The photon-spurion and dispersion theory
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From: Angela Oleandri <oleandri@sif.it>
Date: October 12, 2007 4:07:47 PM GMT+02:00
To: Salvatore Mele <Salvatore.Mele@cern.ch>
Subject: Re: Request for permission to reproduce published material.

Dear Dr Mele,

further to your request, sent in July to Carmen Vasini (herewith attached)
I'm pleased to inform you that the Italian Physical Society is happy to
grant you permission of reproduction of the listed material for your
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The « Photon-Spuriion » and Dispersion Theory.

E. LEADER

CERN - Geneva

(ricevuto il 9 Dicembre 1961)

It has recently been asserted ⁽¹⁾ that the calculation of a process $A+B \rightarrow A+B$ (see Fig. 1) in which one photon is exchanged, by *Dispersion Theory methods*, would

fail to produce the correct result, i.e., would not include the Coulomb interaction term. GOEBEL ⁽¹⁾ has interpreted this as requiring the existence of a genuine « spuriion », i.e., third state of polarization of the photon, at zero energy.

We show in the following, that this is unnecessary and that Dispersion Theory with physical, transverse photons does indeed predict the correct answer ^(*).

As usual define

$$(1) \quad t = -k^2, \quad \text{with} \quad k = p_1 - p'_1.$$

Neglecting inessential factors, unitarity in channel III ($B + \bar{B} \rightarrow A + \bar{A}$) yields

$$(2) \quad \text{Im} \langle p_2, -p'_2 | T | -p_1, p'_1 \rangle \sim \quad \text{for } t \geq 0,$$

$$\sim \int d^4k \sum_{\tau_{\perp}} \langle p_2, -p'_2 | T^{\dagger} | \gamma_k^{(\tau)} \rangle \langle \gamma_k^{(\tau)} | T | -p_1, p'_1 \rangle \delta(k^2) \delta^4(k + p_1 - p'_1),$$

where the polarization sum is over transverse states only,

$$(3) \quad \sim \int d^4k \sum_{\tau_{\perp}} J^{(B)} \cdot \epsilon^{(\tau)} \cdot J^{(A)} \cdot \epsilon^{(\tau)} \delta(k^2) \delta^4(k + p_1 - p'_1).$$

⁽¹⁾ C. GOEBEL: *An elementary particle of zero four-momentum; a genuine spuriion*, University of Wisconsin, preprint (1960).

^(*) Note added in proof. — Dr. GOEBEL has informed me kindly that he is aware of this formal derivation of the correct result.

Gauge invariance, $J \cdot k = 0$, together with the existence of the mass shell condition $\delta(k^2)$ allow us to write ⁽²⁾

$$(4) \quad \sum_{r=1}^4 J^{(B)} \cdot \varepsilon^{(r)} J^{(A)} \cdot \varepsilon^{(r)} \delta(k^2) = \sum_{r=1}^4 J^{(B)} \cdot \varepsilon^{(r)} J^{(A)} \cdot \varepsilon^{(r)} \delta(k^2) = J^{(A)} \cdot J^{(B)} \delta(k^2).$$

Carrying out the trivial integrations in (3) leaves ⁽³⁾

$$(5) \quad \text{Im} \langle p_2, -p_2' | T | -p_1, p_1' \rangle \sim J^{(A)} \cdot J^{(B)} \delta(t) \quad t \geq 0.$$

The amplitude for the original process

$$A + B \rightarrow A + B$$

is then

$$(6) \quad \langle p_1', p_2' | T | p_1, p_2 \rangle \sim \frac{1}{\pi} \int_0^{\infty} \frac{\text{Im} T(t')}{t' - t} dt' \sim \frac{J^{(A)} \cdot J^{(B)}}{k^2}.$$

This agrees with formula (3) of Goebel and can be shown to contain the required Coulomb term ⁽¹⁾.

* * *

I am grateful to Dr. D. AMATI for a discussion on this matter.

⁽²⁾ There is an apparent difficulty if $k=0$ in (4). However one can see from (3) and (6) that the point $k=0$ simply does not appear in the calculation. I am grateful to Dr. J. C. TAYLOR for pointing out this difficulty.

⁽³⁾ One runs into difficulty and obtains the *incorrect* result

$$\text{Im} T(t') \sim J_{\perp}^{(A)} \cdot J_{\perp}^{(B)} \delta(t'),$$

only if one interchanges the order of summation and integration in eq. (3) without sufficient care.