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SOME REMARKS ABOUT INELASTIC ELECTRON-DEUTERON SCATTERING

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Recently important improvements in an understanding of proton and neutron electromagnetic form factors have been made ^{1),2),3)}. In the case of the neutron, the charge form factor, or more precisely F_1 , seems to be positive but the situation is still rather unclear especially at low momentum transfer ⁴⁾.

At present one tries to get information on the neutron form factors from measurements of the differential cross-section for inelastic electron deuteron scattering, and more precisely from the magnitude of the quasi-elastic peak in which the kinematical conditions are as close as possible to those corresponding to scattering of electrons on free neutrons. The main difficulties encountered in such an analysis are the following:

- a) one has to find a reliable theory to express the deuteron data in terms of the proton and neutron form factors. This connection is simple only if rather crude approximations are made ^{5),6)}.
- b) the electron deuteron cross-section at low momentum transfer is strongly dominated by the electron-proton interaction.

In this letter we wish to discuss in some detail the theoretical significance of the different experimental information we can get on electron-deuteron inelastic scattering and, in particular, to insist on some very general restrictions on the cross-sections due to relativistic invariance and one photon exchange assumption. This was investigated by one of us in the general case of inelastic scattering of electrons ⁷⁾ and the result turned out to be the generalization of the one obtained by Lee and Yang ⁸⁾ on the case of neutrino induced reactions.

It was shown that if one observes only the outgoing electron, the differential cross-section in the lab. system has the general structure

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$$\frac{d^2\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[A(q^2, W^2) + \tan^2 \frac{\theta}{2} B(q^2, W^2) \right] \quad (1)$$

where

$$\sigma_{\text{Mott}} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E'^2 \sin^4 \frac{\theta}{2}}$$

E incident energy of the electron

E' final energy of the electron

θ scattering angle of the electron

q four-momentum transfer of the electron

W total energy of the outgoing particles, excluding the electrons (in our case the Np system) in their own centre of mass.

Formula (1) expresses the fact that only two different experiments with the same couple of values for q^2 and W^2 are necessary to determine the two invariant functions A and B. A third experiment will not give any supplementary information on the detailed mechanism of the absorption of the virtual photon by the deuteron but permits to test the one photon exchange assumption. If we now restrict ourselves to the quasi-elastic peak we have a definite relation between q^2 and W^2 :

$$\frac{W^2 - M_D^2 + q^2}{2M_D} = \frac{q^2}{2(M - \epsilon)} - \epsilon \quad (2)$$

where ϵ is the binding energy of the deuterium.

Therefore A and B become functions of the single variable q^2 . It now appears clearly in formula (1) that for fixed q^2 the ratio

$$\frac{d^2\sigma}{dE'd\Omega} / \sigma_{\text{Mott}}$$

is a linear function of $\text{tg}^2 \theta/2$. To test formula (1) one can plot the above ratio for a set of kinematical conditions corresponding to the same momentum transfer ^{*)}. Two possible situations may occur:

- a) the points can be fitted by a straight line within experimental errors,
- b) it is not possible to find a linear fit.

In case a) what is tested is the one-photon exchange assumption and nothing else.

In case b) the one-photon exchange assumption is not valid and it is impossible to find any theory for the virtual photodisintegration of deuterium that leads to a better agreement with experiment.

Now in a given theory of the deuterium quasi-elastic peak the cross-section is a function of the neutron form factors (the proton form factors are furnished by direct experiments). In any theory in which the reaction amplitude is a linear function of the neutron form factors, the knowledge of an experimental value of the deuteron cross-section furnishes an ellipse in the $F_{1n} F_{2n}$ plane. In case a) all ellipses corresponding to a fixed value of q^2 but to different reaction angles θ must necessarily intersect in a unique set of four points which are real or imaginary and in the case b) it is impossible to obtain a unique set of such points of intersection.

*) This method has been applied in the particular case of elastic electron-proton scattering for which the Rosenbluth formula appears as a particular case of formula (1). This analysis has been performed by R.R. Wilson (Proceedings of the Aix-en-Provence Conference, 1961).

The conclusions are the following:

- A) if one obtains a unique set of four interactions for the ellipses, one cannot deduce any information on the validity of the theory used for the deuterium disintegration.
- B) if one does not obtain a unique set of intersections, there are two possibilities:
 - α) the experimental errors are underestimated,
 - β) the one-photon exchange assumption is not valid.
- C) if one does not obtain a unique set of intersections, it is obviously impossible to improve the situation by using a more refined theory for the deuterium, because of the simple fact that all possible corrections to the photodisintegration of the deuterium must satisfy the criterion of linearity.
- D) there is no possible choice, at this stage, between the four solutions which must be equally acceptable or not.

We think that the best method is not to use ellipses but to split the analysis of experimental data into two parts:

- i) we test the one-photon exchange assumption by plotting $\sigma_{\text{peak}} / \sigma_{\text{Mott}}$ versus $\text{tg}^2 \frac{\theta}{2}$, and trying to find the best fit by a straight line $A+B \text{tg}^2 \frac{\theta}{2}$. The uncertainties on A and B are of pure experimental origin. On this plot it is easy to see what complementary measurements are needed to reduce experimental errors.
- ii) one uses a given model to extract F_{1n} and F_{2n} from A, B, and the proton form factors. If it turns out that the data cannot be fitted by real values of F_{1n} and F_{2n} the model must be rejected. If, on the other hand, one gets real solutions, this does not constitute a proof that the model is good.

We want now to illustrate these considerations by analysing the deuterium data at $q^2 = 2.5 \text{ fermi}^{-2}$ and $q^2 = 5 \text{ fermi}^{-2}$. Our purpose is not to obtain new results but to show how our method works. On Figs. 1 and 2 we plot $\sigma(\text{peak deuterium})/\sigma_{\text{Mott}}$, using the data presented by R. Hofstadter at the Aix-en-Provence conference ⁴⁾. Within experimental errors the linearity condition seems to be satisfied at both values of q^2 .

As an example we have used the model of Goldberg ⁵⁾ and Durand ⁶⁾ to extract the electron-neutron cross-section by the formula

$$\frac{d^2\sigma}{d\Omega dE'} = 4.57 \times 10^{-3} \text{ MeV}^{-1} (1 \pm 0.05) \frac{4M^2}{qW} \times \frac{E}{E'} \left(\frac{d\sigma_p}{d\Omega} + \frac{d\sigma_n}{d\Omega} \right) \quad (3)$$

where $\frac{d\sigma_{p,n}}{d\Omega}$ are given by the Rosenbluth formula for electron-nucleon elastic scattering in the same kinematical conditions.

For $q^2 = 2.5 \text{ fermi}^{-2}$ we have plotted

$$\frac{E}{E'} \frac{d\sigma_n}{d\Omega} / \sigma_{\text{Mott}}$$

and, as expected from the preceding test (Fig. 2) and from the structure of equation (3), the linearity condition is satisfied, within experimental errors. By inspection of Fig. 3 one easily sees that it is very difficult to obtain any precise information on electromagnetic neutron form factors because of the magnitude of experimental errors on deuteron and proton cross-sections and the uncertainty on the theoretical formula (3). In particular, the data are consistent with the Bergia, Fubini, Stanghellini,

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Villi model ³⁾ where the parameters are those given by R.R. Wilson ²⁾ from Cornell or Stanford experiments. However, this does not constitute a proof that the model considered is good.

There is only one case where one could reject the Durand model.

Writing

$$\frac{E}{E'} \frac{\sigma_{\text{Neutron}}}{\sigma_{\text{Mott}}}$$

as $A_n + \text{tg}^2 \frac{\theta}{2} B_n$, one easily finds, from the Rosenbluth formula the condition

$$\frac{B_n}{A_n} < 2 + \frac{q^2}{2M^2},$$

for the reality of the neutron form factor. If this inequality were violated a refinement of the model would certainly be necessary.

The above analysis illustrates the difficulties encountered in obtaining the neutron form factors for low q^2 values from the deuterium and proton data. First of all these difficulties are of experimental nature, because in any reasonable model the neutron cross-section appears as the difference between two comparable numbers with rather large uncertainties. It would certainly be extremely useful to reduce appreciably these uncertainties, or to use a direct method of measurement of the neutron cross-section.

The theoretical uncertainties could be reduced by a study of the shape of the quasi-elastic peak for fixed value of the momentum transfer. By a comparison with the various models one could select the most appropriate one. Unfortunately, the experimental difficulty we mentioned just above is still present.

A more satisfactory procedure in all respects would consist in making coincidence experiments between the electron and one of the outgoing nucleons. In this way one can select the events for which the reaction is dominated by process a) or process b), (Fig. 4):

a) behaves like

$$\left(\frac{1}{T_p + \frac{\epsilon}{2}} \right)^2$$

where T_p is the laboratory kinetic of the proton and ϵ the binding energy of the deuteron. So by selecting very low T_p events one can measure the neutron cross-section. This is just an application of the Chew-Low extrapolation procedure ⁹⁾ which was already suggested in the present case by S.K. Bose ¹⁰⁾.

b) for low T_n , where T_n is the neutron laboratory kinetic energy, furnishes the proton cross-section and this can be used as a test of the procedure.

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Fig. 1

Fig. 2

Fig. 3

a.

b.

Fig. 4