squared distributions of the $\Xi\pi$ and $K\bar{K}$ systems by means of the χ^2 test. The probabilities that the observed distributions originate from their corresponding phase-space distributions are <0.0001 for the $\Xi\pi$ system and <0.01 for the $K\bar{K}$ system. The large χ^2 values arise mainly from the single peaks appearing in each curve.

⁴For both the $\Xi\pi$ and $K\bar{K}$ systems the mass resolution is about ± 3 MeV. This resolution has been estimated from the distribution of Λ masses found when the pion and proton from the Λ decay have been fitted to the production vertex.

⁵No subtraction of background has been made in the derivation of these ratios.

REGGE POLE MODEL FOR HIGH-ENERGY p - p AND $\bar{p}$ - p SCATTERING

F. Hadjioannou, R. J. N. Phillips,* and W. Rarita,†

CERN, Geneva, Switzerland

(Received July 27, 1962)

The p - p data from 3 to 30 GeV/ c seem to fit the assumption of a single vacuum Regge pole trajectory¹ (which we label P) and have been analyzed in these terms.²⁻⁵ However, two more trajectories must in fact be important in this region: the ω trajectory to give the big difference between p - p and $\bar{p}$ - p scattering, and a second vacuum trajectory (labelled P') to keep the p - p total cross section roughly constant.⁶ (Igi⁷ has already established the presence of P' from π - N dispersion relations.) Other known trajectories seem less important; the ρ trajectory gives differences between p - p and n - p scattering, but these are small.²

It is important to know if these extra poles spoil the one-pole analysis. We present here a realistic model of p - p and $\bar{p}$ - p scattering, using the P , P' , and ω trajectories, which fits many aspects of the data. This model suggests that one-pole analyses need certain corrections but remain qualitatively valid.

We first assume the scattering is dominated by the spin-averaged amplitude; this is reasonable, at least in the diffraction peak, and leaves a scalar problem. (A complete spinor treatment^{8,9} introduces some modifications to be discussed later.) The amplitude contains terms of the usual form

$$T = \beta(t) P_\alpha(\cos\Theta_t) [1 \pm \exp(-i\pi\alpha)] / \sin\pi\alpha, \quad (1)$$

where $\alpha(t)$ is the corresponding trajectory, t is the invariant momentum transfer squared, P_α is the Legendre function, Θ_t is the scattering angle in the crossed channel, and $\beta(t)$ is the residue function. The signature $\pm$ is $+$ for P and P' , and $-$ for ω . The p - p and $\bar{p}$ - p amplitudes have the forms

$$T \begin{pmatrix} p-p \\ \bar{p}-p \end{pmatrix} = T_P \pm T_{P'} \pm T_\omega \quad (2)$$

in an obvious notation.

From $\beta(t)$ we factor out the statistical weight $(2\alpha+1)$, a factor α to remove⁸ "ghost" singularities at $\alpha=0$ for P and P' , and the threshold dependence^{8,10} $(t-4m^2)^\alpha/(4m^2)^\alpha$ where m is the nucleon mass; any remaining t dependence is ignored. The choice of units to make the threshold term dimensionless is not trivial, for it affects the t dependence. We note that the factor $(2\alpha+1)$ also serves to remove a singularity in P_α at $\alpha = -\frac{1}{2}$. The ω term has no ghost at $\alpha=0$, but we keep the factor α for symmetry; it also helps to fit the data, as explained later.

We now use the asymptotic form of P_α , write $\cos\Theta_t = (2s-4m^2+t)/(t-4m^2)$ where s is the invariant energy squared, and neglect t compared to s . Each pole term becomes

$$T = B\alpha(2\alpha+1)2^\alpha \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)} \frac{1 \pm \exp(-i\pi\alpha)}{\sin\pi\alpha} \left(\frac{s-2m^2}{2m^2} \right)^\alpha, \quad (3)$$

where B is a constant and Γ is the gamma function. Note that $(s-2m^2)/2m = E_L$, the total proton laboratory energy.

We assume the P trajectory is a straight line between $\alpha_P=1$ at $t=0$ and $\alpha_P=0$ at $t=-1$ (GeV/ c)², as suggested by the one-pole analysis; we restrict ourselves to this range of t . Since the P' and ω contributions to the p - p total cross section are to cancel, their trajectories and residues are equal at $t=0$; we continue this symmetry for $t<0$ by choosing these trajectories to coincide on a straight line parallel to the P trajectory. The intercept at $t=0$ is taken to be 0.5 from Igi's work⁷; this value also seems consistent with the variations of total cross sections in the N - N , $\bar{N}$ - N , π - N , K - N , and $\bar{K}$ - N systems. The numerical coefficients are chosen in the ratio $B_P:B_{P'}:B_\omega = 0.45:1:1$ for an approximate fit to

the p - p and $\bar{p}$ - p total cross sections.²⁻⁵ The model is now completely specified.

For a comparison with experiment, we write the differential cross section as

$$d\sigma/dt = (d\sigma/dt)_{\text{opt}} F(s, t) (E_L/m)^{2\alpha} P(t)^{-2}, \quad (4)$$

where the subscript "opt" indicates the minimum forward cross section found from the optical theorem. If only the P trajectory contributes, F is a function of t alone; this is then essentially the formula used to fit p - p data in reference 3. The predictions of our model are shown in Fig. 1; $F(s, t)$ is plotted against t for various fixed values of s , and is compared with $F(t)$ deduced from p - p data between 3 and 30 GeV using the one-pole formula.²⁻⁵

We see that the model agrees qualitatively with p - p data, by giving the initial fall of F as $|t|$ increases. It does not give the subsequent flattening or rise, indicated by a one-pole analysis²⁻⁵ over the whole range 3-30 GeV/c and shown in Fig. 1; however, if the range is restricted to 9-30 GeV/c, the one-pole analysis no longer shows the above marked effect.¹¹ The model also agrees qualitatively with $\bar{p}$ - p data,¹² which

FIG. 1. Predictions of the three-pole model for various fixed values of s : the p - p and $\bar{p}$ - p cases have the same asymptotic limit. "Experimental" results,²⁻⁵ from using the one-pole formula on data from 3-30 GeV/c, are also shown.

show a much narrower diffraction peak than in the p - p case. At $E_L = 3$ GeV and $t = -0.5$ (GeV/c)² the $\bar{p}$ - p peak has fallen by an extra factor 5: our model gives a factor 3 (though it is not really designed for such low energies). We notice a consequence of the model, that as s increases, the shape of $F(s, t)$ shrinks for the p - p case and expands for the $\bar{p}$ - p case: both reach the same asymptotic limit, given by the P term alone.

Two arbitrary features of the model can now be discussed. First it is clear that a different choice of units for the threshold dependence of $\beta(t)$ —which in turn determines the units for s in Eq. (3)—would lead to different shapes for $F(s, t)$. The rough agreement with experiment suggests that our choice, originally made for convenience, may have a deeper significance. Secondly, we can see how the factor α gratuitously included in the ω term helps to fit experiment. For since the cross section for $\bar{p}$ - p is bigger than that for p - p at $t=0$, and since it falls off much faster for $t<0$, the two curves must intersect somewhere; at this point the ω term must vanish, in the three-pole model. The extra factor α_ω insures this vanishing, at $t = -0.5$ (GeV/c)² where $\alpha_\omega = 0$ with our choice of ω trajectory.

Now consider the plot of $\ln[(d\sigma/dt)/(d\sigma/dt)_{\text{opt}}]$ against $\ln(E_L/m)$ at fixed t . The one-pole assumption gives a straight line. The one-pole analysis is made by fitting a straight line to this plot of the data at each value of t ; $\alpha(t)$ and $F(t)$ are found from the slope and the intercept at $\ln(E_L/m) = 0$. Our model, however, gives a curved line, concave upward for the p - p case,¹³ suggesting that the one-pole analysis tends to overestimate both $F(t)$ and $(1-\alpha)$ for the P trajectory.

To illustrate this point, we plot the model predictions for the p - p case between 6 and 25 GeV/c, with nominal 10% uncertainty, and make straight-line fits to these synthetic data, as shown in Fig. 2. The curvature is scarcely visible in this momentum range and might escape detection even with measurements to $\pm 1\%$ (we include some 3-GeV/c points to show the curvature is there, but these are not considered in the fit). Nevertheless, the curvature between 25 GeV/c and ∞ is enough to make appreciable corrections to $\alpha(t)$ as shown in Table I. The determination of $F(t)$ involves a large extrapolation and the corrections here are consequently bigger.

A complete spinor treatment for N - N scattering is formulated by Gell-Mann.^{8,9} With a one-

FIG. 2. One-pole fit to the three-pole model, between 6 and 25 GeV/c, for p - p case. Plot of $\ln[(d\sigma/dt)/(d\sigma/dt)_{\text{opt}}]$ against $\ln(E_L/m)$ at fixed t is fitted by straight lines. Points at 3 GeV/c are shown, but not used in the fit.

pole model there are now two independent residue functions η_1 and η_2 (occurring quadratically) in place of one. It seems impossible to determine both independently from unpolarized differential cross-section measurements alone. However, only η_1 affects the total cross section. If we make assumptions about η_1 equivalent to those we made about $\beta(t)$, and put $\eta_2 = 0$, the cross section differs from the corresponding scalar model by a factor $[1 - t/4m^2]^2$, essentially because of spin-dependent terms. If instead we put $\eta_2 = c\eta_1$, where c is a constant, the additional t dependence asymptotically has the form $(1 - |1 - c|^2 t/4m^2)^2$. Unless c is much larger than 1, the one-pole curve in Fig. 1 is little altered.

With a three-pole spinor model there is much arbitrariness. However, if the three-pole terms

Table I. Results of the one-pole analysis, applied to our three-pole model. Comparison of true and apparent values of α and F .

$-t$ (GeV/c) ²	$\alpha_F(t)$		$F(t)$	
	True	Apparent	True	Apparent
0.1	0.9	0.83	0.68	1.2
0.3	0.7	0.61	0.34	0.74
0.5	0.5	0.40	0.18	0.47
0.7	0.3	0.18	0.10	0.33
0.9	0.1	-0.04	0.06	0.23

differ qualitatively only in signature and we choose the residue functions suitably, we can recover the results of the scalar model simply multiplied by a t -dependent factor as above. Thus a scalar model is not unreasonable for $|t| \ll 4(\text{GeV}/c)^2$.

This provides a limit on the model's range of validity in t . The limits of validity in energy are hard to estimate. However, we are reluctant to take the model seriously below, say, $E_L = 6$ GeV, if only because the forward p - p amplitude develops a large real part in this region, contrary to experiment. At 10 GeV, however, the forward cross section is only 16% above the "optical" value, which agrees with the experimental¹¹ figure, $20 \pm 20\%$. In our model the $\bar{p}$ - p forward amplitude is always purely imaginary.

To summarize, our model uses the following assumptions:

- (i) The problem is essentially scalar.
- (ii) The P , P' , and ω trajectories dominate.
- (iii) The P' and ω terms are symmetrical, so that in the p - p amplitude their imaginary parts cancel, not only at $t = 0$, but also for $t < 0$.
- (iv) The residue functions have the minimum known t dependence (apart from an extra factor in the ω term, needed for symmetry).
- (v) The threshold dependence of residues introduces an arbitrary scale in each pole term, affecting the t dependence; this scale is chosen the same for all three terms.
- (vi) The common trajectory of P' and ω remains parallel to the P trajectory; we take the latter to be a straight line passing through 0 at $t = -1 (\text{GeV}/c)^2$.

We reach the following conclusions:

- (i) The model fits qualitatively what is known about high-energy p - p and $\bar{p}$ - p scattering in its range of validity, $|t| \ll 4(\text{GeV}/c)^2$.
- (ii) Fitting data in the range 6-25 GeV/c by a one-pole formula is likely to overestimate both $[1 - \alpha(t)]$ and $F(t)$ for the P -trajectory, to the ex-

tent shown in Table I. On the other hand, the fit will appear deceptively good.

(iii) Since the curvature in Fig. 2 is so hard to see, the prospects for separating different pole contributions in the region $t < 0$ are poor, at least with presently accessible energies.

We thank Geoffrey F. Chew for extensive consultations and stimulating discussions. We are grateful to Murray Gell-Mann for disclosing to us the contents of his recent manuscript before publication, and to E. Lillithun, A. E. Taylor, and A. M. Wetherell for discussing their latest work with us.

*Permanent address: Atomic Energy Research Establishment, Harwell, England.

†On retirement leave from Brooklyn College, Brooklyn, New York.

¹For recent accounts of the use of Regge poles, see S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. (in press), and in the Proceedings of the 1962 International Conference on High-Energy Physics at CERN (to be published).

²A. N. Diddens *et al.*, Phys. Rev. Letters **9**, 32

(1962).

³A. N. Diddens *et al.*, Phys. Rev. Letters **9**, 111 (1962).

⁴A. N. Diddens *et al.*, Phys. Rev. Letters **9**, 108 (1962).

⁵S. Lindenbaum *et al.*, Phys. Rev. Letters **7**, 185 (1961).

⁶S. D. Drell, in the Proceedings of the 1962 International Conference on High-Energy Physics at CERN (to be published).

⁷K. Igi, Phys. Rev. Letters **9**, 76 (1962).

⁸M. Gell-Mann, in the Proceedings of the 1962 International Conference on High-Energy Physics at CERN (to be published).

⁹V. N. Gribov and I. Ya. Pomeranchuk, Phys. Rev. Letters **8**, 412 (1962).

¹⁰A. O. Barut and D. E. Zwanziger, Phys. Rev. (to be published).

¹¹A. N. Diddens *et al.* (private communication).

¹²Y. Goldschmit-Clermont *et al.*, in the Proceedings of the 1962 International Conference on High-Energy Physics at CERN (to be published).

¹³This curvature is another expression of the shrinking of the $F(s, t)$ pattern. For the $\bar{p}-p$ case the curvature has the opposite sign, and $F(s, t)$ expands.

THEORY OF NONLEPTONIC HYPERON DECAY

S. P. Rosen*

The Clarendon Laboratory, Oxford, England

(Received May 28, 1962)

Several theories¹⁻¹¹ of nonleptonic hyperon decay have been proposed, but only one, due to Pais,¹ is consistent with all of the observed relations¹²⁻¹⁴

$$\alpha_+ \approx \alpha_- \approx 0, \quad (1)$$

$$\alpha_\Lambda \approx -\alpha_0, \quad (2)$$

$$\alpha_{\Xi} \approx -\alpha_\Lambda, \quad (3)$$

between the asymmetry parameters in Σ , Λ , and Ξ decay. Pais uses the baryon doublet approximation¹⁵ and Relation (1) to predict (2). Instead of (3), however, he obtains the weaker prediction:

$$|\alpha_{\Xi}| \approx |\alpha_\Lambda|. \quad (4)$$

Here we propose another theory in which (1), (2), and (3) are derived from time-reversal invariance, the $\Delta T = \frac{1}{2}$ rule,^{12,16} and three doublet symmetries. Since the electromagnetic interaction satisfies two of these symmetries, we are able to make predictions about the weak electromagnetic decays $\Sigma^+ \rightarrow p + \gamma$ and $\Xi^- \rightarrow \Sigma^- + \gamma$.

In the doublet approximation,^{1,15} the isotopic

spin $\vec{T}$ is split into a doublet spin $\vec{I}$ and a K -spin $\vec{K}$,

$$\vec{T} = \vec{I} + \vec{K}, \quad (5)$$

and the baryons are grouped into four $I = \frac{1}{2}$ doublets:

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_2 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix}, \quad N_3 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}, \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix};$$

$$Y^0 = 2^{-1/2}(\Lambda^0 - \Sigma^0), \quad Z^0 = 2^{-1/2}(\Lambda^0 + \Sigma^0). \quad (6)$$

N_2 and N_3 form a $K = \frac{1}{2}$ doublet:

$$N_{23} = \begin{pmatrix} N_2 \\ N_3 \end{pmatrix}, \quad (7)$$

with $K_2 = +\frac{1}{2}$, $-\frac{1}{2}$, respectively, and N_1 and N_4 have zero K spin. The assignments for π and K mesons are $(I=1, K=0)$ and $(I=0, K=\frac{1}{2})$, respectively. To supplement this scheme, we introduce a hypercharge spin $\vec{U}$, with z component

$$U_z = \frac{1}{2}(B + S), \quad (8)$$

and observe that N_1 and N_4 form a $U = \frac{1}{2}$ doublet