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On a contradiction between experimental results on nucleon
form factors and certain symmetries of strong interactions ⁺)

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A number of consequences of the symmetries of strong interactions on the electromagnetic properties of hyperons have been deduced previously. [1-5] The main object of this note is to point out that in some of the symmetrical theories of strong interactions that have been proposed, the contribution of the pion intermediate states to the isoscalar electromagnetic form factors of nucleons and hyperons vanishes at all orders in the strong interactions. This is a consequence of a known theorem [3,6] proved under symmetry assumptions which will be specified later on.

Let us write the hamiltonian of strong and electromagnetic interactions as

$$\begin{aligned}
 H_{strong} = & g_1 \bar{N} \underline{\tau} N \underline{\pi} + g_4 \bar{\Xi} \underline{\tau} \Xi \underline{\pi} + g_2 \bar{\Lambda} \underline{\Sigma} \underline{\pi} \\
 & + g_3 \bar{\Sigma} \underline{T} \Sigma \underline{\pi} + g_5 \bar{N} \Lambda K + g_6 \bar{N} \underline{\tau} \Sigma K \\
 & + g_7 \bar{\Xi} \Lambda K + g_8 \bar{\Xi} \underline{\tau} \Sigma K + h.c. \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 H_{em} = & \int d^3x \bar{\psi} \gamma_\mu A_\mu \psi + \int d^3x \bar{K} \gamma_\mu A_\mu K \\
 = & ie \left[\frac{1}{2} \bar{N} \gamma_\mu (1 + \tau_3) N + \frac{1}{2} \bar{\Xi} \gamma_\mu (\tau_3 - 1) \Xi \right. \\
 & + \bar{\Sigma} \gamma_\mu T_3 \Sigma + \frac{1}{2} \bar{K} (1 + \tau_3) \partial_\mu K - \frac{1}{2} (\partial_\mu \bar{K}) (1 + \tau_3) K \\
 & \left. + \pi T_3 \partial_\mu \pi \right] A_\mu + e^2 \left[\frac{1}{2} \bar{K} (1 + \tau_3) K + \pi T_3^2 \pi \right] A_\mu^2 \quad (2)
 \end{aligned}$$

N, Ξ, K denote the isotopic spinors of the corresponding fields, $\underline{\Sigma}$ and $\underline{\pi}$ are isotopic vectors, $\underline{K} \equiv i \underline{\tau} K$, T_3 is the third component of the isotopic spin-one matrix. In (1) the appropriate γ_5 have been omitted and (2) is to be understood as properly symmetrised.

We shall assume that the strong interactions have the $N - \Xi$ symmetry. In this scheme, specially considered by Salam and Polkinghorne^[7], N and Ξ fields are coupled in a symmetrical way to π 's and K 's, and the $N - \Xi$ mass difference is neglected:

$$g_1 = g_4 \quad g_5 = \pm g_7 \quad g_6 = \pm g_8 \quad m_N = m_\Xi \quad (3)$$

The theorem of interest whose consequences shall be studied can be stated as follows: If the strong interactions have the $N - \Xi$ symmetry the transition amplitude between states containing only pions, photons, and K mesons vanishes at all orders of the strong and electromagnetic interactions for a process in which the sum of the total number of initial and final pions involved is odd.

For completeness we shall sketch the proof. Let us define the transformation $U = GV$ where $G = C \exp i\pi T_2$ is the product of a charge conjugation and a rotation of π around the 2nd axis in isotopic spin space and where V is the transformation,

$$V: \quad \begin{array}{ll} N \rightarrow \Xi & K \rightarrow \dot{K} \\ \Xi \rightarrow -N & \dot{K} \rightarrow -K \end{array} \quad (4)$$

The combined transformation U is then given by

$$U: \quad \begin{array}{lll} N \rightarrow G\Xi & \Sigma \rightarrow G\Sigma & \pi \rightarrow G\pi = -\pi \\ \Xi \rightarrow -GN & \Lambda \rightarrow G\Lambda & K \rightarrow G\dot{K} = -K \\ & & A_\mu \rightarrow A_\mu \end{array} \quad (5)$$

It can be easily seen that H_{strong} is separately invariant under G (from assumed charge independence) and under V (from relations (3) between the coupling constants). On the other hand H_{em} is invariant only under the combined transformation U , the isotopic scalar part

of j_p and the isotopic vector part of J both changing sign under Ξ and under V . We therefore conclude that the complete hamiltonian is invariant under the transformation U if $N - \Xi$ symmetry is assumed. *) The theorem stated above then follows immediately from relations (5) **) It is obvious that if the theorem holds for the $N - \Xi$ symmetry it holds a fortiori for global symmetry in which $g_1 = g_2 = g_3 = g_4$, and K interactions are neglected.

One consequence of the above theorem is that in a theory which has the $N - \Xi$ symmetry or the global symmetry the $\pi^+ \rightarrow 2\gamma$ decay is forbidden [3, 6]. However the fact that π^0 does decay is not so strong a statement against the usefulness of the proposed symmetries since a break in the symmetry, which we know must exist from the $N - \Xi$ mass difference, will allow the decay. Nevertheless it points to the fact that in π^0 decay each graph which contains a $N - \bar{N}$ loop is partially suppressed by the corresponding $\Xi - \bar{\Xi}$ loop. Therefore it might not be meaningless to calculate π^0 decay in lowest order perturbation taking into account the hyperons [8, 9]. One then gets for the π^0 life time

$$\frac{1}{\tau} = \left(\frac{\alpha}{4\pi} \right)^2 \left(\frac{g_1^2}{4\pi\hbar c} \right) \left(\frac{\mu c^2}{\hbar} \right) \left(\frac{\mu}{M_N} - \frac{\mu}{M_{\Xi}} \frac{g_4}{g_1} \right)^2 \quad (6)$$

or

$$\tau = \tau_0 \left[\frac{M_{\Xi}}{M_{\Xi} - M_N g_4/g_1} \right]^2 \quad (7)$$

*) Note that if we have the minus sign in relations (3) then invariance is obtained if the sign is exchanged in the $N - \Xi$ transformations (4) and (5).

**) K mesons can be included in initial and final states because conservation of strangeness requires the total number to be even.

where τ_0 is the perturbation result, neglecting the Ξ . If one takes $g_4 = g_1$, τ is increased over τ_0 by a factor 12. With $g_1^2/4\pi\hbar c = 13$ this gives for the lifetime $\tau \approx 6 \times 10^{-16}$ sec. which, considering the crude approximations, is still compatible with the present experimental upper limit on the π^0 lifetime [10].

Another consequence of the stated theorem concerns the isotopic scalar electromagnetic form factors of the baryons. If the N - Ξ or global symmetry holds, the vertex formed with one photon line and an odd number of pion lines vanishes. On the other hand it is known from G invariance [11] that the intermediate pion states with an even number of pions contribute to the isovector form factors, and those with an odd number of pions contribute to the isoscalar part. The conclusion is then that the assumed symmetries imply that the pion contribution to the isoscalar form factors of nucleons and hyperons vanish at all orders in the strong interactions. In the language of dispersion theory this means that the first mass states which contribute to the isoscalar form factor are the $\bar{K} - K$ pairs. If only these high mass states contribute to the dispersion integrals one would then expect the isoscalar form factor to be nearly constant for small values of the square of the 4 dimensional momentum transfer. By definition it implies a small isoscalar contribution to the mean square radius of both the charge and the magnetization density distributions. This is in contradiction with the large value of the mean square radius of the proton charge distribution and the almost zero value of the corresponding neutron charge distribution found experimentally. [12] Apart from the problem of the mean square radius, the K contribution seems to be unable to account for the isoscalar form factors themselves [11]. The situation

would be specially difficult in the case of global symmetry where the K couplings are assumed to be small.

We wish to emphasize that N - Ξ symmetry is necessary to draw the above conclusions. If for instance only restricted symmetry is assumed, i.e. $g_2 = g_3$, but nothing is said about the equality of N - π and Ξ - π couplings, the vanishing of the pion contribution to the isoscalar form factors does not result any more.

Though one knows that the proposed symmetries can only be approximate it might be hoped that they are still useful concepts when no real K particles are involved in the process considered. The above considerations on the electromagnetic properties of nucleons seem to invalidate such a point of view in the case of N - Ξ and global symmetry, but not in the case of restricted symmetry.

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