$\bar{\sigma}_{YK}(T = \frac{3}{2}) = \sigma_{\pi^+\Sigma^+}, \quad (4.5)$$

$$\bar{\sigma}_{YK}(T = \frac{1}{2}) = \sigma_{\pi^-\Lambda} + \frac{1}{2}[3\sigma_{\pi^-\Sigma^-} - \sigma_{\pi^+\Sigma^+}], \quad (4.6)$$

where

$\sigma_{\pi^+\Sigma^+}$ is the total cross section for $\pi^+ + p \rightarrow \Sigma^+ + K^+$,

$\sigma_{\pi^-\Sigma^-}$ is the total cross section for $\pi^- + p \rightarrow \Sigma^0 + K^0$,
 $\searrow \Sigma^- + K^+$,

$\sigma_{\pi^-\Lambda}$ is the total cross section for $\pi^- + p \rightarrow \Lambda + K^0$.

From the last CERN conference¹² we find

$$\begin{aligned} \sigma_{\pi^+\Sigma^+} &\sim 0.3 \text{ mb}, \\ \sigma_{\pi^-\Sigma^-} &\sim 0.5 \text{ mb}, \\ \sigma_{\pi^-\Lambda} &\sim 0.7 \text{ mb}, \end{aligned} \quad (4.7)$$

as an estimate of the cross sections, and therefore

$$\bar{\sigma}_{YK}(T = \frac{3}{2}) \sim 0.3 \text{ mb} = 0.75/M^2, \quad (4.8)$$

$$\bar{\sigma}_{YK}(T = \frac{1}{2}) \sim 1.3 \text{ mb} = 3.25/M^2. \quad (4.9)$$

19

In order to estimate the λ and γ , we first notice that the threshold for $\Pi^- + p \rightarrow \Lambda + K^0$ is

$$\nu_{L \text{ min}} = 6.6\mu, \quad (4.10)$$

and therefore we can neglect ν_L with respect to ν_L' in Eq. (3.17) over quite a range of ν_L .

There is just one further difficulty and that is if we try to put bounds on the integrals $I_{\frac{1}{2}, \frac{3}{2}}$ using the estimates (4.8)–(4.9) for the cross sections, the integrals diverge logarithmically. However, under our no-subtraction philosophy the original integrals are convergent and therefore we can integrate up to some

¹² *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), pp. 147–157.

$\nu_{L \max}$ with only negligible error. We arbitrarily choose $\nu_{L \max} = 7M$ but since it enters in a logarithm, the precise value is not very important. It is interesting to note that, as in reference 6, only the determination of the $J = \frac{1}{2}$ mixing parameters depends on the use of no-subtraction philosophy. We thus find

$$I_{\frac{3}{2}}(0) = \frac{2}{3\pi} \int_{\nu_{L \min}}^{\nu_{L \max}} \frac{d\nu_{L'}}{\nu_{L'}} \text{Im}[C_{\frac{3}{2}} - C_{\frac{1}{2}}], \quad (4.11)$$

$$I_{\frac{1}{2}}(0) = \frac{4}{3\pi} \int_{\nu_{L \min}}^{\nu_{L \max}} \frac{d\nu_{L'}}{\nu_{L'}} \text{Im}[C_{\frac{1}{2}} - C_{\frac{3}{2}}], \quad (4.12)$$

$$I_{\frac{3}{2}}'(0) = \frac{2}{3\pi} \int_{\nu_{L \min}}^{\infty} \left[\frac{1}{\nu_{L'}} \text{Im}[C_{\frac{3}{2}} - C_{\frac{1}{2}}]' - \frac{1}{M\nu_{L'}^2} \text{Im}[2C_{\frac{3}{2}} + C_{\frac{1}{2}}] \right] d\nu_{L'}, \quad (4.13)$$

$$I_{\frac{1}{2}}'(0) = \frac{4}{3\pi} \int_{\nu_{L \min}}^{\infty} \left[\frac{1}{\nu_{L'}} \text{Im}[C_{\frac{1}{2}} - C_{\frac{3}{2}}]' + \frac{1}{2M\nu_{L'}^2} \text{Im}[C_{\frac{1}{2}} - 4C_{\frac{3}{2}}] \right] d\nu_{L'}, \quad (4.14)$$

where the subscript denotes isotopic spin. Using (2.6), (4.1) and (4.4)

$$I_{\frac{1}{2}}(0) < \frac{4}{3\pi} [\bar{\sigma}_{YK}(T=\frac{1}{2}) + \bar{\sigma}_{YK}(T=\frac{3}{2})] \ln \frac{\nu_{L \max}}{\nu_{L \min}}, \quad (4.15)$$

$$I_{\frac{3}{2}}(0) < \frac{2}{3\pi} [\bar{\sigma}_{YK}(T=\frac{3}{2}) + \bar{\sigma}_{YK}(T=\frac{1}{2})] \ln \frac{\nu_{L \max}}{\nu_{L \min}}, \quad (4.16)$$

$$I_{\frac{1}{2}}'(0) < \frac{2}{\pi\nu_{L \min}^2} [\bar{\sigma}_{YK}(T=\frac{1}{2}) + \bar{\sigma}_{YK}(T=\frac{3}{2})] + \frac{\nu_{L \min}}{3M} [13\bar{\sigma}_{YK}(T=\frac{1}{2}) + 16\bar{\sigma}_{YK}(T=\frac{3}{2})], \quad (4.17)$$

$$I_{\frac{3}{2}}'(0) < \frac{1}{\pi\nu_{L \min}^2} [\bar{\sigma}_{YK}(T=\frac{3}{2}) + \bar{\sigma}_{YK}(T=\frac{1}{2})] + \frac{2\nu_{L \min}}{3M} [8\bar{\sigma}_{YK}(T=\frac{1}{2}) + 7\bar{\sigma}_{YK}(T=\frac{3}{2})]. \quad (4.18)$$

These equations imply

$$\mu^2\gamma_1 < 3.0 \times 10^{-3}, \quad (4.19)$$

$$\mu^2\gamma_3 < 1.5 \times 10^{-3}, \quad (4.20)$$

$$\mu^4\lambda_1 < 4.5 \times 10^{-5}, \quad (4.21)$$

$$\mu^4\lambda_3 < 2.5 \times 10^{-5}. \quad (4.22)$$

Finally, inserting these results in the equations for the

FIG. 1. Graphs contributing to nuclear forces.

ϵ 's, (3.47)–(3.50), one has¹³

$$\epsilon_{33} < 7 \times 10^{-4} (q/\mu)(\omega/\mu), \quad (4.23)$$

$$\epsilon_{31} < 2 \times 10^{-3} (q/\mu)(\omega/\mu), \quad (4.24)$$

$$\epsilon_{13} < 0.01 (q/\mu), \quad (4.25)$$

$$\epsilon_{11} < 0.02 (q/\mu). \quad (4.26)$$

20

To relate these quantities to observables, we give the equation for the longitudinal polarization of the recoil nucleon (2.34) in certain simple cases:

(a) Keeping only S waves and S – P coupling in the $T = \frac{3}{2}$ state,

$$P = -2\epsilon_{13}[1 + \cos\theta]. \quad (4.27)$$

(b) Keeping only the 3–3 state and P – D coupling,

$$P = \frac{-2\epsilon_{33}[1 + \cos\theta][1 - 3\cos\theta]^2}{1 + 3\cos^2\theta}. \quad (4.28)$$

V. CONCLUSION

The main conclusion of our investigation is that, under very general assumptions, violations of parity conservation in low-energy pion physics are expected to be small even if large violations of parity conservation are found in strange-particle interactions.

Another interesting feature of our results is the fact that the most important violations of parity conservation are expected in the $J = \frac{1}{2}$, S – P mixing parameters, whereas the effect is very small for the terms involving the 3–3 resonance. The reason for this is that the parity nonconservation is due to strange-particle effects. These effects take place at a very small distance from the nucleon where only S -wave pions can have any chance of penetrating.¹⁴

¹³ If one uses equality signs in Eqs. (4.23) to (4.26), assumption 12(b) is still valid.

¹⁴ We thank Professor G. Bernardini for a very illuminating discussion on this point.

One might object to the fact that our limits are based on only the strange-particle production cross-section. However, we point out that we have essentially only used the assumption that

$$(\sigma_{tt}-\sigma_{tt})/(\sigma_{tt}+\sigma_{tt})\sim 0.03.$$

Since no violations of parity conservation have yet been observed, for example in $\pi^- + p \rightarrow \Lambda + K^0$,¹³ we feel that this estimate is not unreasonable.

It is also interesting to discuss the possible implications of our results for low-energy nuclear physics. The possible causes of parity nonconservation in $N-N$ scattering are expressed in graphs of the form (b), (c) of Fig. 1. The contribution of the graphs (a) are excluded by the Soloviev-Thirring argument.

The contributions of the graphs of type (c) give a potential of very short range and therefore give rise to small violations of parity conservation.⁴

Our results could be used to estimate the contribution

of all graphs of the type (b) which give rise to potentials of range 2μ and are obviously closely connected to pion-nucleon scattering. Our results indicate that these contributions too are small. Therefore our paper supports the view that the very good experimental evidence for parity conservation in nuclear physics could be reconciled with possible parity nonconservation in strange-particle physics.

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Multipole Singularities of Classical Vector and Pseudovector Fields*

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The general form of the equations of motion of a particle possessing multipole singularities of a neutral vector or pseudovector meson field has been found by Harish-Chandra under the assumption of a divergenceless current. In a recent paper a more general form of the interaction was proposed, and the equations of motion were found for this case for neutral, charged and charge-symmetric fields. In this paper we establish forms of the multipole moments of arbitrary order compatible with these equations, and also of the moments compatible with the more restrictive interactions proposed by Harish-Chandra, which include those of electrodynamics as a special case; they are established under the assumption that the spin of the particle is of constant magnitude and has only spatial components in the rest system, and for pure 2^n -pole moments of constant magnitude ("intrinsic moments") or of variable magnitude ("induced moments," acceleration-dependent forces). In the Appendix the results of a previous paper on singularities of scalar and pseudoscalar fields, which established the general form of the intrinsic moments, are generalized to show the possibility of induced moments of all orders, and it is shown that a particle can carry an arbitrary linear combination of multipole singularities of spin zero and one fields.

I. INTRODUCTION

IN recent years several methods have been developed to obtain finite equations of motion for point particles in spite of the infinities associated with such singularities in classical field theory. The method first used by Dirac¹ for point charges interacting with an electromagnetic field was generalized by Bhabha and Harish-Chandra² to be applicable to any type of linear wave fields which allow a set of conservation laws. A classical

relativistic theory of such wave fields without sources, which after quantization describe neutral or charged particles of integral spin, had been given earlier by Dirac³ and Fierz.⁴ Harish-Chandra⁵ extended Fierz's theory for neutral fields to include sources and succeeded in obtaining the general form of the equations of motion of point multipoles of these fields and the explicit form of these equations for point charges and dipoles in fields of spin zero and one.

In introducing the sources of all fields of nonzero spin Harish-Chandra assumed that they satisfied an equation of continuity. In a recent paper⁶ it was noted

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⁶ P. Havas, Phys. Rev. **113**, 732 (1959); in the following referred to as EM.