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THE EFFECT OF INTERMEDIATE VECTOR BOSONS
ON THE RATE FOR THE DECAY $K \rightarrow e + \nu + \gamma$

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THE EFFECT OF INTERMEDIATE VECTOR BOSONS
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I. INTRODUCTION

Very recently a detailed treatment of the decay mode $K \rightarrow e + \nu + \gamma$ has been given by Neville¹⁾. The purpose of this note is to point out a suggestion not emphasized in Ref. 1), which perhaps could cast light on the existence of the vector bosons as intermediaries in weak interactions.

We show that an intermediate vector boson in K decay could enhance the branching ratio

$$B_\gamma = K \rightarrow e + \nu + \gamma / K \rightarrow e + \nu$$

by a factor of 10^3 over what it would be if there were only inner bremsstrahlung radiation.

The reason that this is at all possible is due to the fact that there is a helicity selection rule at work which makes the $K \rightarrow e + \nu$ decay rate small. This selection rule requires, in order to conserve angular momentum, that the electron must have the opposite helicity to what it would prefer if it had zero mass, and thus the rate for $K \rightarrow e + \nu$ decay is proportional to $(m_e/m_K)^2$ (where m_e and m_K are the electron and kaon masses, respectively).

If one describes the mode $K \rightarrow e + \nu$ by a local interaction of the form

$$\partial_\mu K \bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu$$

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and passes to the channel $K \rightarrow e + \nu + \gamma$ by applying gauge invariance and adding the electromagnetic interaction of the charged particles, then one finds a rate for the above K radiative decay which is still proportional to $(m_e/m_K)^2$ ¹⁾. The γ -ray emission without considering any structure in the weak interaction vertex is referred to as inner bremsstrahlung radiation and has been treated in detail for the process $K \rightarrow e + \nu + \gamma$ by Neville. He finds, as expected, that the rate for inner bremsstrahlung is about 10 times smaller than the direct non-radiative process $K \rightarrow e + \nu$.

What is needed to overcome the electron mass dependence of the $K \rightarrow e + \nu + \gamma$ rate is a structure effect which breaks the helicity selection rule. We consider here a particular kind of structure effect, namely that of an intermediate vector boson, whose existence could enhance the branching ratio B_γ by a factor of 10^3 over the value with only inner bremsstrahlung radiation, for a boson mass not too much larger than the kaon mass.

We give below the branching ratio B_γ , the photon and electron energy spectra, and the angular correlation of photon and electron, computed with a model based on an intermediate vector boson.

II. THE BRANCHING RATIO B_γ DUE TO INTERMEDIATE VECTOR BOSON

We consider a simple model for the decay $K \rightarrow e(\mu) + \nu$ described by a local interaction between the vector meson W_μ and the kaon of the form

$$b W_\mu \partial_\mu K$$

where b is a constant determined by the experimental value of the kaon lifetime. The boson W_μ is coupled to leptons in the usual manner in the form

$$g_W W_\mu \bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu$$

where ℓ is e or μ , and g_W a coupling constant. Because of the propagator W_μ the decay rate for $K \rightarrow \ell + \nu$ with the above interactions

does not depend on the boson mass M providing M and g_W are expressed in terms of the β decay coupling constant as $G_F M^2 = \sqrt{2} g_W^2$.

To compute the branching ratio B_γ we make the following assumptions:

- i) gauge invariance for the derivative couplings;
- ii) that b remains constant in the channel $K \rightarrow e + \nu + \gamma$;
- iii) that there are no other electromagnetic vertices except those coming from the ordinary electromagnetic interaction of the charged particles.

Figure 1a shows the Feynman diagram for $K_{e\nu}$ decay, while Figs. 1b, 1c, and 1d show the Feynman diagrams for $K_{e\nu\gamma}$ decay. The case of the photon emitted from the charged kaon contributes a matrix element proportional to m_e/m_K , and since we are looking for an effect not proportional to m_e we drop such contributions.

In the limit of $M \rightarrow \infty$, the contributions of diagrams 1b, 1c, and 1d, which are not proportional to m_e/m_K , are of the order $(m_K/M)^2$ smaller than those proportional to m_e/m_K . However, if the boson mass M is not too much larger than m_K , the expansion in powers of m_K/M is not correct and the matrix element must be treated exactly.

The specific matrix elements will depend on the choice of the magnetic and electric moments of the W meson. Since nothing is known about these quantities we calculate the rate assuming an arbitrary anomalous magnetic moment μ and quadrupole moment Q .

In that case we have for the matrix element for the process $K \rightarrow e + \nu + \gamma$ the expression ^{*)}

$$\mathcal{M} = \sqrt{4\pi\alpha} \left(2 - \mu - Q \frac{p \cdot K}{2M^2} \right) \left[\left(\frac{b g_W^2}{M^2} \right) \left(m_K^2 - 2p \cdot K - M^2 \right)^{-1} \right. \\ \left. \left\{ (p \cdot e)(p \cdot L) - (p \cdot K)(e \cdot L) \right\} \right]. \quad (1)$$

*) Units are $\hbar = c = 1$ and $\alpha \approx 1/137$, and metric $a \cdot b = a_0 b_0 - \vec{a} \cdot \vec{b}$.

The 4-momentum of kaon and photon are p_μ and K_μ , respectively, $L_\mu = \bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu$, e_μ is the photon polarization and where we have neglected terms proportional to the electron mass.

Calculating the rate from this expression leads to a branching ratio B_γ of the form

$$B_\gamma = \left(\frac{2-\mu}{2}\right)^2 \left(\frac{\alpha}{3\pi}\right) \left(\frac{m_k}{m_e}\right)^2 I\left(\frac{M}{m_k}, \mu, Q\right) \quad (2)$$

where

$$I\left(\frac{M}{m_k}, \mu, Q\right) = \int_0^1 \frac{\left[1 - \left(\frac{Q}{4}\right) \left(\frac{m_k}{M}\right)^2 \frac{x}{(2-\mu)}\right]^2 x^3 (1-x) dx}{\left[x + \left(\frac{M}{m_k}\right)^2 - 1\right]^2} \quad (3)$$

where x is the ratio of the photon energy to the maximum photon energy.

For a specific numerical example we consider the case of zero magnetic moment and zero quadrupole moment, i.e. $\mu = Q = 0$. Table I shows the behaviour of B_γ in this case, and we see that B_γ varies between 120 and 3 for the boson mass lying between the kaon and nucleon mass, respectively. In fact, it takes a boson mass of 1984 MeV to make the ratio B_γ approximately equal to the usual inner bremsstrahlung branching ratio. The results, of course, are sensitive to the anomalous moments of the boson, but we feel that, in view of the current interest in these bosons, this very striking behaviour of B_γ warrants a careful experimental investigation^{*)}.

) Other structure effects such as those due to strong interactions will also give rise to matrix elements which are not proportional to the electron mass. The lighter the mass of the intermediate state, the more important such contributions will be. The lightest known intermediate state is the K, π resonance K^ , with mass about 885 MeV. Some considerations on the contributions of the K^* state have been made by Neville, although no definite calculation has been made. We can, however, see the two following points:

- i) Because of parity conservation only the Feynman diagram of Fig. 1b contributes when the W meson is replaced by K^* . Since B_γ is very sensitive to the mass of the intermediate states, as long as we consider $M < m_{K^*}$ the vector boson should be the dominant contribution.
- ii) Since the leptonic decay rates of the hyperons are smaller by a factor of 12 than the predictions of the universal Fermi interaction it could be that K^* is coupled to leptons with a reduced strength. This would make the K^* contribution to $K \rightarrow e \nu \gamma$ rate even less important.

III. THE ENERGY SPECTRA AND ANGULAR CORRELATION

Let us call:

- r the ratio $(M/m_K)^2$;
- R the decay rate for the process $K \rightarrow e + \nu + \gamma$;
- $R_{\mu\nu}$ the decay rate for the predominant decay mode $K \rightarrow \mu + \nu$;
- $F(x)$ the integrand in the integral (3).

The photon energy differential spectrum is

$$\frac{dR}{dx} = \left(\frac{b g_W^2}{M^2} \right)^2 \frac{\alpha m_K^3 (2 - \mu)^2}{48\pi^2} F(x). \quad (4)$$

For the case where $\mu = Q = 0$ this spectrum becomes

$$\frac{dR}{dx} = 1.811 \times 10^{-2} R_{\mu\nu} F(x). \quad (5)$$

Values of $F(x)$ for r equal to 1, 4 and 16, and $\mu = Q = 0$ are plotted as a function of x in Fig. 2.

The photon energy spectrum as given by Eq. (5) does not have an infra-red divergence since these terms are contained in the inner bremsstrahlung part of the rate which is proportional to m_e and hence is neglected here. This means that for photon energies comparable to the electron mass Eq. (5) is not valid. Note also that the inner bremsstrahlung terms are not enhanced by the presence of intermediate bosons as the dependence on the boson mass drops out just as in the channels $K \rightarrow l + \nu$.

Calling z the ratio of the electron energy to the maximum electron energy, the electron differential spectrum is:

$$\frac{dR}{dz} = \left(\frac{b g_W^2}{M^2} \right)^2 \frac{\alpha m_K^3}{32\pi^2} \left[(2 - \mu)^2 F_1(z) + \frac{Q^2}{8} \left(\frac{m_K}{M} \right)^4 F_2(z) + \frac{(2 - \mu)Q}{2} \left(\frac{m_K}{M} \right)^2 F_3(z) \right] \quad (6)$$

where

$$F_1(z) = -\frac{7}{2} z^2 + \left[2(r+1) + \frac{(r-1)^2}{r-z} \right] z - \left[2z^2 - 2(2r+1)z + \right. \\ \left. + (3r^2 + 1) \right] \ln \left(\frac{r}{r-z} \right), \quad (6a)$$

$$F_2(z) = +\frac{z^4}{12} - \left[\frac{11}{3} - \frac{8}{3}r - 2\frac{(r-1)^2}{r-z} \right] z^3 + \left[\frac{r^2}{2} - 8r + 8 - \frac{6(r-1)^2}{r-z} \right] z^2 + \\ + \left[4r^3 - 6r^2 + 4r - 2 + \frac{4(r-1)^2}{r-z} + \frac{(r-1)^4}{r-z} \right] z - \\ - \left[2(z-1)^2(r-1)(3r-1) - 2(z-1)(4r-1)(r-1)^2 + \right. \\ \left. + (r-1)^3(5r-1) \right] \ln \left(\frac{r}{r-z} \right), \quad (6b)$$

$$F_3(z) = \left[\frac{2(r-1)}{r-z} - \frac{8}{3} \right] z^3 + \left[\frac{7}{2} - 3r - \frac{4(r-1)}{r-z} - \frac{2(r-1)^2}{r-z} \right] z^2 + \\ + \left[3r^2 - 10r + 1 + \frac{2(r-1) + 2(r-1)^2 + (r-1)^3}{r-z} \right] z + \\ + \left[2(z-1)(r-1)(3r-1) - 2(z-1)^2(2r-1) - (4r-1)(r-1)^2 \right] \ln \left(\frac{r}{r-z} \right). \quad (6c)$$

For $\mu = Q = 0$ the differential rate is given by:

$$\frac{dR}{dz} = 2.717 \times 10^{-2} R_{\mu\nu} F_1(z). \quad (7)$$

Figure 3 shows the plot of $F_1(z)$ as a function of z for values of r equal to 1, 4 and 16.

Let ϑ be the angle between electron and photon, $d\Omega$ the solid angle determined by $d\vartheta$. The distribution of ϑ as a function of ϑ and x is

$$\frac{dR}{dx d\Omega} = \left(\frac{b g_W^2}{M^2} \right)^2 \frac{\alpha m_k^3}{2^8 \pi^3} (2B - A) \left[(2 - \mu)^2 + \frac{Q^2 x^2}{16 r^2} - \frac{(2 - \mu) Q x}{2r} \right] \quad (8)$$

where

$$A = \frac{x^3 (1 - x)^3 (1 - \cos^2 \vartheta)}{(x + r - 1)^2 \left[1 - \frac{x}{2} (1 - \cos \vartheta) \right]^4} \quad (8a)$$

$$B = \frac{x^3 (1 - x)^2}{(x + r - 1)^2 \left[1 - \frac{x}{2} (1 - \cos \vartheta) \right]^2} \quad (8b)$$

For $\mu = Q = 0$ we have

$$\frac{dR}{dx d\Omega} = 1.081 \times 10^{-3} R_{\mu\nu} (2B - A). \quad (9)$$

Figure 4 shows the plot of $(2B - A)$ as a function of $\cos \vartheta$ for values of $r = 1$ and $x = 0.5$, $r = 4$ and $x = 0.725$, $r = 16$ and $x = 0.745$. These three particular values of x were chosen because for the corresponding values of r they correspond to the maximum of the photon energy.

*

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Table I

The branching ratio $B_\gamma = K \rightarrow e\nu\gamma / K \rightarrow e\nu$
as a function of the square of the ratio
of boson mass to kaon mass for zero quad-
rupole and anomalous magnetic moments.

Note that the branching ratio $K \rightarrow e\nu /$
 $K \rightarrow \mu\nu = 2.5 \times 10^{-5}$.

B_γ	120	50.0	28.9	19.0	13.5	7.9	5.2	2.7	0.62	0.15
$(M/m_K)^2$	1.00	1.25	1.50	1.75	2.00	2.50	3.00	4.00	8.00	16.00

REFERENCES

- 1) D.E. Neville, Phys.Rev. 124, 2037 (1961).

See also:

A. Kawazawa, M. Sugawara and K. Tanaka, Phys.Rev. 122,
341 (1961). Other references to work in this field
are given in these two papers.

Figure captions

- Fig. 1 : Feynman diagrams for $K \rightarrow e + \nu$ and $K \rightarrow e + \nu + \gamma$ channels.
- Fig. 2 : The function $F(x)$, which is proportional to the photon energy differential spectrum, plotted for $\mu = Q = 0$ for values of $r \equiv (M/m_K)^2$ equal to 1, 4 and 16. To read off values from the curves corresponding to $r = 4$ and $r = 16$ the vertical scale must be divided by 100 and 1000 respectively.
- Fig. 3 : The function $F_1(z)$ of the electron spectrum plotted for $\mu = Q = 0$ for values of $r \equiv (M/m_K)^2$ equal to 1, 4 and 16. To read off values from the curves corresponding to $r = 4$ and $r = 16$ the vertical scale must be divided by 100 and 1000 respectively.
- Fig. 4 : The function $2B - A$ of the angular correlation between photon and electron plotted for $\mu = Q = 0$ for the values $r = 1$ and $x = 0.5$, $r = 4$ and $x = 0.725$, $r = 16$ and $x = 0.745$. For the curves corresponding to $r = 4$ and $r = 16$ the vertical scale must be divided by 10 and 100 respectively.

(a)

(b)

(c)

(d)

Fig.1

Fig.2

Fig.3

Fig. 4