CM-P00056796

THE UNITARY SYMMETRY AND A CONNECTION BETWEEN THE $|\Delta S| \neq 2$ AND
 $|\Delta \vec{I}| = \frac{1}{2}$ RULES

B. d'Espagnat and J. Prentki

CERN - Geneva

A B S T R A C T

The assumption is made that the weak interactions are invariant under a three dimensional rotation subgroup "G" of SU_3 , which of course is not the isospin subgroup. It is then shown that for non-leptonic and leptonic decays, the rule $|\Delta \vec{I}| = \frac{1}{2}$ follows from the requirement $|\Delta S| \neq 2$ and vice-versa. In such a theory these selection rules are equivalent.

I. Introduction and Postulates

A great interest was recently raised by the hypothesis of unitary symmetry ^{1),2)} which, briefly and roughly stated, is the assumption that the bulk of the strong interaction is invariant under the group SU_3 . This obviously sets the problem of investigating the possible links that the weak interaction theory may have to that scheme. Although some preliminary attempts have already been made in this direction ^{3),4)} it has, to our knowledge, not yet been made clear how, exactly, one can hope to establish these links in such a way as to preserve three essential empirical rules of the weak interaction phenomena, namely the $|\Delta S| \neq 2$ and $|\Delta \vec{I}| = \frac{1}{2}$ selection rules for non-leptonic decays and the absence of strange particle decays with an uncharged lepton pair among the decay products. The present article is an attempt in that direction.

Before we begin, it is proper to insist on how uncertain and speculative such attempts necessarily are : a) it is not at all proved that the strong interaction really have anything to do with SU_3 , b) even if they have, the statement that the same is true, in some way, for weak interactions is just a guess and c) granted that a link of some kind does exist between these interactions and SU_3 , the nature of this link is still necessarily a subject for further arbitrary assumptions. Without insisting any further on the fragility of points a) and b), let us now describe our choice for point c), i.e., the special assumptions which we choose to make regarding weak interactions. These are essentially three in number.

- i) Our first assumption is that the weak interaction Lagrangian is invariant under some sub-group "G" of SU_3 . An assumption of this kind has already been made some years ago ⁵⁾ in connection with another theory of strong interactions - due originally to Salam and Polkinghorne - in which the bulk of the strong interactions was assumed to be invariant under the group O_4 (a theory sometimes referred to as : "N - Ξ invariance").

It was shown at that time that - under suitable conditions - a strict $|\Delta \vec{I}| = \frac{1}{2}$ rule can be derived from such a scheme (by "strict" it is meant here that the rule remains strictly valid even when those strong interactions which are not invariant under O_4 are taken into account ; it is violated only by electromagnetic corrections). We still consider as attractive this general idea of deriving isospin - and possibly strangeness - selection rules from a law of invariance : in fact we would consider it as very unlikely that, in this field, the empirical selection rules should not follow from some invariance rule as they usually do in other fields of physics. In present days, however, the emphasis has shifted, for very good reasons, from O_4 to $S U_3$ as a possible group for the main part of strong interactions. A natural thing to do is, therefore, to enquire whether the trick previously used in connection with O_4 also works in the new picture. To be precise, we thus split the strong interactions in two terms : one of them, which may be dominating and which for that reason we call V.S., is invariant in $S U_3$. The other one, M.S., is invariant only in that sub-group of $S U_3$ which corresponds to isospin and hypercharge conservation and which we call "I". Similarly we now require that weak interaction should be invariant in another sub-group of $S U_3$, "G". A supplementary requirement is made that "G" should have the same structure as "I", i.e., should be, essentially, a 3 dimensional rotation group.

- ii) Our second assumption is the one, very commonly made, of partially conserved currents multiplied by their Hermitian conjugates. The motivations for using only conserved or partially - conserved - currents have often been given in the literature and need not be reviewed here. Let it simply be stated that we require the weak interaction to take the form of a sum of products $C C^+$, the C 's being (vector and pseudovector) partially conserved currents, i.e., conserved in the limit that the M.S. interactions go to zero (and also, eventually, the baryon masses). Such currents are 16 in number⁶⁾ (the two currents corresponding to baryon number conservation are not added, here, to this list).

The purpose of the present paper is to show that assumptions i) and ii) establish a one-to-one correspondence between the two empirical rules $|\Delta S| \neq 2$ and $|\Delta \vec{I}| = \frac{1}{2}$ in the sense that if one of them is valid, the other one is then automatically satisfied.

II. Description of SU_3

We describe SU_3 by its eight infinitesimal generators written in canonic form : they are $H_1, H_2, E_{\pm 1}, E_{\pm 2}, E_{\pm 3}$. $\sqrt{3} H_1$ is just the 3rd component I_3 of isospin, $\sqrt{3} E_{\pm 1}$ are, similarly, the operators $I_{\pm} = \frac{I_1 \pm i I_2}{\sqrt{2}}$ in conventional notations. $2H_2$ is the hypercharge U up to an additive constant which depends on the representation. The $E_{\pm 2}$ operators correspond to $\Delta U = \Delta Q = \pm 1$ whereas the $E_{\pm 3}$ operators correspond to $\Delta Q = 0, \Delta U = \pm 1$. For the sake of reference, we give below the connections between the E's and H's and the λ_i of Ref. ²⁾. These are

$$\begin{aligned} \sqrt{6} E_{\pm 1} &= \frac{\lambda_1 \pm i \lambda_2}{2} ; \quad \sqrt{6} E_{\pm 2} = \frac{\lambda_4 \pm i \lambda_5}{2} , \quad \sqrt{6} E_{\pm 3} = \frac{\lambda_6 \pm i \lambda_7}{2} \\ \sqrt{3} H_1 &= \frac{\lambda_3}{2} ; \quad 2 H_2 = \frac{\lambda_8}{\sqrt{3}} . \end{aligned} \quad (1)$$

In any given representation, these operators are represented by matrices acting on a one-column matrix ψ made with the baryons or with the meson fields :

$$\psi = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix} \quad (2a)$$

and

$$\psi = \begin{pmatrix} p \\ n \\ \Lambda \\ \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \\ \Xi^0 \\ \Xi^- \end{pmatrix} \quad (2b)$$

for baryons in the "3" and "8" representations, respectively. The 16 partially conserved vector and pseudovector currents mentioned in assumption ii) of Section I are then of the form :

$$\mathcal{E}_i = \sum_z \bar{\Psi}_z O_\mu^{(2)} E_i \Psi_z ; \quad \mathcal{H}_i = \sum_z \bar{\Psi}_z O_\mu^{(2)} H_i \Psi_z \quad (3)$$

where $r=b$ (aryons) or m (esons), $O_\mu^{(b)} = \gamma_\mu$ or $\gamma_\mu \gamma_5$, $O_\mu^{(m)} = \frac{\vec{\sigma}}{\sigma_{x_\mu}}$ or 0.

The explicit form for the current can be slightly different than that given by (3).

For example, in the pseudovector currents some terms proportional to the derivatives of the meson fields are present. However, all these additional terms transform under SU_3 exactly as $\mathcal{E}_i$ and $\mathcal{H}_i$. This means that irrespective of the representations to which mesons and baryons belong, the current belongs to the representation 8 of SU_3 . That is the only property which is important and essential in our considerations. As we are not at present interested by the space-time properties of the interactions, we forget about these details, omit the symbols $O_\mu^{(r)}$ and write symbolically

$$\mathcal{E}_i = \bar{\Psi} E_i \Psi ; \quad \mathcal{H}_i = \bar{\Psi} H_i \Psi \quad (3')$$

this notation having the further advantage that it does not imply the use of a particular representation nor a decision as to which particles are considered as elementary. Specifically, one has

$$\sqrt{3} \mathcal{H}_1 = \mathcal{J}_3 ; \quad \sqrt{3} \mathcal{E}_{\pm 1} = \mathcal{J}_{\pm} ; \quad 2 \mathcal{H}_2 = \mathcal{U} \quad (4)$$

$\mathcal{J}_{\pm} = (\mathcal{J}_1 \mp i \mathcal{J}_2) / \sqrt{2}$, $\mathcal{J}_3$ being the conserved currents associated with isospin and $\mathcal{U}$ the conserved current associated with hypercharge. One has also

$$\mathcal{J} = \bar{\Psi} X \Psi ; \quad \dot{\mathcal{J}} = \bar{\Psi} \dot{X} \Psi \quad (4')$$

with

$$X = \begin{pmatrix} E_{+2} \\ E_{+3} \end{pmatrix} ; \quad \dot{X} = \begin{pmatrix} -E_{-3} \\ E_{-2} \end{pmatrix} \quad (4'')$$

$\mathcal{J}$ and $\dot{\mathcal{J}}$ are the isospinor pseudoconserved currents corresponding to transitions with $\Delta S = \pm 1$ and $|\Delta \vec{I}| = \frac{1}{2}$.

III. Search for Sub-groups adapted to Weak Interactions

Let us now look for sub-groups "G" of SU_3 adapted to the description of weak interactions in the sense that they fit the demands of assumptions i) and ii) in Section I : weak interactions should be invariant under all operations of "G", they should involve only the partially conserved currents $\mathcal{E}_i$ and $\mathcal{H}_i$ of Section II and, more precisely, they should take the form of sums of products $C C^+$, the C 's being linear combinations of the $\mathcal{E}_i$ and $\mathcal{H}_i$.

Due to the fact that weak interactions with $|\Delta S| = 1$ exist, some of the C 's, at least, are necessarily linear combinations of $\Delta S = 0$ and $\Delta S = 1$ currents : the charged C 's, in particular, are then a combination of $\bar{\psi}_{E_1} \psi$ and $\bar{\psi}_{E_2} \psi$ for $\Delta Q = 1$ and a combination of $\psi_{E_{-1}} \bar{\psi}$ and $\psi_{E_{-2}} \bar{\psi}$ for $\Delta Q = -1$. We therefore set

$$C^\pm \equiv G^\pm \equiv J^\pm \cos \alpha \pm \tilde{J}^\pm \sin \alpha \equiv \sqrt{3} \bar{\psi} (E_{\pm 1} \cos \alpha + E_{\pm 2} \sin \alpha) \psi_{(5)}$$

α being an arbitrary parameter.

The G 's should, because of assumption i), be connected to a sub-group "G" of SU_3 , in much the same way as the J 's are connected to the isospin sub-group "I". We therefore enquire whether the operator

$$G^{(+)} = \sqrt{3} (E_{+1} \cos \alpha + E_{+2} \sin \alpha) \quad (6)$$

can be considered as one of the infinitesimal generator of a 3-dimensional rotation group "G" that would be a sub-group of SU_3 . For that purpose we construct (the definition of $G^{(-)}$ being obvious)

$$G_3 = [G^{(+)}, G^{(-)}] \quad (7)$$

the commutation laws of the E 's and H then give

$$G_3 = \sqrt{3} H_1 \left(1 - \frac{\sin^2 \alpha}{2} \right) + \frac{3}{2} H_2 \sin^2 \alpha - \sqrt{\frac{3}{2}} (E_3 + E_{-3}) \frac{1}{2} \sin 2\alpha \quad (8)$$

and it is easily checked, by the same procedure, that

$$[G_3, G_{\pm}] = \pm G_{\pm} \quad (9)$$

this shows that G_+ , G_- and G_3 are indeed the three infinitesimal generators of a three-dimensional rotation group "G". The existence of this sub-group is not at all surprising; indeed it is easily checked that

$$\vec{G} = R^{-1} \vec{I} R \quad (10)$$

with

$$\begin{aligned} \vec{G} &= (G^+, G^-, G_3); \quad \vec{I} = (I^+, I^-, I_3) \\ R &= e^{i\lambda_3 \alpha} = e^{\sqrt{6} (E_3 - E_{-3})} \end{aligned} \quad (11)$$

R is therefore a (finite) transformation of SU_3 and, correspondingly, the "G" being the transformed group of "I" by means of R , is also a sub-group of SU_3 .

This simple correspondence between "I" and "G" is useful in that it permits to deduce interesting features of "G" from well-known properties of "I". In particular it is well-known that, in "I", the partially conserved currents $\mathcal{E}_i, \mathcal{H}_i$ split into a vector $\vec{J} = (J^{(\pm)}, J_3)$, a scalar U and two spinors ξ and $\bar{\xi} = i\tau_2 \xi^*$. Similarly, in "G", independent linear combinations of the same currents must split into a vector $\vec{J} = (J^{(\pm)}, J_3)$, a scalar V and two spinors ξ and $\bar{\xi}$. The vector $\vec{J}$ is the one that we know already:

$$G^{(\pm)} = \bar{\Psi} G^{(\pm)} \Psi. \quad (12)$$

$$G_3 = \bar{\Psi} G_3 \Psi. \quad (13)$$

in the shorthand notation where the ψ 's etc., have been suppressed, as explained after Eq. (3).

The scalar V can be deduced from $\mathcal{U}$ by means of R or found directly. It is

$$V = \bar{\Psi} V \Psi \quad (14)$$

in the same shorthand notation, with

$$V = \sqrt{3} H_1 \sin^2 \alpha + H_2 (2 \cos^2 \alpha - \sin^2 \alpha) + \sqrt{6} (E_3 + E_{-3}) \frac{1}{2} \sin 2\alpha \quad (15)$$

Similarly the isospinors (in G) ζ and $\dot{\zeta}$ can be written as

$$\zeta = \bar{\Psi} Z \Psi, \quad \dot{\zeta} = \bar{\Psi} \dot{Z} \Psi \quad (16)$$

with $Z = R X R^{-1}$, $\dot{Z} = R \dot{X} R^{-1}$ where X and $\dot{X}$ are defined by (4''). Then obviously

$$Z = \begin{pmatrix} E'_{+2} \\ E'_{+3} \end{pmatrix}, \quad \dot{Z} = \begin{pmatrix} -E'_{-3} \\ E'_{-2} \end{pmatrix} \quad (16')$$

with

$$\begin{aligned} E'_{\pm 2} &= E_{\pm 2} \cos \alpha - E_{\pm} \sin \alpha \\ E'_{\pm 3} &= E_{\pm 3} \cos^2 \alpha - E_{\mp 3} \sin^2 \alpha - \frac{\sqrt{3} H_2 - H_1}{2 \sqrt{2}} \sin 2\alpha \end{aligned} \quad (16'')$$

8.

Finally one should also mention the scalar in SU_3 defined as $\underline{1} = \bar{\psi} \psi$ which of course is a scalar both in I and in G .

With all these currents one can easily construct the most general Lagrangian (with current-current interaction) which is invariant in "G". One has

$$\mathcal{L} = a \vec{g}^2 + b \left(\frac{V}{2} + \kappa \underline{1} \right)^2 + c \bar{\psi} \psi \quad (17)$$

(notice that $\psi^{(\pm)+} = \psi^{(\mp)}$, $\psi_3^+ = \psi_3$, $V = V^+$, $\bar{\psi} = i\tau_2 \psi^*$) (17) is therefore - in accordance with assumptions i) and ii) - our general expression of the non-leptonic leptonic weak interaction Lagrangian.

IV. Proof that $|\Delta S| \neq 2$ implies $|\Delta \vec{I}| < 3/2$ and conversely

We now come to the central point in the paper. It is easy to see that the Lagrangian - more exactly the 5 parameter family of Lagrangians - (16) is still too general to fit the experimental data; it contains $|\Delta S| = 2$ terms and $|\Delta \vec{I}| = 3/2$ terms. One thus has to restrict its generality. In the usual theories this is done by requiring separately both $|\Delta S| \neq 2$ and $|\Delta \vec{I}| < 3/2$. We show that, here, only one of these conditions has to be imposed in the theory and that the other one then automatically follows.

The proof is general and entirely independent of the choice of the representation for baryons and mesons. Call $[\vec{U}, I]_j$ a set of entities belonging to representation "j" of SU_3 ("j" = 1, 3, 3^* , 8, 10, 10^* , 27, etc.) and behaving in subgroup "I" as an entity of isospin I and hypercharge $\vec{U}$. The most general scalar in I that can be constructed with the partially conserved currents (4) is in analogy to (17)

$$S = a \vec{J}^2 + b \left(\frac{U}{2} + ic \underline{11} \right)^2 + c \vec{J}^+ \vec{J} \quad (18)$$

S is of course a linear combination of $[0,0]_j$'s with various values of j. Moreover, it is also - from its construction - a "8"x"8" product (as all partially conserved currents belong to "8"). Now

$$"8" \times "8" = "1" + "8" + "8" + "10" + "10^*" + "27" \quad (19)$$

and we know ¹⁾ that "10", "10*" do not contain any $[0,0]$. Therefore

$$S = \lambda (abcx) [00]_1 + \mu (abcx) [00]_8 + \nu (abcx) [00]_{27} \quad (20)$$

Let us now perform the operation R which leads - see (10) - from subgroup I to subgroup G. S is thereby transformed to $\mathcal{L}$ [Eq. (17)]. Now, a member of a given representation "j" cannot, under R, transform in anything else than members of the same representation "j". $\mathcal{L}$ therefore involves also no "10" and is thus given by

$$\lambda(abcx) [00]_1 + \mu(abcx) [f_1 [00]_8 + f_2 [1\frac{1}{2}]_8 + \dots] \\ + \nu(abcx) [g_1 [00]_{27} + g_2 [1\frac{1}{2}]_{27} + g_3 [21]_{27} + g_4 [1\frac{3}{2}]_{27} + \dots]_{(21)}$$

f_k and g_k being functions of α .

We now must remember that representation "8" contains only terms with $\vec{U} = 0, 1$ and $I = 0, \frac{1}{2}, 1$. We know on the other hand that g_3 is not zero because the general expression (16) with a, b, c, x arbitrary do involve some $|\Delta \vec{U}| = 2$, $|\Delta \vec{I}| = 1$ terms. Under these circumstances a necessary and sufficient condition for $|\Delta S| \neq 2$ (remember $\Delta S = \Delta U$) is

$$\nu(a, b, c, x) = 0 \quad (22)$$

but, then, $\mathcal{L}$ is a linear combination of representations "1" and "8" and therefore involves only $|\Delta \vec{I}| < 3/2$ terms, q.e.d.

The proof also works the other way round so that the absence of $|\Delta \vec{I}| < 3/2$ implies the absence of $|\Delta S| = 2$. The selection rule $|\Delta \vec{I}| < 3/2$ is of course equivalent to the usual rule $|\Delta \vec{I}| = \frac{1}{2}$ for strange particle non-leptonic decays so that a connection of a general nature has been established between the two empirical selection rules $|\Delta S| \neq 2$ and $|\Delta \vec{I}| = \frac{1}{2}$, both non leptonic. In weak interaction theory deduced from $SU_{3\rightarrow}$ and assumption ii) this same connection exists also between the $|\Delta S| \neq 2$ and $|\Delta \vec{I}| = \frac{1}{2}$ leptonic rules, as is well-known. Obviously all properties concerning the empirical rules for non-leptonic and leptonic decays deduced in this section remain valid also when the medium strong interactions are introduced. This is due to the fact that although these interactions break the invariance under the SU_3 group, they do conserve the ordinary isotopic spin I . (In this connection, look for a similar discussion in ref. 5)).

We would like to mention at that point that the proof just given is, in fact, independent of the choice of "G", i.e., of operation R : it obviously applies to any subgroup of SU_3 whose generators can be derived from those of I by means of any operation R of the group SU_3 , i.e., to any subgroup of SU_3 that is isomorphic to SU_2 . The condition, however, that the charge should be conserved also in weak interactions sets limitations on the possible choice of G, so that finally the choice ^{*)}

$$R = \exp [i\alpha \lambda_+ + i\beta \lambda_-]$$

seems the only possible one. The group "G" introduced in the Section III corresponds to the arbitrary choice $\beta = 0$.

^{*)} The transformations with λ_3 and λ_8 allowed by charge conservation are obviously not suitable here because they cannot induce strangeness non-conserving currents.

V. Structure of the $|\Delta S| = 1$ Interaction and Introduction of Leptons

Instead of using a proof based on group theory, it is of course also possible to arrive at the result of Section IV by an explicit calculation which, at the same time, gives a better insight in the detailed structure of such theories. This can still be done without specifying the representation used for baryons and mesons. The result is that condition (22) becomes

$$\frac{a+b}{4} - c = 0 \quad (23)$$

valid, of course, for any α and for $\beta = 0$. The $|\Delta S| = 1$ terms is then :

$$\begin{aligned} \mathcal{L}_w (|\Delta S| = 1) = & \left\{ \frac{3\sqrt{2}}{4} (3a-b) \left[\sqrt{2} (\mathcal{E}_{+1} \mathcal{E}_{-2} + \mathcal{E}_{+2} \mathcal{E}_{-1}) - \left(\mathcal{H}_1 - \frac{1}{\sqrt{3}} \mathcal{H}_2 \right) (\mathcal{E}_{+3} + \mathcal{E}_{-3}) \right] \right. \\ & \left. + \sqrt{6} b x \mathbb{I} (\mathcal{E}_{+3} + \mathcal{E}_{-3}) \right\} \frac{1}{2} \sin 2\alpha \end{aligned} \quad (24)$$

If this is re-expressed by means of $\vec{J}$, ξ , $\mathcal{U}$ and $\mathbb{I}$ it is easily found that this expression is a sum of isospinors (the first terms correspond to the combination $I = \frac{1}{2}$ of $I = 1$ and $\frac{1}{2}$). (24) can be used to construct any specific model one likes. For that purpose one has : a) to choose specific representations for the baryons and mesons, and b) to choose a definite set of values for the parameters a, b, c, x and α satisfying (22). All these schemes have in common the property described in Section IV. Instead of using CC^+ products one can of course, provided that the coefficients a, b, c, x are chosen non-negative, use a description in terms of intermediate bosons. It is then easily seen that the minimum number of such bosons is 5 (2 complex and 1 real field), i.e., one more than in the schizon and vector models. The maximum number is 9.

Leptons can of course be introduced into the scheme in the conventional way, i.e., by adding lepton currents to the charged currents $g^{(\pm)}$ and $j^{(+)}$ and $j^{(-)}$. For special choices of a, b, c , and x it is also possible to add lepton currents to $g_3, j^0, \bar{j}, \bar{V}$ in such a way that $|\Delta S| = 1$ decays with neutral lepton pairs are zero by cancellation.

This procedure for introducing leptonic currents is not satisfactory. What would be natural in a theory of this kind would be to consider the leptons as forming a basis for a 3 representation of SU_3 . Then, however, neutral lepton currents seem impossible to avoid and this, as is well-known, leads to contradictions with experiments.

VI. Conclusion

Only three theories of weak interactions have been considered that can be described by a current-current interaction [we mean $\sum J_{(k)\mu} J_{(k)\mu}^+$ - assumption ii), paragraph 7] or by intermediary vector bosons.

It has been shown :

- a) that a qualitatively correct description of the non-leptonic weak interaction phenomena is compatible with the further requirement that the theory be invariant under some subgroups G of SU_3 , different from the isospin group but also isomorphic to SU_2 ;
- b) that all such theories have in common the property that for them the two empirical selection rules $|\Delta S| \neq 2$ and $|\Delta I| = \frac{1}{2}$, both leptonic and non-leptonic are equivalent requirements in the sense that the validity of one of these rules automatically implies the validity of the other one.

Conclusions a) and b) are independent from any other details of the models. They remain valid also when the medium strong interactions, which are only I-invariant, are switched on.

R E F E R E N C E S

- 1) Ikeda, Oyawa and Y. Ohnuki, Prog.Theor.Phys. 22, 715 (1959);
Y. Yamaguchi, Prog.Theor.Phys.Suppl. No.11, (1959);
J. Wess, Nuovo Cimento 10, 15 (1960);
M. Gell-Mann, "The eightfold way", preprint.
- 2) M. Gell-Mann, "Symmetries of baryons and mesons", submitted to Phys.Rev.;
Y. Neeman, Nuclear Physics, in press;
A. Salam and J. Ward, Nuovo Cimento, in press.
- 3) S.L. Glashow and M. Gell-Mann, "Gauge theory of vector particles",
report CTSL 28, (1961).
- 4) L.A. Radicati and D. Speiser, to be published.
- 5) B. d'Espagnat, J. Prentki and A. Salam, Nuclear Physics 5, 441 (1958).
- 6) M. Gell-Mann, La Jolla Conference on Strong and Weak Interactions,
June 1961.
- 7) R.E. Behrends, J. Dreitlein, C. Fronsdal and B.W. Lee, "Simple groups
and strong interaction symmetries", to be published.