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THE PHASES OF THE PROTON'S ELECTROMAGNETIC
FORM FACTORS IN THE TIME-LIKE REGION

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ABSTRACT

It is shown that from the known electromagnetic form factors of the proton in the space-like region and the measurement of the differential cross-section of the process $p + \bar{p} \rightarrow e + \bar{e}$, information can be obtained concerning both the modulae and phases of form factors for all time-like arguments including the unphysical region.

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1. Introduction

Recently it has been suggested ¹⁾ to study proton-antiproton annihilations in flight into lepton pairs, in order to extract the electromagnetic form factor of the proton for time-like four-momenta of the photon. Whereas in the space-like region so far explored ²⁾ the form factors are real, in the time-like region these are evidently complex. Unfortunately, the differential cross-section for unpolarised incoming and outgoing particles depends only on the modulae of certain linear combinations of the form factors. For further reference we reproduce the expressions of the $p\bar{p} \rightarrow e\bar{e}$ differential cross-section in the centre-of-mass system ¹⁾

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{8} \frac{\alpha^2}{E\sqrt{E^2-M^2}} \left[|G_1|^2 (1 + \cos^2\theta) + \frac{M^2}{E^2} |G_2|^2 \sin^2\theta \right] \quad (1)$$

$$G_1 = F_1 + F_2, \quad G_2 = F_1 + \frac{E^2}{M^2} F_2 \quad (2)$$

where E = antiproton c.m. energy, θ = c.m. angle between $\bar{e}$ and $\bar{p}$, M = proton mass, F_1 and $F_2/2M$ the electric, respectively the magnetic, form factors of the proton which are functions of $z = 4E^2$. So (1) does not determine the complex functions

$$F_1(z) = |F_1(z)| e^{i\phi_1(z)} \quad \text{and} \quad F_2(z) = |F_2(z)| e^{i\phi_2(z)}$$

It is the purpose of the present note to point out that using (1) in connection with the analyticity properties of the form factors, information can be obtained concerning the modulae and phases of the form factors and their structure in the unphysical region $0 < z < 4M^2$.

2. General formula for the phase

In order to see how this aim can be reached, let us consider the following somewhat simplified problem. Let $f(z) = |f(z)|e^{i\varphi(z)}$ be a complex function :

- i) analytic in the complex z plane with a cut along the positive real axis running from a to ∞ ;
- ii) having no complex zeros and;
- iii) satisfying the condition

$$f(z^*) = f^*(z) \quad (3)$$

What can we say about the phase $\varphi(z)$ of $f(z)$ if we know its modulus $|f(z)|$ along the whole ^{*)} real axis? Knowing the modulus, we could express the real part of the function in terms of its imaginary part. Inserting it in the dispersion relation which can be written due to i), we would find a non-linear integral equation for the imaginary part. This equation would, however, be very complicated to handle, and it is therefore convenient to study the function $\log f(z)$ rather than $f(z)$ itself.

According to our assumptions $\log f(z)$ is analytic in the upper half plane. Taking the imaginary part of Cauchy's formula for $\log f(z)$ along the contour C in Fig. 1 :

Fig. 1

*) Evidently because of (3) $|f(z)|$ has no discontinuity along the cut.

and using the relation

$$\log f(z) = \log |f(z)| + i \delta(z) \quad (4)$$

we find for real z ,

$$\delta(z) = \delta(z_0) - \frac{(z-z_0)}{\pi} p \int_{-\infty}^{+\infty} \frac{\log |f(z')| dz'}{(z'-z)(z'-z_0)} \quad (5)$$

where one subtraction has been performed at $z = z_0$ in order to ensure the convergence of the integral along the real axis and the vanishing of the integral along the semicircle ^{*}).

It is easy to check that, e.g., for $f(z) = (1-z)^{-\frac{1}{2}}$ which satisfies all our restrictions i) - iii), (5) really gives $\delta(z) = \pi \theta(z-1)$.

3. Electromagnetic form factors of the proton

In order to apply the general formula (5) to the special case of the electromagnetic form factors of the proton, let us first discuss if the linear combinations G_1 and G_2 satisfy the conditions i) - iii) under which (5) was derived.

Condition i) is evidently satisfied because of the well-known analyticity properties of F_1 and F_2 ³⁾. Condition ii) for G_1 and G_2 cannot be proved out of general principles.

It may, however, be remarked in this context that at the solution of the Omnès equation ⁴⁾ which usually appears in dispersion theoretic treatments of vertex functions ⁵⁾ the trick of studying the logarithm rather than the function itself is also applied, so that similar restrictions are to be made.

^{*}) This point will be discussed in more detail below.

It should be emphasized that complex zeros of $f(z)$ lying sufficiently far from the real axis cause no trouble.

Condition iii) just states the well-known "reality" property of the form factors. It ensures also the unambiguity of the solution of our problem. Indeed, two analytic functions $f(z)$ and $g(z)$ having the same modulae along the real axis can differ only by a factor of the form $e^{i\alpha(z)}$, with α an analytic function, real values along the real z axis. If now both f and g have to satisfy (3), $e^{i\alpha(z)}$ also has to satisfy (3) which implies $\alpha = \text{constant}$. This constant is then eliminated by the subtraction performed in (5). It is clear that for $f(z)$ increasing at infinity not stronger than a polynomial (which is the case for form factors ^{5),6)}, one subtraction is sufficient in (5). Even for an exponential behaviour of $f(z)$, a finite number of subtractions is sufficient, but these would not introduce additional parameters since $f(z)$ is known along the whole negative real axis. The way how (5) has to be applied for the case of the form factors is the following.

As we have just seen, G_1 and G_2 obey to conditions i)...iii). Now measuring $\frac{d\sigma}{d\cos\theta}$ for $p\bar{p} \rightarrow e\bar{e}$, by (1) we determine $|G_1|$ and $|G_2|$ for $E \geq 2M$. From the ep scattering experiments $|G_1|$ and $|G_2|$ are known for $E \ll 0$. In the region $0 < E < 2M$ one has an idea about the shape of $|G_1|$ and $|G_2|$ because of the known resonances that will dominate this region and thus an analytic ansatz with certain parameters can be made. Inserting these $|G_1|$ and $|G_2|$ into (5), one can compute the phases ϕ_{G_1} and ϕ_{G_2} of G_1 and G_2 . As a test both G_1 and G_2 have to be real along the negative real axis. G_1 and G_2 , and consequently by (2) F_1 and F_2 , are thus completely (modulae and phases) determined along the whole real axis. So one can compute the difference of their phases $\Delta = \phi_{F_1} - \phi_{F_2}$ which can be measured in a $p\bar{p} \rightarrow e\bar{e}$ experiment with detection of polarisations ¹⁾. Comparing the computed values of Δ with its experimental values at different energies, is then a means of finding the right parameters for the ansatz of F_1 and F_2 in the unphysical region $0 < E < 2M$. Besides this, the reality of F_1 and F_2 on the negative real axis has always to be checked. The sensitivity of the method outlined in the present note evidently depends on the amount of structure contained in the physical region of the form factors. The generalisation of the results to other vertex functions is obvious.

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