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REABSORPTION OF FAST SECONDARIES IN NUCLEI

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A B S T R A C T

The reabsorption of particles in the same nucleus where they were produced, is investigated. The model supposes incoherent interaction of the nucleons with the primary particle and with the secondaries and is limited to such high energies where this assumption holds good. An absorption coefficient is calculated, which relates the actual number of secondaries leaving the nucleus to the number of secondaries which would leave it if there were no reabsorption. This coefficient is a function of the atomic number, of the production and absorption cross-section per single nucleon and of the angle of emission.

This coefficient together with the total inelastic cross-section of the whole nucleus, which also is calculated as a function of the atomic number, yields all the relevant information.

I. Introduction

So far the multiple production of particles in high energy collisions has been described by statistical theories. In comparing these calculations with experiments one meets in general the difficulty that the calculations refer to elementary processes (pp or π - p collisions) whereas the targets are normally not hydrogenic.

The discrepancies between theory and experiment [e.g. ¹⁾], in particular regarding the antiproton production, may partly be due to reabsorption of the particles in the same nucleus where they were produced. Since not many detailed experiments with hydrogen targets exist, it may be worthwhile to try to estimate the reabsorption effect by a simple model. This will be helpful in attempts to find the true limitations of statistical theories of multiple production by separating off the disturbing effects of the presence of nuclear matter.

As we shall see, it will not be possible to treat the problem in full generality, we rather shall restrict to limiting cases : large angles and high momenta of secondaries in the lab. system.

A genuine quantum theoretical model of a particle entering a nucleus, producing secondaries (pions, antinucleons, hyperons, etc.) which in turn are absorbed and/or scattered inside the nucleus, would be a complicated thing. Fortunately, we wish to deal here with high energies as e.g. 25 GeV primary lab. energy of incident protons. Then even the secondaries have momenta of the order of several GeV/c.

Let p be a typical momentum [we use units $\hbar = c = 1$; the nucleon mass M is the unit mass; $p = 1$ corresponds to 0.936 GeV/c], then the corresponding de Broglie wavelength is $\lambda = \frac{1}{p}$ and the typical length inside the nucleus is of the order of $\lambda_0 \approx \frac{1}{\mu} \approx 7$ (μ = pion mass). Obviously, the incident (or secondary) particle is scattered incoherently by the individual nucleons if $\lambda \ll \lambda_0$ or if

$$\frac{\lambda_0}{\lambda} \approx 7p \gg 1 \quad (1)$$

This is certainly true for the primary particle. As to the secondaries, we shall limit ourselves to discuss the reabsorption effect only for lab. momenta of secondaries of the order of several MeV/c , i.e. we use the "impulse approximation" for the whole process. This is justified by the condition (1). In this limit we also get rid of the influence of the Pauli principle, since practically everything happens far above the Fermi momentum.

We may now treat the incoming and the secondary waves as being absorbed exponentially inside the nucleus and may add up intensities rather than amplitudes. We feel that, if one would put the problem in the precise quantum mechanical form and solve it, one would finally be able to describe the solution approximately like this :

The incident wave penetrates through the nucleus and is partially absorbed, the absorption coefficient being a function of the nucleon - nucleon cross-section and the nuclear density. At a given point P inside the nucleus the intensity is then well defined and so is the production process which takes place there. From P the secondaries emerge with a well-defined energy-angle distribution and are partially absorbed in turn. At a given angle and momentum in the lab. system one will then observe those particles, which, coming from P , are not absorbed on their way through the nucleus. Finally one integrates P over the whole nucleus.

There are three points to be considered :

- i) The intensities in question (primaries and secondaries) will not be absorbed in proportion to the relevant total cross-section, since elastic and quasi-elastic (small energy loss and small angles) scattering will have practically no effect. The appropriate cross-section determining the absorption will therefore be somewhat smaller than the inelastic one at the energy in question.

- ii) Neglecting quasi-elastic scattering of the primaries implies that we do not consider what happens to the secondaries produced in these processes. Since they will in general be emitted under small angles, whereas secondaries with large angles come mostly from collisions with higher inelasticity, we should restrict the application of this model to not too small angles in the lab. system (above $10^\circ - 15^\circ$, say).
- iii) If, according to the above idea, we ask only for that part of the secondaries which reaches the nuclear boundary without being absorbed, we obviously neglect multiple scattering of the secondaries and the fact that secondaries of higher momenta and different angles can - by the "absorption" - be scattered into the angle and momentum interval under observation in the experiment. We therefore must restrict the application of the model to high momenta of the secondaries. This is qualitatively seen by the following argument which is basically the same as that leading to the Boltzmann equation in statistical mechanics.

In Fig. 1 we draw a typical momentum spectrum of the secondary particles which are originally produced at the point P inside the nucleus. Consider those particles which go in a given direction and have a momentum $\approx p$ inside Δp . On their way through the nucleus a part of them will be absorbed (this is described by the model). Let the number reaching the nuclear boundary be given by the area of region I. However, the absorbed particles do (in general) not really disappear; they may be only scattered and in such a scattering they possibly will even create more secondaries of the same kind or others (as for instance pions will do). They therefore will be absorbed from Δp but reappear somewhere at lower momenta. The same happens to those of initial momenta $> p$. Therefore the interval Δp will partly be refilled by those particles and their secondaries which are absorbed from higher momenta and other angles: they fill region II. The total number of

particles in Δp , which finally leave the nucleus, is proportional to the area of I + II. The contribution in region II is approximately given by an integral of the sort

$$J = \Delta p \cdot \int_p^\infty W(p', p) \frac{dn}{dp'} dp'$$

where $W(p', p)$ is the probability density for a particle of momentum p' undergoing a scattering from which a particle of the same kind but momentum p emerges. The function W is very complicated: it contains the whole description of production processes, namely multiplicities, angular and momentum distribution of the particles created in the "absorption" of one of the secondaries. It would of course be possible to make reasonable assumptions about W and calculate the integral J . By iterating this procedure one would obtain the total contribution to II. That would lead to a cascade theory which we do not wish to work out.

Since the area of region II is roughly given by the integral J , it follows that it is negligible for sufficiently large p (but may overcompensate the absorption for small p).

In Appendix I we discuss this quantitatively using simplifying assumptions. The result is that, if 25 GeV primary protons hit nuclei, the contribution of area II may be neglected for all secondaries with momenta (lab.) $\gtrsim 4$ -5 GeV/c.

Since also the condition of incoherent scattering is fulfilled for such momenta, we feel that under these restrictions the model may work rather safely.

In the next section we give some details.

II. The model

We choose the notation according to Fig. 1. Let a ray of an incoming wave enter at point B, create the secondary particle in question at P and let this secondary leave the nucleus at C.

Let further be :

$n_A(r)$	the nuclear density in Nucleons/Volume as a function of r , the distance from the centre of the nucleus, and A , the number of nucleons.
σ_1	the elementary cross-section describing the attenuation of fast primaries. This cross-section is smaller than the inelastic cross-section, as e.g. protons of 20 GeV will approximately still produce the same spectrum of antiprotons at 16° as protons of 25 GeV do.
σ_2	the elementary cross-section describing the absorption of secondaries considered.
σ_c	the elementary cross-section of the primaries for central collisions, of which we assume that they are described by the statistical theory.
j_0	the intensity of the primary beam.
$f(\vartheta, p)d\omega dp$	the number of secondaries in question produced per central collision. This function may be taken e.g. from results of the statistical theory ²⁾ , it depends on the kind of particles considered.

It follows then that the intensity at P is

$$j(P) = j_0 \exp \left[-\sigma_1 \int_B^P n_A(s) ds \right] \quad (2)$$

The number of central collisions in dV_P becomes

$$j(P) \cdot \sigma_c \cdot n_A(P) \cdot dV_P \quad (3)$$

and the number of secondaries [observed at the counter] coming from dV_P is

$$j(P) \sigma_c n_A(P) f(\vartheta, p) d\omega dp dV_P \exp \left[-\sigma_2 \int_P^C n_A(s) ds \right] \quad (4)$$

The values which $n_A(s)$ takes on along the paths leading from B to P to C follow from elementary geometry and from the given density distribution $n(r)$ (Fig. 2); they are determined by the position of P and the angle ϑ . The density distribution $n(r)$ was chosen in such a way that the integrals in the exponentials could be carried out and at the same time agree sufficiently with the charge density found by electron scattering³⁾. $n(r)$ is shown in Fig. 3. With the above definitions and from the equations (2) to (4) one can calculate the following quantities which, all via $n_A(r)$, depend on the atomic number A of the target nucleus considered.

- i) The total number $F_A(\vartheta, p)$ of secondaries (of a particular kind) per unit momentum and solid angle in the direction ϑ coming from a target nucleus with A nucleons. *)

*) Though all following quantities depend on σ_1 and/or σ_2 we shall never write this variable in the arguments of the functions explicitly.

$$F_A(\vartheta, p) d\omega dp = f(\vartheta, p) d\omega dp \cdot \sigma_c \cdot j_0 \times$$

$$\times \int_{(A)} dV_P n_A(P) \exp \left[-\sigma_1 \int_B^P n_A(s) ds - \sigma_2 \int_P^{C'} n_A(s) ds \right] \quad (5)$$

where $\int_{(A)} dV_P$ means the integral over the whole nuclear volume.

- ii) The absorption coefficient $\alpha_A(\vartheta)$ defined as the ratio of the actually observed number of secondaries (angle ϑ) coming from a target nucleus A, to the corresponding number if there were no absorption

$$\alpha_A(\vartheta) = \frac{\int_{(A)} dV_P n_A(P) \exp \left[-\sigma_1 \int_B^P n_A(s) ds - \sigma_2 \int_P^{C'} n_A(s) ds \right]}{\int_{(A)} dV_P n_A(P) \exp \left[-\sigma_1 \int_B^P n_A(s) ds \right]} \quad (6)$$

- iii) The cross-section of the whole target nucleus with respect to central collisions

$$\sigma_A = \sigma_c \cdot \int_{(A)} dV_P n(P) \exp \left[-\sigma_1 \int_B^P n_A(s) ds \right] \quad (7)$$

$$\equiv \sigma_c \cdot A_{\text{eff}}(\sigma_1) \quad (7a)$$

(definition of A_{eff} = the effective number of nucleons)

σ_A shows the expected behaviour, namely $\lim_{\sigma_1 \rightarrow \infty} \sigma_A = \pi R^2$ (R = nuclear radius) and $\lim_{\sigma_1 \rightarrow 0} \sigma_A = \sigma_1 \cdot A$. In the first case all absorption takes place in the surface of that hemisphere where the beam enters, in the second case the cross-sections of the individual nucleons add.

We add a few remarks :

the first quantity $F_A(s, p)$ relates the predictions of the statistical theory to what will be observed if not hydrogen but another target substance is used. If the integral on the r.h.s. of (5) is known, one can compare experiments with arbitrary target materials to the predictions of the statistical theory and thereby increase largely the material needed to improve the latter.

From the definitions it follows

$$F_A(s, p) = \left[\alpha_A(s) \right] \cdot \left[A_{\text{eff}}(\sigma_1) \right] \cdot \left[\sigma_c \cdot j_0 \cdot f(s, p) \right] \quad (8)$$

We will present as results curves of the quantity $\alpha_A(s)$ and of $A_{\text{eff}}(\sigma_1)$ as functions of A , σ_1 , σ_2 .

Numerically it turns out that $\alpha_A(s)$ is independent of σ_1 for $\sigma_1 \lesssim 30$ mb. On the other hand A_{eff} is by (7) independent of σ_2 and σ_c . Therefore the presentation in the form (8) is instructive as it contains three factors each of which depends only on one of the cross-sections.

Since σ_2 is used as a parameter for $\alpha_A(s)$ one may easily account for a momentum dependence and for different particles (pions, antinucleons, deuterons, etc.).

We remind the reader to the limitations stated in the introduction :

the primary energy must be so large that one can experimentally obtain sufficiently many secondaries under lab. angles $\gtrsim 10^\circ$ to 15° and lab. momenta $\gtrsim 4-5$ GeV/c since only these secondaries meet the various conditions of consistency discussed in the introduction.

III. Results

We give here the results in form of curves for $\alpha_A(\vartheta)$ for $\vartheta = 16^\circ$ as a function of A and σ_2 . As was mentioned before, the numerical calculation showed that $\alpha_A(\vartheta)$ is independent of σ_1 for $\sigma_1 \lesssim 30$ mb. This seems not unplausible from the definition. The σ_1 dependence "cancels" so to speak in numerator and denominator.

Also the effective number of nucleons A_{eff} as a function of A and σ_1 is given by a graph. $\overline{A}_{\text{eff}}$ is independent of ϑ .

The curves for $\alpha_A(\sigma_2)$ and $A_{\text{eff}}(A, \sigma_1)$ are found in Figs. 4 and 5 respectively.

IV. Discussion

We wish to use our results to separate off the nuclear effects from the observations and we shall discuss two cases in which the experimental data disagree distinctly with the predictions of the statistical theory. These two cases concern the ratio antiprotons to negative pions and that of deuterons to protons, both resulting from p-AL collisions at a proton lab. energy of 25 GeV.

From eq. (8) it follows that the relation between the experimental ratio of the intensities of two different kinds of particles a and b emitted into the same momentum and angle (lab.) and the corresponding ratio predicted by the statistical theory should be

$$\frac{F_A^{(a)}(\vartheta, p, \sigma_2)}{F_A^{(b)}(\vartheta, p, \sigma_2)} = \frac{\alpha_A^{(a)}(\vartheta, \sigma_2)}{\alpha_A^{(b)}(\vartheta, \sigma_2)} \cdot \frac{f_a(\vartheta, p)}{f_b(\vartheta, p)} \quad (9)$$

- if
- i) the predictions of the statistical theory are correct and
 - ii) our present model of reabsorption works well.

Here the left hand side is what the experimental ratio ought to be under these suppositions, whereas the ratio of the f 's is that of the statistical theory.

We wish to suppose that (ii) is fulfilled and that the statistical theory is to be blamed for the eventually remaining discrepancy.

At this point we may discuss this working hypothesis by the remark that there are other mechanisms than ours conceivable which might describe the interaction of fast nucleons with nuclei differently, namely the "tunnel" model, where one assumes that the incoming proton punches a tunnel

through the nucleus and all the nuclear matter contained in that tunnel melts together with the incoming particle to a single fire-ball which decays after having left the nucleus [see Fig. 6]. Whereas in our model [see Fig. 2] the assumption is that the incoming proton interacts once and that the multiple production of secondaries happens instantaneously; the idea of the tunnel model is that the emission of secondaries is a process which takes more time than necessary for "punching the hole". The two models are opposite limiting cases and there seems no a priori preference to either of them. Both models are able to explain qualitatively why the experimental lab. momentum spectra are weaker than those predicted for pp collisions by the statistical theory. Consider pions : in our model high-energy pions undergo a scattering inside the nucleus with the result that they lose energy and furthermore create some new low-energy pions. Thus there are less high-energy and more low-energy pions than in pure pp collisions. In the tunnel model the average fire-ball contains a number of nucleons which is proportional to $A^{1/3}$ and therefore moves as a whole much slower than the centre-of-mass system in a pure pp collision. The transformation of the statistical predictions from the c.m. to the lab. is based, however, on the assumption of a pp collision. Therefore, the predicted spectrum contains more high-energy and less low-energy pions than expected in the tunnel mechanism. To these kinematical reasons adds the fact that the mean kinetic energy of pions in the fire-ball rest system will be somewhat lower than in the pure pp collision, as so many more nucleons share the available energy.

It is well possible that the truth lies somewhere in the middle between our impulse approximation model and the tunnel model. A decision between the two models, or - as the tunnel model has not yet been worked out in detail and no predictions are available - a confirmation of the present model could be provided by measuring the experimental ratio of the yield of a given kind of particle (given angle and momentum) for several different targets (of equivalent thickness in atoms/cm²). From (8) we obtain :

$$\frac{F_A(s, p, \sigma_2)}{F_{A'}(s, p, \sigma_2)} = \frac{\alpha_A(s, \sigma_2)}{\alpha_{A'}(s, \sigma_2)} \cdot \frac{\sigma_A}{\sigma_{A'}} = \frac{\alpha_A(s, \sigma_2)}{\alpha_{A'}(s, \sigma_2)} \cdot \frac{A_{\text{eff}}}{A'_{\text{eff}}} \quad (10)$$

Such an experiment seems not to be difficult and it will give some useful information about the internuclear processes.

For the time being we assume now that our model is good. Table I shows the results which are obtained if one reads off the correction factor $\alpha_A^{(a)}/\alpha_A^{(b)}$ from Fig. 4, using (more or less estimated) values of the inelastic cross-section σ_2 at the relevant energies, namely :

$$\begin{aligned} \sigma_{\bar{p}} & (4.6 \text{ GeV/c}) \approx 51 \text{ mb} \\ \sigma_{\bar{n}} & (4.6 \text{ GeV/c}) \approx 22 \text{ mb} \\ \sigma_p & (5.3 \text{ GeV/c}) \approx 35 \text{ mb} \\ \sigma_d & (5.3 \text{ GeV/c}) \approx 60 \text{ mb} \end{aligned}$$

[In Appendix II these values are derived from experiments. We take the full inelastic cross-section, since every inelastic scattering of the secondary in question will throw it out of the momentum interval considered. The elastic cross-section will hardly contribute to this mechanism, as it is mostly diffractive.]

The value of σ_1 was taken to be about 35 mb, which together with the assumed nuclear density distribution led to satisfactory over-all agreement of σ_A [see eq. (7)] with the total absorption cross-sections for 25 GeV protons of several target nuclei⁴⁾.

T A B L E I

Comparison of results of the statistical theory corrected for nuclear absorption with experimental values ; all at 16° lab. angle.

	①	②	③	④	⑤
	$\alpha^{(a)} / \alpha^{(b)}$	f_a / f_b	$F^{(a)} / F^{(b)}$	exp. ratio	remaining discrepancy $\frac{③}{④}$
antiprotons at 4.6 GeV/c Al target	0.57	0.20	0.114	0.012	9.5
deuterons protons at 5.3 GeV/c Pt target	0.67	0.011	0.0073	0.024 ± 0.003	0.30
deuterons protons at 5.3 GeV/c Al target	0.64	0.011	0.0070	0.017 ± 0.002	0.41

Column ① shows the correction factor according to eq. (9).

Column ② shows the ratio expected for pp collisions according to the statistical theory ^{2),10)}

Column ③ shows the ratio expected for p-nucleus collisions

Columns ② and ④ were taken from references ^{1),11)}.

The results laid down in Table I are rather meagre. We believe however that the application of our absorption model to the ratio antiproton / π^- is quite meaningful. The remaining discrepancy with the experiments should then be due to the fact that the antinucleon production decreases rapidly with decreasing available energy, whereas the pion production is much less sensitive. It is generally assumed that most collisions at 25 GeV are peripheral. In such collisions the available energy for particle production is smaller than the total c.m. kinetic energy and antinucleons are not produced in them, whereas pions are even at large lab. angle. One would conclude then that roughly 10 per cent of all 25 GeV pp collisions are central enough to release almost the whole c.m. kinetic energy for production of secondaries.

As for the deuteron / proton ratio we feel quite uncertain. Our correction makes it worse here and we do not know whether this means that the predictions of the statistical theory about deuteron production disagree more with the experiment than was thought hitherto, or that our model of reabsorption fails in this case. This is well possible since the effect of refilling the momentum interval in question by scattering of other secondaries of higher momenta (as discussed in the introduction and Appendix I) may be non-negligible just in the deuteron case [see Figs. 1 and A1]. It is true that a deuteron of the momentum considered (5.3 GeV/c) will with large probability be destroyed by scattering and the same will happen to deuterons with higher momenta. Therefore deuterons will not refill region II in Fig. 1 by being scattered down individually from higher momenta. But all the secondary nucleons of high momenta (above ≈ 5.5 GeV/c) which come from point P in Fig. 2 and which scatter again may in this secondary process well create deuterons. It has been shown¹⁰⁾ that if the deuterons are produced in the elementary pp or pn interaction, the total ratio deuterons to protons is nearly independent of the primary energy :

$$\frac{\sigma_{d\text{-}prdt.}}{\sigma_{inel.}} \approx \begin{array}{ll} 0.013 \text{ at } 2.3 \text{ GeV} & \text{primary } p - \text{ energy (kin., lab.)} \\ 0.024 \text{ at } 25 \text{ GeV} & \end{array}$$

The deuteron momentum (5.3 GeV/c) considered above corresponds to only 2.65 GeV/c momentum per nucleon, which lies much below the limits of 4 - 5 GeV/c stated in the introduction. Therefore probably a great deal of the absorbed deuterons is replaced by others which were created in secondary processes of the nucleons crossing the nucleus. In order to avoid this effect, one should use the present model for (large angles, as here, but furthermore) higher deuteron momenta, such that the momentum per nucleon satisfies our condition that implies 8 - 10 GeV/c deuteron momentum. Unfortunately such measurements do not exist.

In any case, the experimental ratio (Al and Pt targets) seems to have the tendency to stay constant¹¹⁾, whereas the theoretical ratio (pp collision) rises to about 0.045 at 8 GeV/c¹⁰⁾. There at least our correction factor works in the right sense.

One may assume that the refilling effect cancels out to some extent, if one compares what should happen to the ratio deuterons / protons in different materials. We obtain from Table I

$$\frac{\text{ratio d/p for Pt}}{\text{ratio d/p for Al}} = \frac{0.67}{0.54} = 1.05$$

whereas the experimental value¹¹⁾ is 1.41 ± 0.34

Though the theoretical ratio is larger than one, it remains below the experimental one.

As to ratios of other particles one sees that there are neither large discrepancies between the statistical theory predictions (pp collisions) nor between the relevant cross-sections σ_2 . Therefore no spectacular corrections can be expected.

As a general result we may state that the hope - the discrepancies concerning the antiproton production might be explained by nuclear absorption - is destroyed.

Experiments on multiplicities under equivalent conditions but with different target materials - including hydrogen wherever possible - would be interesting.

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A P P E N D I X I

The effect of scattering of particles into the momentum interval under observation

If one looks at the lab. spectra of secondaries produced in pp collisions at 25 GeV, one finds that above their maximum they can fairly well be represented by a curve of the form $e^{-\beta p}$ [see the results of the statistical calculations²⁾ which in this respect agree well enough with the experiment¹⁾] which eventually, at some $p_{\max}$, is rather suddenly cut off.

Consider therefore the following simple model [see Fig. A1 and Fig. 1 main text] :

- i) A spectrum of the form $e^{-\beta p}$ is cut sharply at $p_{\max}$.
- ii) At any energy the attenuation factor is the same constant α (this is confirmed by our results in the main text), therefore in Fig. A1 we have $\text{area I} = \alpha \cdot e^{-\beta p'} \cdot \Delta p'$.
- iii) All the particles absorbed from $\Delta p'$ (their number being $\Delta p'(1-\alpha)e^{-\beta p'}$) are scattered into lower momenta such that they are uniformly distributed over a region p'/B to p'/A . [This is of course not true ; but the statistical theory curves²⁾, which give e.g. the distribution of nucleons scattered from 26 GeV/c into the various other momenta, tell us that most of the particles absorbed at a high energy reappear (or cause other particles to appear) in a rather narrow interval of much lower energies (around 5 GeV/c). The above assumption is then that the bulk of them is scattered with constant density into the region from p'/B to p'/A , which means that the energy after the "absorption" is within certain limits proportional to that before the "absorption"].

This model gives then ;

$$\text{Area I (p)} = \Delta p \cdot \alpha \cdot e^{-\beta p}$$

$$\text{Area II (p)} = \frac{\Delta p (1-\alpha)}{(\frac{1}{A} - \frac{1}{B})} \int_{Ap}^{Bp} \frac{dp' e^{-\beta p'}}{p'} \quad \left\{ \begin{array}{l} \text{if } Bp \geq p_{\max}, \text{ the upper limit} \\ \text{is } p_{\max} \\ \text{if } Ap \geq p_{\max}, \text{ Area II(p)} = 0 \end{array} \right.$$

$$\approx \frac{\Delta p \cdot B (1-\alpha)}{p (B-A)} \frac{1}{\beta} (e^{-\beta Ap} - e^{-\beta Bp})$$

$$\frac{\text{Area II}}{\text{Area I}} \approx \frac{1-\alpha}{\alpha} \cdot \frac{B e^{-\beta p (A-1)}}{\beta p (B-A)} \cdot (1 - e^{-\beta p (B-A)}) \quad \left[= 0 \text{ if } Ap \geq p_{\max} \right]$$

Now α is for most nuclei of the order of 0.2 - 0.5 thus the first factor is between 1 and 4.

From the curves of the statistical theory²⁾ one finds :

$$\beta_{\bar{N}} \approx 1.15$$

$$\beta_N \approx 0.9$$

$$\beta_K \approx 0.8$$

$$\beta_\pi \approx 0.8$$

hence

$$\beta \approx 1$$

Furthermore the nucleon spectrum of the statistical calculations for 26 GeV/c pp collisions²⁾ gives $\sqrt{\text{for } p' = 26}$

$$p'/B \approx 1 \text{ and } p'/A \approx 15 \text{ hence}$$

$$B \approx 26, \quad A = 1.75$$

With these numbers we find :

$$\frac{\text{Area II}}{\text{Area I}} \lesssim n \cdot \frac{1-\alpha}{\alpha} \cdot \frac{e^{-0.75p}}{p} (1 - e^{-24p})$$

where we have added a factor n to take into account somehow the multiplicity of the reactions at p' ($n \approx 2-3$ for pions and very small for anti-nucleons).

Even in the worst case ($n = 3, \alpha = 0.2$) we see that for $p \gtrsim 4$ GeV/c the ratio Area II / Area I becomes $\approx \frac{12}{80} = \frac{1}{6.7}$. For higher p this becomes rapidly smaller.

A P P E N D I X I I

Derivation of the numerical values for the absorption cross-sections σ_2

We consider here σ_2 to be identical with σ_{inel} .

$$(a) \quad \sigma_{\bar{p}} \left(4.6 \frac{\text{GeV}}{c} \right) \approx 51 \text{ mb}$$

One finds in reference ⁹⁾, that $\sigma_{\text{total } \bar{p}p} (4.6 \text{ GeV}/c) \approx 65 \text{ mb}$

Data on $\bar{p}p$ cross-sections [separately for elastic, inelastic and total] are available up to 1.8 GeV/c in reference ⁵⁾. There exists a relation ⁶⁾ of the form

$$\frac{\sigma_{in}^2}{\sigma_{el}} \leq 4\pi R^2$$

where R is the range of interaction. One may define by the equation

$$\frac{\sigma_{in}^2}{\sigma_{el}} = 4\pi \varrho^2 \quad (\text{A II, 1})$$

a radius ϱ which should be approximately energy-independent. Then one writes

$$\sigma_{inel} \equiv \sigma_{tot} - \sigma_{el} = \sigma_{tot} \left(1 - \frac{\sigma_{in}}{4\pi \varrho^2} \right) \quad (\text{A II, 2})$$

calculates ϱ from (A II, 1) at 1.8 GeV/c and finds $\sigma_{inel}(4.6 \text{ GeV}/c)$ from the measured σ_{tot} and eq. (A II, 2). This yields the above value of 51 mb.

$$(b) \quad \sigma_{\pi^-} \left(4.6 \frac{\text{GeV}}{c} \right) \approx 22 \text{ mb} .$$

This value was taken directly from reference ⁷⁾.

$$(c) \quad \sigma_p \left(5.3 \frac{\text{GeV}}{c} \right) \approx 35 \text{ mb} .$$

In reference ⁸⁾ $\sigma_d(\text{pp}, 5.3 \text{ GeV}/c) = 10 \text{ mb}$ was measured, whereas reference ⁹⁾ gives for the total cross-section at the same momentum

$$\sigma_{\text{tot}} = 45 \text{ mb} , \text{ hence our value for } \sigma_p .$$

$$(d) \quad \sigma_d \left(5.3 \frac{\text{GeV}}{c} \right) \approx 60 \text{ mb} \quad (?)$$

This is the most doubtful value, since we do not know how the deuteron looks like during the process considered, nor do we know its cross-section. The above value was found in the following way :

According to the philosophy of deuteron production in pp (or pn) collisions proposed in reference ¹⁰⁾, the deuteron is born in a state of extreme contraction : namely, it has to escape from the interaction volume $\Omega_0 = \frac{4\pi}{3} \left(\frac{1}{\mu} \right)^3$ [$\mu = \text{pion mass}$] and therefore the probability for its creation was taken (apart from phase space factors) to be proportional to the integral of the squared deuteron wave function over the volume Ω_0 , which is the probability to find it in a so much compressed state in configuration space. The diameter of Ω being $2/\mu$ one expects from the uncertainty principle momenta up to $p_0 \approx \mu/2$ and the velocity distribution of the proton and neutron in this new borne deuteron will extend up to $\beta_0 \approx \frac{p_0}{(1+p_0^2)^{1/2}}$. Thus the time necessary to extend to its normal radius r_d will be of the order of $t \approx \frac{r_d}{\beta_0} = r_d \sqrt{1 + 4/\mu^2} \approx 14 r_d$.

This time is large compared to the time the deuteron needs to cross the nucleus and to leave it (if not destroyed). We adopt therefore the geometrical cross-section of the volume Ω_0 as the absorption cross-section σ_2 of the deuteron. One finds ≈ 60 mb . The true value could well be larger.

R E F E R E N C E S

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Fig.1 *The balance of the particle number in a momentum interval Δp .*

Region I: Particles in Δp which are not absorbed

Region II: Particles scattered into Δp from higher momenta

Region I+II: Number of particles being observed in Δp

Fig. A1 A model for studying the effects of scattering of particles into the momentum interval under observation.

Fig. 2

The model of absorption (see text)

Fig. 3

Nuclear density distribution

$$r_1 = (1.07 A^{1/3} - 1.5) \cdot 10^{-13} \text{ cm}$$

$$d = 3.0 \cdot 10^{-13} \text{ cm}$$

Fig. 4

Fig.5 The effective number of nucleons

Fig. 6 The tunnel model