PRODUCTION OF ANTIDEUTERONS, LIGHT NUCLEI AND HYPERFRAGMENTS

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HIGH-ENERGY PP-COLLISIONS

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A B S T R A C T

Using the statistical theory of particle production the partial cross-sections for the creation of antideuteron, light nuclei and hyperfragments are calculated. Rough estimates of multiplicities, centre-of-mass energy spectra and lab. momentum spectra at several angles are given.

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I. Introduction

In the secondary beams emerging from targets hit by high energy ($\sim 20-30$ GeV) protons one observes deuterons, H^3 , and He^3 of surprisingly high momenta ¹⁾. As far as the target is not hydrogen, the processes leading to the formation of these particles are certainly rather complex. An attempt to describe the deuteron production as the result of the interaction of the nucleons participating in the cascade initiated by the incoming proton was recently made by Butler and Pearson ²⁾. A model, in which the incoming nucleon hits a deuteron, formed by fluctuations inside the nuclear matter was proposed by Blokhintsev ³⁾ some years ago. A third mechanism was investigated by means of the statistical theory ⁴⁾. In that model it was assumed that the deuterons emerge from the elementary interaction

$$p+N \rightarrow d+\text{pions}+\text{others}. \quad (1)$$

All these models were not able to provide a quantitative account of the real mechanism and fail in some respect or other. It is conceivable that all three mechanisms intermingle in a complicated way.

Until recently, it was doubtful, whether the "elementary deuteron production" described in Equ. (1) was of any importance at all, but now its presence has been established experimentally in a hydrogen bubble chamber by B.S. Zorn at Brookhaven ⁵⁾. The experiment was done with 2.0 GeV primary protons and gives a deuteron production cross-section in agreement with that predicted for 2.3 GeV primary energy ⁴⁾, while ratio $\sigma(d+\pi)/\sigma(d+2\pi)$ is experimentally about four times smaller than predicted. As there seems to be an indication that $d+\pi^++\pi^0$ is partly produced via $d+\rho \rightarrow d+\pi^++\pi^0$ this discrepancy may be due to having neglected that channel in the statistical calculation.

2.

Another confirmation of the existence of reaction (1) comes from a difference measurement with polyethylene and carbon targets carried out by J. Oostens at Saclay ⁶⁾. In view of the fact that for nuclei, all proposed mechanisms seem to produce results which give about the correct orders of magnitude, and regarding the complications involved, we shall restrict ourselves to the elementary processes of the type (1). Though it will certainly contribute to the deuteron production in nuclei and though one may estimate the absorbing effect of nuclear matter ⁷⁾ on deuterons formed in this process, we shall make no attempt to calculate anything for the usual target materials:

all calculations and results of this paper refer to pure proton-proton collisions only, as measurable in H₂-bubble chambers or in difference methods.

If the centre-of-mass (CM) energy is sufficiently large for the production of pairs of nucleons, then further reactions similar to (1) become possible:

[The He³ does probably not exist ⁸⁾, but has been included tentatively in the analysis.] If double pair production becomes possible, the following reactions can take place:

the reaction (5) is given by $p+p \rightarrow \text{He}^4 + 2\bar{B} + \text{others}$ (5)

$$p+p \rightarrow \left\{ \begin{array}{c} \text{H}^4 \\ \text{He}^4 \end{array} \right\} + 2\bar{B} + \text{others}. \quad (6)$$

In reactions (2) to (6) the symbol B means baryons (N, N*, hyperons, Ξ), $\bar{B}$ antibaryons, $\bar{d}$ antideuterons, while "others" means pions and K-mesons (such that isospin and strangeness is conserved).

The present paper deals with the production of

$\text{He}^4, \text{He}^3, \text{H}^3,$

$\text{He}_{\Lambda}^4, \text{H}_{\Lambda}^4, \text{He}_{\Lambda}^3, \text{H}_{\Lambda}^3,$

$\bar{d}.$

The other particles B, $\bar{B}$, K, $\bar{K}$ are not considered, since their rate of production in these reactions is negligible compared to the yield in "ordinary" reactions ^{9),10)}. Deuterons have been treated in ⁴⁾ and will not be discussed here.

We consider pp-collisions with ~ 25 GeV primary energy (CERN-PS, Brookhaven AGS), where all reactions (2) to (6) are possible, and shall calculate yields and spectra according to the methods outlined in ^{4), 9), 11)}. We shall discuss the justification of this treatment and of the statistical theory below. Presently we only add the remark that there is an experimental indication that reaction (2) contributes a non-negligible amount even if Be and Al targets are used: if one measures the yield of He^3 and of $\bar{d}$ as a function of the primary proton energy, then the production of He^3 increases with the primary energy in a similar way as the antiproton production does ¹²⁾ - which seems to indicate a correlation between He^3 and $\bar{p}$ such as provided by

the mechanism (2). The deuteron yield, on the other hand, decreases somewhat with increasing primary energy, which shows that it seems not to be coupled to the antinucleon production.

This throws some light on the question, which of the possible deuteron-producing processes dominates in nuclei: if one considers the statistical weight of all deuteron producing reactions in the elementary process (1) (unpublished material underlying the results in Ref. ⁴) then one finds that the majority of deuterons are created along with antibaryons (about 97.7 %). If therefore the mechanism (1) would dominate the d-production in Be and Al, we should expect a similar behaviour as we find for He^3 . A rather strange interpretation of this is that possibly the deuterons mainly are created in the interplay of nuclear effects, whereas the He^3 comes largely from the elementary reaction (2). Certainly, kinematical effects influence the measurements (the momenta of secondary particles were kept fixed when the primary energy was varied), but qualitatively the effect seems to be real.

According to our proposition mentioned above, we shall not pursue this question any further and consider only "p-targets".

II. The model

We shall first make the hypothesis that

the statistical theory is a suitable tool to calculate the production of compound particles

and derive the consequences of this supposition. Secondly, we shall critically discuss the validity of the statistical theory in this particular case.

- 1) If the statistical theory is assumed to be valid, then one can calculate the production of bound states along the lines explained in 4), 9), 11), 19); the bound state is considered as a particle, which has the same right to appear in the final state as any "elementary" particle, but which may have additional (internal) degrees of freedom. One of these degrees of freedom is its structure (wave function Ψ). If such a bound state of mass, say M , is to be created in a very small region Ω of interaction, then it must be created in a state of extreme contraction. The probability for this equals the probability for the creation of an "ordinary" particle with the quantum numbers (mass M , spin, isospin) times the probability that this particle is momentarily contained in a volume Ω . This latter probability is

$$\int_{\Omega} |\Psi|^2 dV.$$

Thus, if Ω is the "natural interaction volume"

$$\frac{4\pi}{3} \left(\frac{1}{m_{\pi}} \right)^3,$$

then for the deuteron production one should use

$$\Omega_d = \Omega \cdot \int_{\Omega} |\Psi_d|^2 dV \approx \Omega \cdot \frac{\Omega}{V_d} \quad (7)$$

There are several arguments ^{13), 14), 15)}, which all lead to this conclusion or to similar expressions ^{*)} [e.g., to $|\psi(0)|^2$ instead of the integral]. The numerical difference is small in any case, if reasonable types of wave functions are considered. We shall add here a further, almost purely kinematical argument, which, though it cannot prove the formula used in Ref. ⁴⁾, it makes it at least very plausible.

Intuitively the idea is that in a nucleon-nucleon collision, in which several particles are produced, two of the final nucleons can emerge in a bound state only if extremely favourable kinematical conditions are fulfilled. Namely, the binding energy of a deuteron is so small in comparison to the usual kinetic energies of the final particles that one feels it will not be strong enough to melt the two nucleons together, except if they accidentally are created in states of almost equal momenta - such that they fly nearly parallel and with practically equal velocities and would - should there be a binding force or not - stay near together for some time in any case (see Fig. 1). The condition for this is that the relative momenta are small; they must be contained in that region where the momentum distribution of the nucleons in the deuteron is essentially different from zero. From this point of view the phase space governs essentially the situation and only in those few cases, where, already for pure statistical reasons, two nucleons happen to stay together, can the small force between them come into action. Either it leads then not to a bound state - this case is contained in the statistical weight for free nucleons (which occasionally might have small relative momenta) or it leads to a bound state. As this latter case arises, however, from the same kinematical situation as the case of small relative momenta in the free nucleon case, its probability can be calculated by determining that part of the total phase space, in which this kinematical condition is fulfilled - but: since it describes a bound state, it is something entirely new and has its own statistical weight as a new channel.

*) J.S. Kowacs ¹³⁾ seems to be the first author who considered d-production in the framework of the statistical theory.

We now calculate the probability for two nucleons emerging with a small relative momentum. We shall use a method, which has already been employed in the calculation of the probability of pair annihilation in the final state ¹⁸⁾ but formulate it here using the invariant phase space description ²²⁾ which is more elegant.

The invariant phase space density can be written ²³⁾

$$R_n(m_1, \dots, m_n; P) = \int \delta^4(P - \sum_{i=1}^n p_i) \prod_{i=1}^n \delta(p_i^2 - m_i^2) d^4 p_i \quad (8)$$

where P is the total four-momentum of all n -particles. Let us divide the particles into two groups, one with particles $i = 1 \dots l$, the other containing the rest. Each group has as total four-momentum P_l and P_{n-l} as well as a "rest mass" $M_l^2 \equiv P_l^2$ and $M_{n-l}^2 \equiv P_{n-l}^2$ respectively. We ask for a situation which is shown in Fig. 1 for the case $l = 2$. We obtain from Equ. (8) by splitting the δ -function

$$R_n = \int \delta^4(P - P_l - \sum_{i=l+1}^n p_i) \prod_{i=l+1}^n \delta(p_i^2 - m_i^2) d^4 p_i \times \\ \times \int \delta^4(P_l - \sum_{i=1}^l p_i) \prod_{i=1}^l \delta(p_i^2 - m_i^2) d^4 p_i d^4 P_l$$

Introducing a further factor 1 in the form

$$1 = \int_0^\infty \delta(P_l^2 - M_l^2) dM_l^2$$

we may write

$$R_n(m_1, \dots, m_n; P) =$$

$$= \int_0^\infty dM_l^2 \int \delta^4(P - P_l - \sum_{i=l+1}^n p_i) \prod_{i=l+1}^n \delta(p_i^2 - m_i^2) \delta(P_l^2 - M_l^2) d^4 p_i d^4 P_l \int \delta^4(P_l - \sum_{i=1}^l p_i) \prod_{i=1}^l \delta(p_i^2 - m_i^2) d^4 p_i$$

8.

According to the definition (8) this is nothing else than

$$R_n(m_1, \dots, m_n; P) = \int_0^\infty dM_\ell^2 \cdot R_{n-\ell+1}(M_\ell, m_{\ell+1}, \dots, m_n; P) \cdot R_\ell(m_1, \dots, m_\ell, P_\ell)$$

As R_n is relativistically invariant, we may calculate each individual R in the rest system of the group of particles concerned. Hence in the overall CM-system

$$R_n(m_1, \dots, m_n; E) = \int_0^\infty dM_\ell^2 R_{n-\ell+1}(M_\ell, m_{\ell+1}, \dots, m_n; E) \cdot R_\ell(m_1, \dots, m_\ell; M_\ell) \quad (9)$$

We apply this formula to the case of $\ell = 2$; $m_1 = m_2 = m$; $M_\ell = M$ (see Fig. 1). Then

$$\begin{aligned} R_2(m_1, m_2, M) &= \int \delta(M - \varepsilon_1 - \varepsilon_2) \delta(\vec{p}_1 + \vec{p}_2) \frac{d\vec{p}_1}{2\varepsilon_1} \frac{d\vec{p}_2}{2\varepsilon_2} \quad ; \quad \varepsilon = \sqrt{p^2 + m^2} \\ &= \frac{\pi}{2M} \sqrt{M^2 - (2m)^2} \end{aligned}$$

hence

$$R_n(m_1, \dots, m_n; E) = \pi \int_{2m}^\infty dM \sqrt{M^2 - (2m)^2} \cdot R_{n-1}(M, m_3, \dots, m_n; E) \quad (9')$$

Now the invariant mass of the two considered particles m_1 and m_2 is

$$M^2 = (\varepsilon_1 + \varepsilon_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 = 4\varepsilon^2 = 4(m^2 + p^2) \quad (10)$$

if $\mathcal{E}$ and p are taken in the two-body rest frame. Therefore, we obtain the still rigorous formula

$$R_n(m_1, \dots, m_n; E) = 4\pi \int_0^\infty \frac{p^2 dp}{\sqrt{p^2 + m^2}} R_{n-1}(M(p), m_3, \dots, m_n; E) \quad (9'')$$

where $M(p)$ is given by (10). We now picture the production of a deuteron in the following way:

- (i) we introduce in (9'') a weight factor $f(p)$ for the p integration, which limits p to those values which are in accordance with the deuteron structure. Remember that p is the magnitude of momentum of either nucleon in the deuterons rest system.
- (ii) as the two nucleons considered stay near to each other because of their limited relative momentum, the nuclear forces between them can act, while between all the other particles they do not, since these particles rapidly reach large mutual distances. Therefore, in the energy balance, the potential energy of these two nucleons must be taken into account. This can be done roughly by replacing

$$M(p) \rightarrow M(p) + V(p) = M^*(p)$$

where $V(p)$ is essentially the Fourier transform of the potential $V(r)$ and $V(p) \leq 0$ for all relevant momenta.

With these two changes we obtain the "partial phase space" for deuteron production

$$\delta R_n(m_1, \dots, m_n; E) = R_{n-1}(M_d, m_3, \dots, m_n; E) \cdot 4\pi \int_0^\infty \frac{f(p) p^2}{\sqrt{p^2 + m^2}} dp \quad (11)$$

Here the function $R_{n-1}(M^*(p), \dots)$ has been taken out of the integral at some average value $M^*(\bar{p}) = M(\bar{p}) + V(\bar{p})$. This average value should clearly be the mass of the deuteron M_d . It remains to evaluate the p -integral. Let the structure of the deuteron be roughly described by

$$|\psi|_d^2 \sim e^{-\frac{r^2}{2r_0^2}}$$

it does not matter in this consideration that this is a bad wave function. We define then the volume of the deuteron by

$$V_d \equiv 4\pi \int r^2 e^{-\frac{r^2}{2r_0^2}} dr = (2\pi r_0^2)^{3/2}.$$

The corresponding momentum distribution will be governed by the Fourier transform of $|\psi|^2$, namely

$$f(p) = e^{-\frac{p^2}{2/r_0^2}}.$$

With this $f(p)$ we obtain for the integral in (11)

$$4\pi \int_0^\infty \frac{p^2 e^{-\frac{p^2}{2/r_0^2}}}{\sqrt{p^2 + m^2}} dp = \left(\frac{2\pi}{r_0^2}\right)^{3/2} \cdot \frac{1}{\bar{\epsilon}} = \frac{(2\pi)^3}{\bar{\epsilon} \cdot V_d}.$$

Here $\bar{\epsilon}$ is some mean value of $(p^2 + m^2)^{1/2}$. As p extends from zero to about $2/r_0$ only, we have $\bar{\epsilon} \approx m$. Therefore,

$$\delta R_n(m_1, \dots, m_n; E) = \frac{(2\pi)^3}{\bar{\epsilon} V_d} \cdot R_{n-1}(M_d, m_3, \dots, m_n; E) \quad (11')$$

We now consider the probability for finding n -particles, two of which are in a bound state: this is given (apart from irrelevant factors) by

$$P_{n,d}(E) \equiv K^n \delta R_n(m_1, \dots, m_n; E) \quad (12)$$

where K is a certain parameter which plays in the formulation with the invariant phase space the same rôle as the interaction volume Ω does in the non-invariant formulation. Combining (11') and (12)

$$\left. \begin{aligned} P_{n,d}(E) &= K^{n-2} \left[\frac{K^2 (2\pi)^3}{\bar{\varepsilon} V_d} \right] R_{n-1} \equiv K^{n-2} \cdot K_d \cdot R_{n-1} \\ K_d &= \frac{K^2 (2\pi)^3}{\bar{\varepsilon} V_d} \end{aligned} \right\} \quad (12')$$

K_d is defined by this equation.

We now translate this to the non-invariant formulation, which was used in the actual calculations. As the calculated probabilities must be the same in both descriptions, we have quite generally

$$P_n = K^n R_n = \left[\frac{\Omega}{(2\pi)^3} \right]^n \mathcal{P}_n$$

The invariant phase space R_n and the non-invariant $\mathcal{P}_n$ are related to each other by

$$\begin{aligned} R_n &= \int \delta^4(P - \sum p_i) \frac{d\vec{p}_1 \dots d\vec{p}_n}{2\varepsilon_1 \dots 2\varepsilon_n} \\ &= \frac{1}{\prod_{i=1}^n 2\varepsilon_i} \int \delta^4(P - \sum p_i) d\vec{p}_1 \dots d\vec{p}_n = \frac{1}{\prod_{i=1}^n 2\varepsilon_i} \mathcal{P}_n \end{aligned}$$

The $\bar{\epsilon}_i$ are defined by this equation and physically they are mean energies of the particles. This, if different sorts of particles are present, leads then to the correspondence

$$\frac{K_i}{2\bar{\epsilon}_i} = \frac{\Omega_i}{(2\pi)^3}.$$

Rewriting (12') in the non-invariant formulation, we obtain with this correspondence

$$P_{n,d}(E) = \left[\frac{\Omega}{(2\pi)^3} \right]^{n-2} \cdot \frac{\Omega_d}{(2\pi)^3} \cdot \rho_{n-1} \quad \left. \vphantom{\frac{\Omega_d}{(2\pi)^3}} \right\} \quad (12'')$$

with

$$\Omega_d = \frac{(2\pi)^3 K_d}{2\bar{\epsilon}_d} = \frac{(2\pi)^3 (2\pi)^3 K^2}{2\bar{\epsilon}_d \cdot \bar{\epsilon} \cdot V_d}.$$

Since $\bar{\epsilon} \approx m$ was the average energy of a nucleon in the deuteron while $\bar{\epsilon}_d$ is the average energy of the deuteron in the CM-system of the n-particles, we have roughly $\bar{\epsilon}_d \approx 2\bar{\epsilon}$. [The deuterons are non-relativistic in the CM.] Then, using again the correspondence between K and Ω , we obtain

$$\Omega_d \approx \left[\frac{(2\pi)^3 K}{2\bar{\epsilon}} \right]^2 \cdot \frac{1}{V_d} = \Omega \cdot \frac{\Omega}{V_d} \quad (7')$$

with Ω being the natural interaction volume. This Ω_d is the same as defined in Equ. (7). Exactly the same consideration can be carried through for He^3 , He^4 , etc., starting from Equ. (9) with $l = 3, 4$ etc.

and introducing suitable form factors for these nuclei. This result and its whole derivation shall not prove the formula or justify its use to calculate the deuteron yield.

The conclusive argument to use the statistical theory with the above interaction volume is the one stated above at the beginning of this paragraph (p. 5). The consideration just carried through aims rather to show: if the statistical theory (with the above deuteron interaction volume) gives unexpectedly large numbers of deuterons in high-energy pp-collisions⁴⁾, then this is not in contradiction with the intuitive argument that a deuteron will appear only if the kinematical conditions favour it. In fact, the quantitative formulation of this intuitive argument leads to exactly the formula used. Then the unexpectedly large number of deuterons (in experiment and calculation) only teaches us that our intuition is right concerning the mechanism, but it is wrong in guessing how often in the average the special kinematical conditions are met. Intuition fails easily if applied to numerical estimates in n-dimensional spaces.

Once the principle is accepted, the calculation is straightforward along the lines described in Refs. 9), 11). One needs for the computation the following parameters of the particles which, in the case of hyperfragments, are not all well established (the existence of He_{Λ}^3 is rather doubtful).

TABLE 1

Parameters of particles ⁸⁾ [$\Omega = 1.365$]

Particle	Spin	Isospin	$\int_{\Omega} \psi ^2 dV$	Ω_{eff}	Remarks and References
D	1	0	0.17	0.23	16)
H ³	$\frac{1}{2}$	$\frac{1}{2}$	0.28	0.38	8)
He ³	$\frac{1}{2}$	$\frac{1}{2}$	0.28	0.38	8)
He ⁴	0	0	0.36	0.49	17)
H ³	$\frac{1}{2}$	0	0.2	0.27	guessed volume
He ³	$\frac{1}{2}$	1	0.1	0.14	He ³ does probably not exist
H ⁴	0	$\frac{1}{2}$	0.3	0.41	guessed volume
He ⁴	0	$\frac{1}{2}$	0.3	0.41	guessed volume

For the antideuteron $\int_{\Omega} |\psi|^2 dV$ has been calculated using an S-wave function (including a hard core), given by Hulthén and Hedin ¹⁶⁾ (of course, the structure of $\bar{d}$ is the same as that of d).

For He⁴ the value was calculated from the r.m.s. radius value in the following way:

Denote with R_{d, He^4} the r.m.s. radii of deuteron and He⁴ respectively [taken from Herman and Hofstadter ¹⁷⁾] and with $r_{d, \text{He}^4} \equiv \alpha \cdot R_{d, \text{He}^4}$ those radii, which define a volume $V \equiv \frac{4\pi}{3} r^3$ in such a way that

$$\frac{\Omega}{V} = \left(\frac{\chi_{\pi}}{r}\right)^3 \equiv \int_{\Omega} |\psi|^2 dV$$

Since for the deuteron the integral has been computed r_d and with it also α is defined. Taking the same α for He^4 we arrive at

$$\int_{\Omega} |\psi|_{\text{He}^4}^2 dV \equiv \left(\frac{\lambda_{\pi}}{\alpha R_{\text{He}^4}} \right)^3 = 0.36$$

Similarly, the value for He^3 was found, using $R_{\text{He}^3} = 1.83 \text{ f}$ (p. 18 in Ref. ⁸). We took the same value for H^3 .

The hypernuclei are somewhat larger than the corresponding nuclei (calculations on $\text{N-N-}\Lambda$ with reasonable assumptions lead to a H^3 bound state but not to a He^3 bound state ⁸). But as no reliable numbers exist, we guessed the values by assuming that

$$V_{\text{He}^4} < V_{\text{He}^4_{\Lambda}} < V_{\text{He}^3} < V_{\text{H}^3_{\Lambda}} < V_d < V_{\text{He}^3_{\Lambda}} \quad [\text{if the latter exists}].$$

As the parameter

$$\int_{\Omega} |\psi|^2 dV \equiv \frac{\Omega}{V}$$

enters only as a simple factor in the multiplicities (i.e., partial cross-sections for the production of these particles) the numbers used are more than accurate enough in view of the fact that in the best case we can hope only to predict orders of magnitude.

Errors in spin and isospin would also not have a large numerical influence, since, as we shall see below, there are uncertainties in the theory which are presumably larger than a factor of the order of two. But as the computer programmes require definite numbers, we choose tentatively those of Table 1.

A last point to be mentioned is the final state annihilation: if any nuclear particle emerges with a similar velocity (vector) as any anti-baryon such that the two remain near to each other for a while, then they may annihilate in the final state. The probability for this is calculated in the same way as above the kinematical probability for a bound state. A more detailed discussion is found in Appendix II of Ref. ¹⁸⁾. A consistent treatment of this final state interaction in a similar way as in ¹⁸⁾ was, however, not possible here, without increasing unreasonably the computing work. Some typical cases were checked, namely

$$\left. \begin{array}{l} \bar{d}+4N+n\pi \\ (\bar{d}+d)+2N+n\pi \\ (\bar{d}+\text{He}^4)+n\pi \end{array} \right\} n = 0, 1, 2, \dots$$

where in the first case $\bar{d}$ annihilates with any N , and in the other cases it was supposed that the particles in parenthesis annihilate with each other (not always completely, of course). It was found that the annihilation rate depends on the meson number n and on the reaction in question, but that it would be sufficient to take the effect roughly into account by saying that the multiplicities calculated without final state annihilation will be multiplied by ~ 0.6 to give the numbers of particles escaping this annihilation, i.e., the observable numbers. The value 0.6 was the average over the cases listed above.

- 2) So far we discussed the implications of the assumption that the statistical theory is suitable for the calculation of production of bound states in pp-collisions.

We shall now explain why and to what extent we believe that this assumption holds. We repeat again that we do not mean the p-nucleus collision, where a variety of different mechanisms intermingle, but the pure p-p collision.

Recently, at the "International Conference on Theoretical Aspects on Very High-Energy Phenomena", held at CERN in 1961, the statistical theory has been critically reviewed¹⁹⁾ with the main result that, if it is considered as a means to calculate averaged transition probabilities etc., it contains two essentially different parts: one is the phase space describing the kinematics; it can be computed to any precision required. The other is the averaged matrix element squared, about which we know practically nothing. If it could be determined, then the statistical theory would make rather limited but exact predictions (e.g., on multiplicities, branching ratios and CM-energy spectra averaged over all angles) and these predictions would even include the effects of peripheral collisions and/or asymmetric initial states (e.g., $\bar{N} + p \rightarrow \dots$). The actual situation is, however, that the averaged matrix element squared is replaced by a factor $\prod_i \Omega_i^{n_i}$ where the n_i are "interaction volumes" and the n_i particle numbers. This is such a crude ansatz that it cannot possibly account for those reactions in which the kinematical situation is very different from a statistical equilibrium, namely for the peripheral collisions. We shall, therefore, assume that peripheral collisions are better described by models specifically designed for them²⁰⁾ and that the present form of the statistical theory accounts mainly for the contributions coming from central collisions. The model proposed by Amati, Fubini, Stanghellini and Tonin²⁰⁾ is a generalization of the simple peripheral model, which is contained in it. It deals with a situation pictured in Fig. 2, namely an iterated one-meson exchange. We mention this model here, because it seems to be able to describe many of the observed facts and one could think of trying to use it in place of the statistical theory, because at high energies nearly all collisions will be "peripheral". But it seems not possible to use this model (without overstressing it) in precisely those reactions in which we are presently interested, namely those in which baryon-antibaryon pairs are created: either these pairs emerge from

one single bubble in Fig. 2 - then this bubble must have a high excitation energy; or the pair comes from two different bubbles - then a virtual baryon line must combine the two vertices. The latter holds also in the case of deuteron production if no pairs are involved. It may be that the model will give reasonable answers if it is overstressed that way. This has not yet been worked out. In any case, we seem to meet here a class of reactions which should be adequately described by the statistical theory. The only question is, whether these reactions happen as often as one would expect from an unrestricted application of the statistical theory to all reactions.

It has been found (at 25 GeV) that, while the pion and K-multiplicities, as predicted from the statistical theory, are in good agreement with the experiments, the production of antiprotons is about 10 times smaller than calculated (see ¹⁹⁾; a factor of the order of two may be attributed to annihilation in the nucleus where the antinucleon was created ⁷⁾). As we saw above, the pair production lies presumably outside the possibilities of peripheral models and could be described by the statistical theory. If that is so, then one can conclude from the numerical value of the discrepancy that for 25 GeV pp-collisions about 90 % are peripheral and only ~ 10 % are central. The question arises, whether this distinction is a constant or a dynamical one in the following sense:

if there exists something like a nucleon core, which is essentially different from the meson cloud, then one could expect core-core collisions to be practically 100 % central and the rest to be more or less peripheral. Then the ratio of these two $\sigma_{\text{centr.}}/\sigma_{\text{inel.}}$ should be given by the core radius and be independent of the energy.

If, however, the nucleon is essentially only a cloud of virtual particles (with a point-like source) whose densities and masses (K-mesons, baryon-antibaryon pairs) increase towards the centre,

then it may depend on the CM-energy which collisions will be "central" and which will be "peripheral" and these concepts have a dynamical meaning. On the basis of such an assumption it was estimated that at 25 GeV primary energy about 10 % of the pp-collisions would be central; that is, in these 10 % most of the CM-energy is available for particle production and the statistical theory is applicable ¹¹⁾.

These two pictures lead to different predictions in the case of all those processes, where two pairs are involved, namely $\bar{d}$ and He^4 production.

In the first case, one should expect that all multiplicities which depend on the creation of pairs will be smaller than the predictions of the statistical theory by roughly a factor ten, should the number of pairs involved be one or two.

In the second case, the creation of two pairs will require more "centrality" than the single pair creation and consequently $\bar{d}$, He^4 , H^4_{Λ} and He^4_{Λ} multiplicities should differ from the experimental findings by roughly a factor hundred rather than ten as the antiprotons do (whose majority comes from single pair creation) ^{*)}.

*) It should be mentioned here that from unpublished numerical results of the 25 GeV pp-collision analysis by the statistical theory ⁹⁾ it can be concluded that at this energy ≈ 97.7 % of all deuterons are created along with an antibaryon (if the statistical theory is able to describe this process). Since the antibaryon production is, for the above explained reasons, 10 times smaller than predicted, the same should happen to the deuterons. Hence the numbers predicted for 25 GeV (not those for 2.3 GeV) in Ref. ⁴⁾ should be too large by a factor 10. It would be interesting to check this experimentally.

The production of H^3 , He^3 and the corresponding hyperfragments goes, however, almost exclusively via those channels in which one single baryon-antibaryon pair is created: namely, the present calculation gives for the ratio

$$\frac{\text{number of } H^3 \text{ and } He^3 \text{ created together with one pair}}{\text{number of } H^3 \text{ and } He^3 \text{ created together with two pairs}} \approx 200 .$$

Therefore, the calculated multiplicities of H^3 , He^3 and their hyperfragments should be too large compared to the future experimental findings by roughly the same factor ten as the antinucleon production rate.

III. Results (only for pp-collisions at 25 GeV)

We present here the results following from the parameters of Table 1 (p.14) and the application of the statistical theory in three groups:

- 1) the total multiplicities,
 - 2) the normalized centre-of-mass kinetic energy spectra,
 - 3) lab.-momentum spectra for $\bar{d}$, H^3 , He^3 and He^4 at a few angles.
- 1) The total multiplicities, that is, the number of particles created per inelastic pp-collision, are given in Table 2. The first column shows the computer output obtained with the parameters of Table 1.

The second column lists these numbers reduced by a factor ~ 0.6 to take into account the final state annihilation (see above, p.16).

The third column gives those values which one obtains by reducing everything in which two NN -pairs are involved, by a further factor ~ 10 . This would correspond to having hard core in the nucleons whose collisions are "central" whereas core-cloud and cloud-cloud collisions are peripheral. The ratio peripheral: central is then roughly 10:1 (see discussion on p. 18-20) for all kinds of reactions.

Finally, the fourth column contains those values which are obtained if one considers the double-pair creation to require more "centrality" than the single-pair creation: $\bar{d}$, He^4 , H^4_Λ and He^4_Λ can be created only together with two pairs and accordingly their multiplicities have been reduced by a factor hundred, while H^3 , He^3 and the corresponding hyperfragments are mostly created together with one pair. Their multiplicities have not been reduced further.

TABLE 2

Multiplicities per inelastic event (25 GeV pp)

Particle	(1) Computer output	(2) After final state annihil.	(3) If fixed hard core	(4) Double pairs require more "centrality" ***)	Remarks
$\bar{d}$	1.79×10^{-4}	1.1×10^{-4}	1×10^{-5}	1×10^{-6}	
$H^3 + He^3$	1.86×10^{-3}	1.1×10^{-3}	1×10^{-4}	1×10^{-4}	*)
He^4	1.42×10^{-5}	8.5×10^{-6}	1×10^{-6}	1×10^{-7}	
H^3_{Λ}	3.82×10^{-4}	2.3×10^{-4}	2×10^{-5}	2×10^{-5}	
He^3_{Λ}	4.49×10^{-4}	2.7×10^{-4}	3×10^{-5}	3×10^{-5}	**)
$H^4_{\Lambda} + He^4_{\Lambda}$	6.84×10^{-6}	4.1×10^{-6}	4×10^{-7}	4×10^{-8}	*)

Remarks:

- *) the sum of the multiplicities is given, their ratio is roughly 1:1;
- ***) this hyperfragment does probably not exist ⁸⁾;
- ***) these multiplicities have been used for the calculation of the lab.-spectra, Figs. 7-9.

The corresponding partial cross-sections, e.g., $\sigma_{\bar{d}}$ are found by multiplying the numbers in columns (3) or (4) into the total inelastic pp-cross-section at 25 GeV, which is roughly 30 mb.

- 2) The normalized centre-of-mass kinetic energy spectra are displayed in Figs. 3-6 in the form

$$\frac{\text{mean particle number per 100 MeV energy interval}}{\text{mean multiplicity}} = \frac{1}{\langle n \rangle} \cdot \frac{\langle \Delta n \rangle}{\langle \Delta \epsilon \rangle}$$

versus kinetic energy in MeV. As the curves are normalized to unity, one has to multiply them by the numbers found in column (4) (or (3)) of Table 2 to obtain the number of particles per 100 MeV and per inelastic pp-collision at 25 GeV.

The curves have been obtained from the histograms resulting from the calculation by applying a smoothing procedure (least square method ²¹⁾). The effect of final state annihilation has not been included. The spectra are by definition averages over all angles in the centre-of-mass system. Nothing can be said about the angular distribution ¹⁹⁾, but it should be fairly isotropic.

- 3) The momentum spectra in the lab. system are given in Figs. 7-9 for $\bar{d}$, H^3+He^3 and for He^4 respectively. They have been obtained from the centre-of-mass spectra by a Lorentz transformation under the additional assumption of isotropy in the CM-system. Represented is the function

$$\frac{\text{absolute number of particles at the angle } \theta_{\text{lab}}}{\text{steradian} \cdot [\text{GeV}/c] \cdot \text{event}} = \frac{\partial^2 N}{\partial p \partial \Omega}$$

versus momentum p in GeV/c.

The numbers are absolute numbers per inelastic collision and, therefore, contain the calculated multiplicities. We choose those of column (4) of Table 2 which are based on the assumption of a mechanism in which double pair creation requires much more "centrality" than single pair creation. These numbers represent in the best case crude estimates only.

H^3 and He^3 have been treated throughout the calculation as one particle (namely as isospin doublet) and the curves give the sum of their spectra. Very roughly each contributes one half.

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FIGURE CAPTIONS

Fig. 1 The kinematical situation favouring bound state creation.

Fig. 2 The iterated one-meson exchange model.

Fig. 3 CM-spectrum of $\bar{d}$.

Fig. 4 CM-spectrum of H^3 , He^3 , H_{Λ}^3 , He_{Λ}^3 .

Fig. 5 CM-spectrum of He^4 .

Fig. 6 CM-spectrum of H_{Λ}^4 and He_{Λ}^4 .

Fig. 7 Lab-spectrum of $\bar{d}$.

Fig. 8 Lab-spectrum of H^3+He^3 .

Fig. 9 Lab-spectrum of He^4 .

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig.5

Fig.6

FIG. 7

FIG. 8

FIG. 9