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LOW MOMENTUM TRANSFER REACTIONS IN COMPLEX NUCLEI AT HIGH ENERGY

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ABSTRACT

The radiochemical cross-sections for simple spallation reactions at very high energy are calculated from the elementary nucleon-nucleon and pion-nucleon interactions using the impulse approximation and are expressed in terms of nuclear form factors. Elimination of the nuclear form factors by comparison of pion and proton induced reactions may give information on the (π π) interaction.

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I. Introduction

The purpose of the present article is to express the cross-section for certain very simple spallation reactions in terms of the elementary interactions in the nucleus as well as to draw attention to the possibility of using simple spallation reactions as a crude tool for studying effects of very low momentum transfers in pion-nucleon and nucleon-nucleon collisions.

Spallation reactions induced by fast strongly interacting particles usually involve a very profound change of the initial nucleus, the charge and mass numbers of the residual nucleus differing by many units from the original one. The main part of spallation reactions is reasonably well described by an initial cascade of a few fast particles followed by a long evaporation chain. The large number of evaporation particles (of the order 10-20) gives in this case the dominating characteristics to the process, and consequently most of the information on the initial interaction is lost by averaging and folding.

In contrast to this majority of spallation products, there is a small group of residual nuclei, which are characterized by small changes of the initial nucleus. They are mainly those in which the initial nucleus (Z, A) is transformed into the residual nucleus $(Z + \Delta Z, A + \Delta A)$ with $\Delta Z \leq 1$ and $\Delta A \leq 1$. We can subdivide these reactions into :

- 1) $\Delta Z \leq 1, \Delta A = 0$, i.e., the mass number is unchanged and equivalent to a net transfer of pions only. We refer to these as pion exchange reactions.
- 2) $\Delta Z \leq 1, \Delta A = 1$, i.e., the mass number has changed by one unit so that these reactions must involve the net exchange of one nucleon. This group contains in particular the nucleon capture reaction ^{*)} and the "knock-out" reaction. We refer to these as nucleon transfer reactions.

^{*)} Nucleon capture obviously occurs only for nucleon projectiles.

Since the complex nucleus is a very loosely bound system when investigated with many GeV particles, the pion exchange and nucleon transfer reactions are very rare events. The residual nucleus must be left with an excitation below the threshold for particle emission, ≈ 8 MeV, as otherwise further nuclear break-up immediately follows by particle emission. This very strong restriction on the excitation energy of the residual nucleus implies correspondingly strong restrictions on the momentum transfer: the momentum distribution of such a lowly excited residual nucleus is nearly the same as in the ground state. The maximum effective momentum transfer in the collision is thus of the order of twice the Fermi momentum $p_F \sim \mu$ ^{*)}, which is very small. Consequently, the simple spallation reactions tend to select the part of the nucleon-nucleon and pion-nucleon interaction in the nucleus which has momentum transfers to the target nucleon of the order μ . This is of particular interest in the case of the pion-exchange processes, since it is known that the contribution from the one-pion exchange is very important for these momentum transfers due to the neighbourhood of the pole in the pion propagator. The pion exchange spallation reactions can therefore in principle be used for crude studies of effects of one-pion exchange. The crudeness is due to the fact that nuclear effects cannot be entirely eliminated. In addition, there are experimental limitations to the information which can be extracted chemically: the price paid for the easy study of these very rare events ^{**)} is the averaging of momentum transfers over the nuclear momentum distribution ^{***)} .

*) We use units so that $\hbar=c=1$; μ denotes the pion mass.

**) The cross-sections are detected either by the induced radioactivity or by chemical separation of elements followed by mass separation of isotopes, which both can be done for quite small cross-sections.

***) There have, however, recently been studies of the angular distribution of the recoil nucleus by radiochemistry, so that the momentum transfer to the residual nucleus can, in principle, be measured by radiochemical methods.

In Section 2 we will discuss the magnitude of the "pion-exchange" spallation cross-section in terms of the elementary interactions of low momentum transfer in the nucleus. The discussion is based on a Born approximation with absorption effects in the incident and outgoing channel. The nuclear effects are obtained as a mixed form factor. In Section 3 we calculate the dominating contributions to the elementary cross-section corresponding to simple types of one-pion exchange reactions. The one-pion exchange diagrams should be of particular importance in the very low momentum transfer reactions we are discussing due to the neighbourhood of the pole in the pion propagator. In Section 4 we discuss the possibility of eliminating the nuclear form factors by comparing proton and pion induced reactions and the possibility of obtaining information on the $(\pi\pi)$ scattering from the radiochemical cross-sections.

2. The nuclear cross-section

At present very little experimental data are available on the "pion-exchange" spallation reactions at energies of the order of 25 GeV. In view of this and in view of the fact that our main interest in these spallation reactions is the relative cross-section of proton and pion induced reactions, we will only aim at rather crude estimates of the magnitude of the radiochemical cross-sections. In this section we will relate these cross-sections to the elementary pion-nucleon and nucleon-nucleon interactions.

The observation of a quasi-stable residual nucleus by radiochemical techniques implies a nucleus which has been left with so low an excitation energy that further particle emission cannot occur, i.e., in practice at an excitation energy below the threshold for neutron emission, B. This is

a very strong restriction on the final states, since a nucleon is very loosely bound to the nucleus from the view-point of a many GeV particle. The delicate balance in nuclear binding is very easily upset already by one inelastic scattering of the incident particle in the nucleus; further inelastic scattering almost inevitably leads to nuclear break-up. It is therefore natural to describe the "pion-exchange" radiochemical cross-sections in perturbation theory. In addition to the special inelastic scattering considered, the incident and outgoing waves are elastically scattered and undergo attenuations due to other inelastic processes. These effects are naturally described in the optical approximation. The discussion is simplified by the distinguishability of the projectile from the target nucleons due to the high bombarding energy.

The optically distorted incident wave of momentum k_1 is denoted by $\phi_{\vec{k}_1}^{(+)}$; the (time reversed) wave of the outgoing particles is denoted by $\phi_{\vec{k}_2}^{(-)} v(\xi)$, where the first part refers to the motion of their c.m., and $v(\xi)$ describes their internal motion in terms of the internal co-ordinates ξ . The initial nuclear state is denoted by $|0\rangle$ and that of the residual nucleus by $|n\rangle$.

In the spirit of high-energy reactions, we describe the nucleon-nucleon (or pion-nucleon) interaction in the impulse approximation, i.e., by the free particle scattering matrix t_i for the i th target nucleon. The Born approximation matrix element is thus

$$M_{n0}(\vec{k}_1, \vec{k}_2, \xi) = \langle \phi_{\vec{k}_2}^{(-)} v(\xi) n | \sum_i t_i | 0 \phi_{\vec{k}_1}^{(+)} \rangle \quad (1)$$

where we have labelled the matrix element also by ξ , so as to remind that the same final nuclear state $|n\rangle$ corresponds to many different channels of the outgoing products. At high energy the free nucleon-nucleon or pion-nucleon scattering matrix is well approximated by a local operator:

$$t_i = t(\vec{r} - \vec{r}_i) \delta(\vec{r} - \vec{r}') \delta(\vec{r}_i - \vec{r}'_i) \quad (2)$$

where $\vec{r}$ and $\vec{r}'$ refer to c.m. variables of incident and outgoing particles, $\vec{r}_i$ and $\vec{r}'_i$ to target nucleons - which is to say that the Fourier transform of t_i is only a function of the momentum transfer $\vec{q}$. The free particle scattering matrix is still spin and isospin dependent, as well as permitted to create secondary particles.

The scarce experimental material hardly justifies any detailed treatment of the optical distortion of the waves. The refraction in the medium is negligible and the main optical effects are due to absorption into inelastic channels. We will for the time being neglect the absorption and come back to it later.

Without absorption the incident and outgoing channels are simply described by plane waves

$$\begin{aligned} X_{\vec{r}_1}^{(+)} &= u_1 e^{i\vec{K}_1 \cdot \vec{r}} \\ X_{\vec{r}_2}^{(-)} &= e^{i\vec{K}_2 \cdot \vec{r}} \end{aligned} \quad (3)$$

The factor u_1 is the spin wave function of the incident particle; corresponding functions for the outgoing wave are contained in the function $v(\xi)$ describing the internal motion of the outgoing particles. Writing the free scattering matrix as

$$t_i = \delta(\vec{r} - \vec{r}') \delta(\vec{r}_i - \vec{r}_i') \int e^{i\vec{q}(\vec{r} - \vec{r}_i)} t(\vec{q}) d\vec{q} \quad (4)$$

the matrix element $M_{no}(\xi)$ becomes in terms of the momentum transfer $\vec{q} = \vec{k}_1 - \vec{k}_2$

$$M_{no}(\xi) = (2\pi)^3 \langle n v(\xi) | t(\vec{q}) \sum_i e^{i\vec{q} \cdot \vec{r}_i} | u_1 0 \rangle \quad (5)$$

The measurements of radiochemistry only give yields of states in the residual nucleus without specifying what particles are leaving the nucleus. It is therefore at this point appropriate to square the matrix element of Equ. (5) and sum over all final states compatible with the residual nucleus in state $|n\rangle$. We furthermore average over spin directions as ordinarily in a free collision process of unpolarized projectiles. The cross-section to final state n , $\sigma(n)$, is thus in terms of the elementary cross-section of momentum transfer $\vec{q}$, $d\sigma^{(\lambda)}/d\vec{q}^2$,

$$\sigma(n) = \int \frac{d\sigma^{(\lambda)}}{d\vec{q}^2} \left| \langle n | \sum_i e^{i\vec{q} \cdot \vec{r}_i} \tau_i^{(\lambda)} | 0 \rangle \right|^2 d\vec{q}^2 \quad (6)$$

where $\vec{\tau}_i$ is the isospin operator and λ refers to changes in target nucleon charge ^{*)}. The last factor of Equ. (6) is the square of a mixed

*) Since the radiochemically observed cross-sections nearly always involve charge changes $\lambda = \pm 1$, we will not discuss the special case of no charge change though the analysis closely follows the same lines as here.

form factor $F_{n0}^{(\lambda)}(\vec{q})$ for the nuclear states n and 0 and contains the nuclear structure effects.

$$F_{n0}^{(\lambda)}(\vec{q}) = \langle n | \sum_i e^{i\vec{q} \cdot \vec{r}_i} z_i^{(\lambda)} | 0 \rangle \quad (7)$$

The total observed cross-section is that obtained by summing over all final states n of the residual nucleus which decay predominantly by γ -emission to the radiochemically observed state, i.e., the ground state in most cases. The sum over n is thus in practice for most nuclei carried out for n with excitation energy E_n^* smaller than the neutron binding B ; for higher excitation particle emission will dominate over γ -emission. Thus the radiochemical cross-section is

$$\sigma_{rc} = \int \frac{d\sigma^{(\lambda)}(\vec{q})}{d\vec{q}^2} \sum_{\substack{n \\ E_n^* < B}} |F_{n0}^{(\lambda)}(\vec{q})|^2 d\vec{q}^2 \quad (8)$$

The common treatment of a sum of squared mixed form factors over final states is to apply closure and express the result in terms of two-body pair correlation functions. This treatment cannot be used in Equ. (8) since only a small part of the final states occur in the sum. We can, however, obtain an estimate of the cross-section on the basis of a single particle description. From the shell model it is clear that the form factor only can involve particle-hole excitations. Thus, denoting by $|a\rangle$ and $|b\rangle$ the unoccupied particle state and the hole state single particle wave functions and by ϵ_a and ϵ_b the corresponding excitation energies

$$\sum_{\substack{n \\ E_n^* < B}} |F_{n0}^{(\lambda)}(\vec{q})|^2 = \sum_{\epsilon_a + \epsilon_b < B} |\langle a | e^{i\vec{q} \cdot \vec{r}} | b \rangle|^2 \quad (9)$$

It is clear from Equ. (9) that the radiochemical cross-sections for these very high energy reactions still will in part reflect low energy nuclear shell structure. No experimental evidence is at present available for or against shell effects; since in the rest of our discussion we will aim at eliminating nuclear factors we will just estimate the sum of the form factors using the Fermi gas picture. We, therefore, consider the initial and final nuclei as Fermi spheres with a Fermi energy $\mathcal{E}_F = p_F^2/2M$, the same for neutrons and protons ^{*)}. The operator $e^{i\vec{q}\cdot\vec{r}}$ displaces the Fermi sphere by $\vec{q}$; we have to seek the condition that an initial particle state $\vec{p}_i$ is below the Fermi sphere and the final state $\vec{p}_f$ above the Fermi sphere so that the excitation energy $(\vec{p}_f^2 - \vec{p}_i^2)/2M < B$. It is shown in Appendix 1 that in this case

$$\sum_{E_i^* < B} |F_{no}^{(\lambda)}(\vec{q})|^2 \simeq \begin{cases} \frac{A}{2} \frac{3}{16} (B/\mathcal{E}_F)^2 \frac{p_F}{q} & ; \quad 0 < q^2 < 4 p_F^2 \\ 0 & q^2 > 4 p_F^2 \end{cases} \quad (10)$$

Since the momentum transfer is very low for non-vanishing contributions to this form factor it is convenient to substitute $T = q^2/2M$, the non-relativistic kinetic energy of a target nucleon after collision when it initially was in its rest system. Substitution of Equ. (10) in Equ. (8) then yields

$$\sigma_{nc} \simeq \frac{A}{2} \frac{3}{16} \left(\frac{B}{\mathcal{E}_F} \right)^2 \int_0^{4\mathcal{E}_F} \frac{d\sigma^{(\lambda)}}{dT} \left(\frac{\mathcal{E}_F}{T} \right)^{1/2} dT \equiv \frac{A}{2} \frac{3}{16} \left(\frac{B}{\mathcal{E}_F} \right)^2 4\mathcal{E}_F < \frac{d\sigma^{(\lambda)}}{dT} > \quad (11)$$

*) The results remain valid to a good approximation even when the neutron and proton Fermi energies are different.

This expression has resulted from an impulse approximation using plane waves. On the other hand the incident wave is certainly distorted on entering the nucleus mainly due to absorption in the nuclear medium.

In order to take absorption into account we first observe that both the incident projectile and the c.m. of the outgoing particles to a very good approximation follow a straight trajectory through the nucleus. Furthermore, we observe that the outgoing particles are produced in such a small cone in the laboratory system that they will spread by less than a nucleon diameter within the nucleus. They can thus approximately be considered as one strongly interacting entity with a total cross-section σ_2 for inelastic scattering. Consequently the absorption can approximately be accounted for in terms of the nuclear density $\omega(\vec{r})$, the total inelastic cross-sections in entrance and exit channels σ_1 and the impact parameter b with the beam direction as the x-axis, as

$$\begin{aligned}\phi_{\vec{R}_1}^{(+)} &= u_1 e^{i\vec{K}_1 \cdot \vec{r} - \frac{1}{2} \int_{-\infty}^x \omega(x, \rho) \sigma_1 dx} \\ \phi_{\vec{R}_2}^{(-)} &= e^{i\vec{K}_2 \cdot \vec{r} - \frac{1}{2} \int_x^{\infty} \omega(x, \rho) \sigma_2 dx}\end{aligned}\quad (12)$$

The reduction introduced in the Fermi gas estimate of the cross-section can then be obtained by calculating the effectively participating part of the nucleus which is a fraction R of the total nuclear volume. From Equ. (12) and the nuclear density distribution

$$R \approx \frac{\int \omega(\vec{r}) \exp \left\{ - \int_{-\infty}^x \sigma_1 \omega(x, \rho) dx - \int_x^{\infty} \sigma_2 \omega(x, \rho) dx \right\} d\vec{r}}{\int \omega(\vec{r}) d\vec{r}} \quad (13)$$

To estimate the reduction factor R of Equ. (13) we take σ_2 equal to σ_1 and assume a) a constant nuclear density distribution, which underestimates R due to lack of diffuseness in the nuclear surface; b) a Gaussian density distribution, which over-estimates R due to a too diffuse surface.

$$R_{cd} = \frac{3}{R^3} \left\{ \frac{1}{4\sigma_1^3 \omega_0^3} - e^{-2\sigma_1 \omega_0 R} \left(\frac{R^2}{2\sigma_1 \omega_0} + \frac{R}{2\sigma_1^2 \omega_0^2} + \frac{1}{4\sigma_1^3 \omega_0^3} \right) \right\} \approx \frac{\pi}{\sigma_1^3 \omega_0^2 A} \quad (14a)$$

$$R_G = \frac{2\pi}{3} \frac{\langle r^2 \rangle}{\sigma_1 A} \quad (14b)$$

The two estimates represent rather extreme assumptions about the nuclear density distribution, but in spite of this, they differ by less than a factor 2 for $A < 70$. They should thus suffice as an orientation in this region, in spite of the fact that their values are quite different for very large A . As an illustration of the reduction effect the numerical value of R is for $A = 64$, $\sigma_1 = 30$ mb and $r_0 = 1.2$ F : $R_{cd} = \frac{1}{15}$ and $R_G = \frac{1}{9}$.

In this section we have aimed at two things; 1) to show that the radiochemical cross-section approximately can be written as a nuclear form factor multiplied with the elementary cross-section at low momentum transfers. Elimination of the nuclear term can thus give information on the elementary process; 2) to obtain a crude estimate of the cross-section, so as to verify that the description of the reaction in a distorted wave impulse approximation is not in contradiction with reality. Some numerical comparisons will be made in Section 4.

3. The elementary process

We assumed in the preceding section that the processes under consideration are generated from collisions of the incoming particle with one of the nucleons of the target nucleus. The logical next step in order to get numerical values for the radiochemical cross-sections would consist in introducing experimental values of $d\sigma^{(\lambda)}/dT$ in Equ. (11). This cannot be done, however, because of the present scarcity of experimental knowledge of the elementary cross-section.

Fortunately, there seems to be a way of calculating $d\sigma^{(\lambda)}/dq^2$, at least for low values of q^2 , which are those we are interested in. This is possible using the so-called peripheral interactions ¹⁾ which are due to the following simple physical effect: the interaction between two colliding strongly interacting particles comes in general from the exchange of virtual systems (e.g. π -mesons) which can be thought to belong to the field of either particle. The peripheral interaction corresponds to the transfer of the simplest of these virtual systems, namely one single π -meson. Of course, the exchanged π -meson can either be directly emitted and absorbed, thus generating an elastic scattering or can lead to the creation of new particles (mainly π -mesons) both at the point of emission and at that of absorption. In order to fix ideas consider the graphs drawn in Fig. 1. We will always consider the target nucleon as entering from the left in the lower vertex. The typical peripheral effect comes from the propagator of the virtual pion which is $(q^2 + \mu^2)^{-1}$ (μ being the pion mass) and gives a characteristic big cross-section for small q^2 . In principle, we could consider any number of outgoing particles in the upper and lower vertices of Fig. 1.

There are, however, two general objections against considering too many particles coming out from the lower vertex. First there would be many low energy pions which would therefore have a big probability of interacting again before leaving the nucleus. Secondly, the emerging nucleon would not have a spectrum peaked to very low ($\lesssim 30$ MeV) energy in the system in which it is initially at rest, as it is the case when it comes out alone or with one π -meson. Therefore, we will limit ourselves to consider the graphs of Fig. 1, since the other graphs lead to reactions which are not compatible with a quasi-stable final nucleus.

The formula for $d\sigma^{(\lambda)}/dq^2$ obtained from the first graph was first given by Chew and Low²⁾ and has been used many times in the literature. We write it down for ease of reference^{*)}:

$$\frac{d\sigma^{(\lambda)}}{dq^2} = \frac{f^2}{\pi p_i^2 \mu^2} \frac{q^2}{(q^2 + \mu^2)^2} \int_{m+\mu}^{g(q^2)} \omega R(\omega) \sigma(\omega) d\omega \quad (15)$$

where $f^2 = 0.08$, p_i is the lab. momentum of the incoming proton, and ω the total energy of the upper bulk of particles in their c.m. system.

Furthermore

$$g(q^2) = \left[m^2 - W^2 \frac{q^2}{2m^2} + W(W^2 - 4m^2)^{1/2} \left(\frac{q^4}{4m^4} + \frac{q^2}{m^2} \right)^{1/2} \right]^{1/2} \quad (16)$$

$$R(\omega) = \frac{1}{2} \left[\omega^4 - 2\omega^2(m^2 + \mu^2) + (m^2 - \mu^2)^2 \right]^{1/2} \quad (17)$$

*) The remainder of this section refers to nucleon-nucleon scattering. The corresponding πN scattering results [Eqs. (18) - (20) excepted] are obtained simply by replacing the nucleon mass m by the pion mass μ .

W being the total c.m. energy, m the nucleon mass, and $\sigma(\omega)$ the total pion-nucleon cross-section at a c.m. energy ω . When very large values of ω are involved, $\sigma(\omega)$ can be taken constant and equal to 30 mb. When only low energies are involved one must take for $\sigma(\omega)$ the resonant elastic cross-section which is given by ³⁾ (units $\frac{\hbar}{4} = c=1$)

$$\sigma_{\pi^+p}(\omega) = \frac{2\pi}{q_1^2} \frac{\Gamma^2}{(\omega - \omega_r)^2 + \Gamma^2/4} \quad (18)$$

where

$$q_1 = \frac{1}{\omega} R(\omega) \quad (19)$$

is the c.m. momentum and

$$\Gamma = \frac{2(aq_1)^3}{1 + (aq_1)^2} \gamma \quad (20)$$

$$a = 0.88 \mu^{-1}$$

$$\gamma = 0.41 \mu$$

The elastic low-energy cross-section for other charge states can be obtained multiplying Equ. (18) with numerical coefficients. For example :

$$\sigma(\pi^-p \rightarrow \pi^-p) = \frac{1}{9} (\pi^+p \rightarrow \pi^+p).$$

The radiochemical cross-section σ_{rc} as calculated from (11) and (15) is drawn in Fig. 2, where the effect of the reduction factor has not been included. We pass now to the graph of Fig. 1b.

Let p_1 and p_2 be the four-momenta of the initial nucleons, q and q_2 those of the final pion and nucleon respectively in the lower vertex and $Q = \sum_i Q_i$ the total four-momentum of the upper group of particles. Then the S-matrix element for the Fig. 1b diagram is

$$S_{fi} = \frac{i}{2\pi} \int d^4k \sqrt{2k_0} S_1 \frac{1}{k^2 + \mu^2} \sqrt{2k_0} S_2 \quad (21)$$

where k is the four-momentum of the virtual pion and k_0 its energy component. S_1 and S_2 are the S-matrix elements for the processes

$$p_1 - k \rightarrow Q$$

$$p_2 + k \rightarrow q + q_2$$

respectively. We neglect the dependence of S_1 and S_2 on the mass of the virtual pion. Introducing an M-matrix element related to S (in the case of elastic πN scattering) in the following way

$$S_2 = (2\pi)^{-2} \left[\frac{m^2}{4k_0 q_0 q_{20} p_{20}} \right]^{1/2} \delta^{(4)}(q + q_2 - k - p_2) M_2 \quad (22)$$

and using the relation

$$|M_2|^2 = (2\pi)^2 \frac{4}{m^2} \omega^2 \frac{d\sigma_2}{d\Omega} \quad (23)$$

where $d\sigma/d\Omega$ is the angular c.m. cross-section for πN -scattering, we can prove the following formula

$$d\sigma = 2 \frac{(2\pi)^4}{W p} \frac{R(u^2) \sigma_1(u^2)}{(\Delta^2 + \mu^2)^2} \omega^2 \frac{d\sigma_2}{d\Omega} \delta^{(4)}(Q - P_1 - \Delta) \frac{d^3\vec{Q}}{Q_0} \frac{d^3\vec{q}}{q_0} \frac{d^3\vec{q}_2}{q_{20}} du^2 \quad (24)$$

where p is the initial momentum in the c.m. system, $\Delta = q + q_2 - p_2$, $u^2 = -Q^2$, and $R(u^2)$ is defined by a formula identical to Equ. (17) when ω is replaced with u . From Equ. (24) performing the integration over the δ -function in the c.m. system, we get

$$\frac{d\sigma^{(1)}}{dq^2} = \frac{2(2\pi)^3}{W p^2} \int du^2 R(u^2) \sigma_1(u^2) \left(Q \frac{dq_{20} dQ_0 d(\cos\alpha)}{[c^2 - b^2 - Q^4 + 2a^2b]^{\frac{1}{2}} (\Delta^2 + \mu^2)^2} \frac{\omega^2}{d\Omega} \right) \quad (25)$$

where

$$\begin{aligned} a &= W - Q_0 - q_{20} \\ b &= Q^2 + q_2^2 + \mu^2 + 2Qq_2 \cos\theta \cos\alpha \\ c &= 2Qq_2 \sin\theta \sin\alpha \end{aligned}$$

$Q_0(q_{20})$ is the total energy of the upper group of particles (lower nucleon). $Q = [Q_0^2 - u^2]^{\frac{1}{2}}$ and $q_2 = [q_{20}^2 - m^2]^{\frac{1}{2}}$; θ and α are the scattering angles of the vectors $\vec{q}_2$ and $\vec{Q}$ respectively and $\cos \theta$ is a function of q^2 . The relation is

$$\cos \theta = \frac{q^2 + 2m^2 - Wq_{20}}{2q_2 P} \quad (26)$$

Furthermore the 4 momentum transfer

$$\Delta^2 = -u^2 - m^2 + WQ_0 - 2PQ \cos \alpha \quad (27)$$

The region of integration in Equ. (25) is given in Appendix 2. Since we are interested in very high energies (several GeV) the range of integration in Equ. (25) is rather large even for small q^2 . Therefore, we have taken an average value of $d\sigma_2/d\Omega$ over the energies and the angles. Averaging over angles simply gives

$$\left\langle \frac{d\sigma_2}{d\Omega} \right\rangle = \frac{1}{4\pi} \sigma_{2 \text{ tot}}$$

and averaging over the energies means taking a constant $\sigma_{2 \text{ tot}}$. In actual numerical computations also σ_1 has been taken flat.

4. Conclusion

In the preceding paragraphs we have made a rough description of some of the simplest spallation reactions based on perturbation theory using optically distorted incident and outgoing waves, and the impulse approximation for the elementary interaction. The cross-section is then obtained in terms of nuclear form factors multiplied by nucleon-nucleon scattering matrix elements corresponding to very low momentum transfer. Few measurements exist as yet of the radiochemical cross-sections in question at energies at which our description is expected to be valid and to our knowledge those existing refer to proton induced reactions.

At 25 GeV Rudstam et al.⁴⁾ have determined the proton induced cross-sections for Cu^{65} going into Ni^{65} to be 130 μb and that of Cu^{65} going into Zn^{65} to be 450 μb . Using the estimates of the nuclear factor from the Fermi gas picture [Equ. (10)] with an absorption reduction factor 1/10 in accordance with the results of Eqs. (14a,b) and with the total nucleon-nucleon cross-section equal to 30 mb we find for the diagram 1a a cross-section of 80 μb , which is in satisfactory agreement with the experimental values in view of the roughness of the approximations. It is to be expected that the actual cross-section will turn out to be bigger (of a factor of two or so) than our estimate, mainly due to contributions of graphs like that in Fig. 1b. The difference by over a factor 3 in the yield of Ni^{65} and Zn^{65} is easily understood as an effect of nuclear shell structure, since Cu^{65} has only one proton but several neutrons outside the $f^{7/2}$ shell. In forming Ni^{65} only one proton can thus be transformed into a neutron without breaking of shells, while in forming Zn^{65} each of the several neutrons can be changed into a proton without breaking of shells. Since breakage of shells usually corresponds to rather high excitation energies, the corresponding states are largely incompatible with the requirement of excitation energy below neutron binding energy. The larger number of neutrons than protons outside the closed shell thus definitely favours the formation of Zn^{65} , an effect that cannot be included by a simple Fermi gas estimate. It is thus clear that low energy nuclear physics will significantly affect the nuclear form factors in these high energy reactions, but a detailed description must await further experimental results.

Our main object in this article is, however, not to calculate the exact cross-sections of the simple spallation reactions but rather to obtain information on the elementary interactions by exploiting the strong restrictions on the momentum transfer eliminating the effects of nuclear physics. Suppose that a pion is substituted for the incident proton in Fig. 1. By this substitution all nuclear physics effects associated with the lower vertex remain the same; the optical distortion in incident and outgoing channels will also be very similar since a) both pions and nucleons are strongly interacting with total nucleonic cross-sections which are close to each other at high energies; b) the outgoing particles are also absorbed in a very similar manner, since they differ mainly by a baryon in the two cases, and since they will stay together closely as one single, strongly interacting object during their passage through the nuclear matter; the outgoing particles only spread over the order of a nucleon diameter inside the nucleus, since their c.m. moves rapidly. Consequently, the nuclear matrix elements must be expected to be very similar in both cases and they are indeed identical if all optical effects are neglected. The elimination of the nuclear matrix elements is according to Equ. (11) approximately achieved by taking the ratio of the two corresponding pion and proton induced radiochemical cross-sections

$$\frac{\sigma_{rc}(\pi)}{\sigma_{rc}(p)} \approx \frac{\langle d\sigma_{\pi}^{(N)}/dT \rangle}{\langle d\sigma_p^{(N)}/dT \rangle} \quad (28)$$

where the averages again refer to the low momentum transfer region $0 < T < 4\varepsilon_F$, i.e., to momentum transfers $0 < q < 2p_F \approx \mu$. The elimination of the nuclear matrix elements can be made better by a detailed distorted wave calculation. According to Equ. (28) the ratio of the simple pion and proton induced cross-sections are approximately independent of target nucleus, which provides a

test of our description. It should be observed that the insensitivity of the total cross-sections to energy permits the use of pion and proton incident energies which are rather different; the elimination of the nuclear matrix elements can still be approximately achieved.

The interest of the ratio Equ. (28) stems from the expectation that the one-pion exchange diagram dominates the low momentum transfer cross-section, as was discussed in the previous sections. Comparison of pion and proton induced diagrams in Fig. 1 shows that the upper vertices can be expressed in terms of $(\pi\pi)$ - and (πp) total cross-sections (slightly off the mass shell). The cross-section for colliding particles X and X' going into C and C' at the upper and lower vertices is for momentum transfer q ⁵⁾

$$\frac{d\sigma_{XX' \rightarrow CC'}}{dq^2} = \frac{2}{(2\pi)^3} \frac{1}{P_{in} u^2} \frac{1}{(q^2 + M^2)^2} \int dW dW' P_W W^2 \sigma_{\pi+X \rightarrow C}(q^2, W) P_{W'} \cdot W'^2 \sigma_{\pi+X' \rightarrow C'}(q^2, W') \quad (29)$$

where the total c.m. energies of the emerging groups of particles are W and W' . Expressed in terms of incident particles a proton induced reaction has W given by

$$W = (P_W^2 + m^2)^{1/2} + (P_W^2 + \mu^2)^{1/2} \quad (30)$$

and similarly for W' . The variables q^2 , W and W' related by energy momentum conservation

$$q^2 W^2 \left(1 - \frac{W'^2 + W^2 + 2m^2 \Delta^2}{W^2} \right) = (W^2 - m^2)(W'^2 - m^2) \quad (31)$$

The quantity $p_{iu} = |\vec{p}_{iu}|$ is the initial momentum of the projectile, U is its energy. At high incident energies and small momentum transfer by the intermediate pion we can replace $\sigma_{\pi+X \rightarrow C}(q^2, W)$ by a constant, provided we sum over all final C compatible with C' . The constant will be $\sigma_{\pi+X}^{\text{tot}}(W)$. This constant can be taken outside the integration. The ratio of the radiochemical cross-sections of Equ. (28) thus becomes

$$\frac{\sigma_{rc}(\pi)}{\sigma_{rc}(p)} = \frac{\langle \int dW dW' k_W W^2 k_{W'} W'^2 \sigma_{\pi+X' \rightarrow C'} \rangle_{u_\pi}}{\langle \int dW dW' k_W W^2 k_{W'} W'^2 \sigma_{\pi+X' \rightarrow C'} \rangle_{u_p}} \frac{\sigma_{\pi\pi}^{\text{tot}}(u_\pi)}{\sigma_{pp}^{\text{tot}}(u_p)} \quad (32)$$

The average over the lower vertex will become the same for high energy incident protons and pions of the same c.m. energy, and can otherwise be calculated from kinematics using explicit assumptions about $\sigma_{\pi+X' \rightarrow C'}$. It is therefore possible to obtain estimates of the total $(\pi\pi)$ -cross-section from radiochemistry measurements, if the proposed reaction mechanism can be established experimentally.

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A P P E N D I X 1

Estimate of $\sum_{E_n^* < B} |F_{n0}^{(\lambda)}(\vec{q})|^2$

The sum of squares of mixed form factors over states of excitation energy less than the neutron binding energy B is simply obtained as follows for a Fermi gas of particles confined to the nuclear volume:

assume the Fermi gas to contain $A/2$ neutrons and $A/2$ protons and that a proton is switched into a neutron or vice versa in the reaction. We have found (Equ. (8)) that

$$\sigma_{nc} = \int \sum_{E_n^* < B} |F_{n0}^{(\lambda)}(\vec{q})|^2 \frac{d\sigma^{(\lambda)}}{d\vec{q}^2} d\vec{q}^2 = \frac{A}{2} \int \left(\frac{d\vec{p}_i}{\frac{4\pi}{3} p_F^3} \right) \frac{d\sigma^{(\lambda)}}{d\vec{q}^2} d\vec{q}^2 \quad (A1.1)$$

where $\vec{p}_i$ is the momentum of a target nucleon and p_F is the Fermi momentum. The following restrictions are imposed on the integration variables, denoting the final momentum by $\vec{p}_f = \vec{p}_i + \vec{q}$:

$p_i < p_F$ that the initial state is occupied

$p_f > p_F$ that the final state is unoccupied

$\frac{p_f^2 - p_i^2}{2M} \leq B$ that the excitation energy is less than B .

We observe that $d\sigma^{(\lambda)}/d\vec{q}^2$ does not depend on the direction of $\vec{q}$, which gives the relation

$$\frac{d\sigma^{(\lambda)}}{d^3\vec{q}} = \frac{1}{2\pi\sqrt{q^2}} \frac{d\sigma^{(\lambda)}}{d\vec{q}^2} \quad (A1.2)$$

Changing variables from $\vec{q}^2$ to $\vec{q}$ in (A1.1) we therefore have

$$\sigma_{nc} = \frac{A}{2} \left(\frac{4\pi}{3} P_F^3 \right)^{-1} \iint \frac{d\sigma(\lambda)}{d\vec{q}^2} \frac{1}{2\pi\sqrt{\vec{q}^2}} d\vec{q} d\vec{P}_i \quad (\text{A1.3})$$

Changing integration variables to $\vec{p}_i$ and $\vec{p}_f$ observing

$$\vec{q}^2 = \vec{P}_i^2 + \vec{P}_f^2 - 2|\vec{P}_i| |\vec{P}_f| \cos \theta \quad (\text{A1.4})$$

yields

$$\begin{aligned} \sigma_{nc} &= \frac{A}{2} \left(\frac{4\pi}{3} P_F^3 \right)^{-1} 2\pi \int_0^{P_F} \int_{P_i}^{\infty} P_i dP_i P_f dP_f \int_{-1}^1 \frac{d\sigma(\lambda)}{d\vec{q}^2} \frac{1}{\sqrt{\vec{q}^2}} d(2P_i P_f \cos \theta) = \\ &= \frac{A}{2} \left(\frac{4\pi}{3} P_F^3 \right)^{-1} 2\pi \int_0^{P_F} \int_{P_i}^{\infty} P_i dP_i P_f dP_f \int_{(P_f - P_i)^2}^{(P_f + P_i)^2} \frac{d\sigma(\lambda)}{d\vec{q}^2} \frac{d\vec{q}^2}{\sqrt{\vec{q}^2}} \end{aligned} \quad (\text{A1.5})$$

$P_f^2 - P_i^2 \leq 2mB$

An excellent approximation to Equ.(A1.5) is obtained by observing that the neutron binding energy in practice always is well below the Fermi energy

ϵ_F , i.e., $B < \epsilon_F = p_F^2/2M$. We can thus write

$$\frac{\Delta p}{P_F} \equiv \frac{p_f - p_i}{P_F} \leq \frac{2mB}{P_F(p_f + p_i)} \simeq \frac{B}{2\epsilon_F} \quad (\text{A1.6})$$

Thus $\Delta p = (p_f - p_i) \ll p_f$, so that we can replace the integral

$$\frac{(p_f + p_i)^2}{(p_f - p_i)^2} \int \frac{d\sigma^{(\lambda)}}{d\vec{q}^2} \frac{d\vec{q}^2}{\sqrt{\vec{q}^2}}$$

by

$$\int_0^{4P_F^2} \frac{d\sigma^{(\lambda)}}{d\vec{q}^2} \frac{d\vec{q}^2}{\sqrt{\vec{q}^2}}$$

which is independent of p_i and p_f . Furthermore

$$\int_0^{P_F} \int_0^\infty p_i dp_i p_f dp_f \frac{(2mB)^2}{8} \quad (A1.7)$$

$p_i^2 - p_f^2 \leq 2mB$

Using these two results in Equ.(A1.5) gives

$$\sigma_{nc} \simeq \frac{A}{2} \frac{3}{16} \left(\frac{B}{\varepsilon_F} \right)^2 \int_0^{4P_F^2} \sqrt{\frac{P_F^2}{\vec{q}^2}} \frac{d\sigma^{(\lambda)}}{d\vec{q}^2} d\vec{q}^2 \quad (A1.8)$$

From Equ. (A1.8) compared to Equ. (A1.1) we therefore immediately obtain

$$\sum_{E_n^* < B} |F_{n0}^{(\lambda)}(\vec{q})|^2 \simeq \begin{cases} \frac{A}{2} \frac{3}{16} \left(\frac{B}{\varepsilon_F} \right)^2 \sqrt{\frac{P_F^2}{\vec{q}^2}} & \text{for } 0 < \vec{q}^2 < 4P_F^2 \\ 0 & \text{for } \vec{q}^2 > 4P_F^2 \end{cases} \quad (A.19)$$

This result has been obtained assuming equal number of neutrons and protons but is insensitive to the assumption of different Fermi energies for these.

A P P E N D I X 2

We remark that the phase-space differential in Equ. (24), apart from du^2 , is exactly that of a final state containing three particles with momenta $\vec{Q}$, $\vec{q}$ and $\vec{q}_2$. The presence of du^2 indicates, however, that we have to perform the integration over the "mass" of one of the final particles, since $u^2 = Q_0^2 - \vec{Q}^2$.

We proceed therefore in two steps: first we find the region of integration, fixed q^2 , for the variables q_{20} , Q_0 and $\cos \alpha$. Secondly, we perform the integration over the "mass" of the upper group of particles.

The region of integration, fixed q^2 and u , is

$$m \leq q_{20} \leq \frac{1}{2W} \left[W^2 + m^2 - (u + \mu)^2 \right] \quad (\text{A2.1})$$

$$u \leq Q_0 \leq \frac{1}{2W} \left[W^2 + u^2 - (m + \mu)^2 \right] \quad (\text{A2.2})$$

$$\frac{1}{2W} \left[W^2 + (\mu + m)^2 - u^2 \right] \leq Q_0 + q_{20} \leq W - \mu \quad (\text{A2.3})$$

$$\cos(\theta + \alpha) \leq f(Q_0, q_{20}, u^2) \leq \cos(\theta - \alpha) \quad (\text{A2.4})$$

where

$$f(Q_0, q_{20}) = \frac{1}{2Qq_2} \left[W^2 + m^2 + u^2 - \mu^2 - 2W(Q_0 + q_{20}) + 2Q_0 q_{20} \right] \quad (A2.5)$$

and $\cos \Theta$ (function of q^2) is given by formula (26) of the text. Equations (A2.1) to (A2.3) ensure that the energies of the final state particles are not smaller than the corresponding masses and not bigger than their maximum permissible values. (A2.4) ensures the physically values of all the c.m. angles by restricting $\cos \Theta$ and $\cos \alpha$ to be between -1 and $+1$.

After one has integrated over q_{20} , Q_0 and $\cos \alpha$ there remains the integration over u^2 . The limits for fixed q^2 can be shown to be

$$(m + \mu)^2 \leq u^2 \leq f_1(q^2) \quad (A2.6)$$

where

$$f_1(q^2) = \left[(W^2 + m^2 - 2Wg(q))^{1/2} - \mu \right]^2 \quad (A2.7)$$

$$g(q^2) = \frac{W(q^2 + 2m^2) - \sqrt{W^2 - 4m^2} \sqrt{(q^2 + 2m^2)^2 - 4m^4}}{4m^2} \quad (A2.8)$$

The formulae of this Appendix refer again to the incoming nucleon case. Minor modifications have to be performed when the incoming particle is a pion.

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FIGURE CAPTIONS

Fig. 1 The most important diagrams for the elementary interaction. In the figure p_2 and q_2 refer to nucleons, q and k to pions; p_1 can be either a nucleon or a pion depending on the projectile. The system of outgoing particles Q refers to any such system in agreement with conservation laws.

Fig. 2 The radiochemical cross-section σ_{rc} divided by A (nucleon number) and B^2 (neutron binding energy squared) vs. laboratory energy of projectile. The reduction factor R [see Equ. (14)] is not included.

Fig. 1

Fig. 2