CM-P00056803

REST ENERGY, REST MASS. AND NON-COVARIANT
ELECTRODYNAMICS

J.S. Bell
CERN - Geneva

A B S T R A C T

The experimental agreement between electron mass and electron-positron annihilation energy provides a stringent restriction on recent non-covariant formulations of electrodynamics.

3115/TH.250
8 February 1962

I. Introduction

To eliminate the divergences of field theory Heitler et al.^{1),2),3)} have proposed a non locality which is not Lorentz invariant. It is then found that the "mass" of a particle, defined by the energy momentum relation

$$E^2 = m^2 c^4 + p^2 c^2 \quad (1)$$

is momentum dependent. It seems that the sort of variation predicted would not have shown up in the so far very rough tests of (1) with electrons⁴⁾.

But the proposed theory retains charge conjugacy and PCT. From either of these m is the same for positrons and electrons. Then the energy balance in positron-electron annihilation can be used as a rather accurate verification of the constancy of m for low momenta.

II. Theory

For low momenta we expand in powers of p^2 :

$$m = m(o) - b(p^2/2m(o)c^2) + \dots \quad (2)$$

where b would be zero in a Lorentz invariant theory. The corresponding energy is

$$E = E_o + p^2/2\bar{m} + \dots \quad (3)$$

where

$$E_o = m(o)c^2 \quad (4)$$

2.

is the rest energy, and

$$\bar{m} = m(o)/(1-b) \quad (5)$$

we call the rest mass. It is $\bar{m}$ which controls the acceleration of slow particles by applied forces : E_o is an irrelevant additive constant in the energy. The customary relation between rest mass and rest energy does not hold :

$$E_o \neq \bar{m}c^2 \quad (6)$$

In fact,

$$\frac{\bar{m}c^2 - E_o}{\bar{m}c^2} \approx b \quad (7)$$

if b is small.

In the cut-off theory it is the "self-mass" which is velocity dependent. Perturbation calculations have been made for electrons ^{2),3)}. If $m(o) = m_o(1+a)$, where m_o is the bare mass, then with a cut-off at about the nucleon mass

$$a \approx .03 \quad (8)$$

The ratio (b/a) depends critically on the details of the theory. With one version ²⁾

$$b = \frac{1}{6} a \quad (9)$$

and with another ³⁾

$$b = \frac{1}{36} a \quad (10)$$

Correspondingly

$$b \approx .005 \text{ or } .007 \quad (11)$$

Recently ⁵⁾ a new structure for the cut-off has been proposed by Arnous which may make b very much smaller for electrons.

Calculations have not yet been made for particles with strong interactions. Clearly much larger effects are to be expected, and correspondingly less accurate experiments are relevant. But we do not go into that here.

III. Experiment

The rest energy E_0 appears as photon energy in the annihilation process. Assume, with Heitler et al., the conventional energy wavelength relation for photons. Then the wavelength of each of the two photons is

$$\lambda_A = hc/E_0 \quad (12)$$

The measurement gives ^{6),10)}

$$(\lambda_A / \Lambda) = (24.213 \pm .003) \cdot 10^{-11} \text{ cm} \quad (13)$$

where Λ (≈ 1.002) is the Siegbahn calibration constant of crystal spectrometry.

The corresponding quantity constructed with $\bar{mc}^2$,

$$\lambda_c = \frac{hc}{mc^2} = \frac{h}{mc} \quad (14)$$

appears in the Compton scattering of low-energy photons on stationary electrons :

$$\lambda_c = \delta\lambda (1 - \cos \theta) \quad (15)$$

where $\delta\lambda$ is the wavelength increase for a photon scattered at an angle θ . This measurement ⁷⁾ has been made only with rather less precision than (13) :

$$\lambda_c = (24.18 \pm .04) 10^{-11} \text{ cm} \quad (16)$$

From (7) and (13) then

$$b \approx \frac{\lambda_A - \lambda_c}{\lambda_A} \approx .003 \pm .0015 \quad (17)$$

Better accuracy can be obtained by combining measurements of (e/mc) and (h/e) . The cyclotron frequency can be obtained from the gyromagnetic ratio of the proton ⁸⁾ and the proton magnetic moment in Bohr magnetons ⁸⁾

$$\begin{aligned} \frac{e}{mc} &= \frac{2.67520 \pm .00002}{1.521032 \pm .000003} \\ &= 1.75881 \pm .00002 \end{aligned} \quad (18)$$

The ratio (h/e) is measured directly in the short wavelength limit of the continuous X-ray spectrum. According to Cohen et al. ¹⁰⁾ the most reliable result is

$$\frac{hc^2}{\lambda_e} = 12370.8 \pm 1.0 \quad (19)$$

From (18) and (19), with $c = (2.997930 \pm .000003) 10^{10}$,

$$(\lambda_c/\Lambda) = (24.209 \pm .002) 10^{-11} \text{ cm}$$

So

$$b \approx (.0002 \pm .0002) \quad (20)$$

Consider finally the value for the Compton wavelength according to the last (1955) analysis of Cohen et al.¹⁰⁾. This is based on the least squares adjustment of a body of data which does not include the annihilation energy. Subject to reservations given below, we identify it with what we have called λ_c . They give

$$\begin{aligned} (\lambda_c/\Lambda) &= \frac{24.2626 \pm .0002}{1.002039 \pm .000014} \\ &= 24.2132 \pm .0002 \end{aligned} \quad (21)$$

This implies

$$b \approx 0 \pm .0001 \quad (22)$$

IV. Discussion

Some tacit assumptions of the above analysis should be made explicit.

A significant departure from zero of b as computed in (17), (20), or (22), would certainly imply a breakdown somewhere of conventional theory. But for the measured quantity to be simply the energy-mass difference (7), the successive results (17), (20), (22) require that increasing areas of conventional

theory do not break down. The two simple processes involved in (17) require essentially only the kinematics of photons. Heitler et al. explicitly preserve these. For (20) one needs also the notions of macroscopic electromagnetism involved in electrical measurements, and the closely related law of motion for an electron in a magnetic field. These have not been considered by Heitler et al. But any modification here would have to be reconciled with the apparent possibility of measuring currents consistently, on the basis of the usual laws, to a few parts in a million ^{9),10)}, and with the fact that the ratio of electric to magnetic charge units equals the velocity of light within an experimental error of fifty parts in a million ^{9),10)}. Finally, for (22), the least squares adjustment relies heavily on formulae, such as for spectral fine structure, which would almost certainly be modified in a non-covariant theory.

It has been implicit throughout that the laboratory is effectively at rest in the preferred frame, or aether, of the non-covariant theory. Motion through the aether would be partly compensated by contraction and dilation of measuring equipment, but compensation would not be complete ⁴⁾. Clearly such motion might complicate, but could hardly conceal, the effect we have discussed.

With these reservations, it seems reasonable to take (20) as the present experimental limit on a possible mass-energy difference for electrons. This would rule out the preliminary versions of the non-local theory giving (11).

REFERENCES

- 1) W. Heitler, Proceedings of 1961 Solvay Conference.
 E. Arnous, W. Heitler and Y. Takahashi, Nuovo Cimento 16, 671 (1960).
 E. Arnous and W. Heitler, Nuovo Cimento 11, 443 (1959).
 E. Arnous, W. Heitler and L. O'Raifeartaigh, Nuovo Cimento 16, 785 (1960).
- 2) L. O'Raifeartaigh, Helv.Phys.Acta. 33, 783 (1960).
- 3) L. O'Raifeartaigh and Y. Takahashi, Helv.Phys.Acta. 34, 554 (1961).
- 4) P.S. Farago and L. Janossy, Nuovo Cimento 5, 1411 (1957).
- 5) W. Heitler, private communication.
- 6) D.E. Muller, H.C. Hoyt, D.J. Klein and J.W.M. Du Mond, Phys.Rev. 88,
 775 (1952).
- 7) P.A. Ross and P. Kilpatrick, Phys.Rev. 46, 668 (1934).
- 8) We take the most recent (and apparently most accurate) values quoted
 in (9). They are due respectively to Driscoll and Bender,
 Hardy and Purcell. The units are c.g.s. Gaussian.
- 9) J.H. Sanders, The Fundamental Atomic Constants, Oxford Univ. Press 1961.
- 10) E.R. Cohen, K.M. Crowe and J.W.M. Du Mond, The Fundamental Constants
 of Physics, Interscience Pub. Inc., New-York 1957.