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FORBIDDEN WEAK INTERACTIONS AT VERY HIGH ENERGIES

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In the search for possible ways to study forbidden (second order) weak interactions, the photoproduction of a μe -pair in the field of a nucleus and the process $e^- + e^- \rightarrow \mu^- + \mu^-$ at very high energies have been considered. By taking advantage of the coherence effect in the first process a total cross-section of the order 10^{-40} cm^2 is obtained for 300 GeV photons assuming the weak interaction cut-off $\Lambda = 50 \text{ GeV}$. Due to the large mass difference between the electron and the μ -meson the energy distribution for the μ -mesons is very strongly peaked for nearly maximal energy, which distinguishes this process from possible competing processes. This is of importance, experimentally, as one has no method of distinguishing a very fast electron from a very fast μ -meson at present. The fact that nearly all the electrons come out with an energy less than 25 MeV in "weak pair production" enhances, therefore, the chances to observe the process.

The weak pair production depends only logarithmically on the energy of the incoming photon. As a consequence of the weak energy dependence an experiment would be more easily done with a high intensity machine rather than with a very high energy machine.

The second process considered in this paper gives a cross-section of the order 10^{-43} cm^2 at 1 GeV c.m. energy far beyond what can be experimentally detected in a foreseeable future and is, therefore, of academic interest only.

1. Introduction

When the universal Fermi interaction of the V-A type was originally proposed ¹⁾ there were several apparent inconsistencies between the predictions of the theory and the experimental results available at that time. Since then all these difficulties have been overcome and the theory seems to be in excellent agreement with experiments, possibly with the exception of the leptonic decays of hyperons ²⁾. The domain of applicability of this theory based on a point-like four fermion interaction still remains an open question, however, as one so far only has experimental verification in the low energy region from decay processes. It may be that the point-like structure for the basic interaction only is a good approximation in the low energy limit for a more "complicated" interaction. This would be the case, for example, if there exist intermediate vector bosons coupled to the weak interaction currents. The non-locality of the four fermion interaction due to such intermediate bosons would not be experimentally detectable in the energy region covered by decay processes if the masses of these bosons are sufficiently large. On the other hand if the basic interaction really is point-like then the theory based on the interaction Hamiltonian

$$\mathcal{H} = \frac{G}{\sqrt{2}} \bar{J}_\mu \times J_\mu^* \quad (1.1)$$

$$J_\mu = (\bar{A}\rho) + (\bar{n}\rho) + (\bar{e}\nu) + (\bar{\mu}\nu) + \dots$$

is a true field theory and we would expect that conventional field-theoretic methods could be applied. It turns out, however, that the theory in this form is non-renormalizable and strongly divergent. This has been taken as evidence against the assumption that (1.1) is a proper foundation for a genuine field theory ³⁾. One argues that (1.1) treated to lowest order in perturbation theory is a good approximation at low energies and that higher order corrections are meaningless.

To answer the question raised above whether the basic interaction is a contact four fermion interaction or not, it is necessary to extend the study of weak interactions to processes where the structure of the underlying interaction shows up. This could be done by studying basically low energy processes which can occur only due to a more complicated structure of the interaction or as a result of higher order effects. Investigations along these lines have been carried out by several authors ⁴⁾. Another way would be to study weak interactions at very high energies where the internal structure of the interaction becomes important. The experimental difficulties in such a programme are enormous as cross-sections for possible higher order processes are expected to be frustratingly small. Also the possibilities to reveal the basic structure of weak interactions in this way were surveyed in reference ⁴⁾.

In spite of the inherent difficulties in a field theory of weak interactions based on an interaction Hamiltonian of the form (1.1) we shall approach the problem with (1.1) as a starting point. However, the fact that the theory is strongly divergent, is taken as evidence that there is somehow a modification of the basic interaction at distances smaller than a characteristic length corresponding to a cut-off of about 50 GeV ⁵⁾, where weak interaction form factors become important. As a first approximation we shall represent these form factors by a common cut-off introduced in the theory by a Feynman regularization of the propagators.

Possible processes to yield information on the structure of weak interactions can only involve leptons as these do not participate in strong interactions. The presence of strong interactions must be avoided for two reasons in our case; strong interactions would obscure the theoretical interpretation of the process and further introduce an effective cut-off around the nucleon mass, that is far below the 50 GeV cut-off that possibly governs the weak interactions. The choice of leptonic processes is further limited by the necessary requirement that they must not have electromagnetic counterparts in which case the effect of the weak interactions will only be an unobservable correction to the cross-section. This means, for example, that the process

$e^+e^- \rightarrow \mu^+\mu^-$ is excluded as it can proceed through diagram a) as well as b) in Fig. 1. Of these, diagram a) is obviously the dominant.

Fig. 1

Feynman diagrams for $e^+e^- \rightarrow \mu^+\mu^-$

A process fulfilling all requirements above is $e^-e^- \rightarrow \mu^-\mu^-$ which to lowest order only can proceed through the following diagram

Fig. 2

Feynman diagram for $e^-e^- \rightarrow \mu^-\mu^-$

A straightforward computation of this diagram including the effects of the Pauli principle gives

$$\sigma \approx \frac{116 G^4 \mathcal{M}^4}{3 (2\pi)^5} \cdot |\bar{q}|^2 \quad (1.2)$$

where G is the usual unrenormalized weak interaction coupling constant, Λ the cut-off energy, and $\vec{q}$ the three momentum of the incoming electron in the c.m. system. If we take $\Lambda = 50$ GeV and $|\vec{q}| = 1$ GeV/c (corresponding to 4000 GeV/c in the lab. system if the target electron is at rest there) we obtain

$$\sigma \approx 1.5 \times 10^{-43} \text{ cm}^2$$

far beyond what can be experimentally detected in a foreseeable future.

It is obvious from this example that just any second order process fulfilling our basic requirements above will not do. We must find some mechanism to enhance the cross-section by a factor $10^3 - 10^4$ in order to reach the region where one can hope for experimental verification of the predictions. Such a mechanism is the coherence effect in electromagnetic processes. We shall here exploit the possibilities to take advantage of the coherence effect in the study of forbidden weak interactions.

In particular, we shall study the weak pair production

$$\gamma + Z \rightarrow \mu^- + e^+ + Z$$

in the field of a nucleus, here represented by Z . The pair creation can only occur if an absorber of three momentum is available as follows from energy momentum conservation. That the process is coherent means that the nucleus acts as a whole and represents the origin of a Coulomb field from a charge Ze . In order that the nucleus shall act as a whole, the momentum transfer to the nucleus must be small compared to the inverse of the radius, that is

$$|\vec{Q}| \lesssim \frac{m_\pi}{A^{1/3}} \quad (1.3)$$

where $\vec{Q}$ denotes the momentum transfer to the nucleus, m_π is the inverse pion Compton wavelength and A is the mass number of the nucleus. For larger momentum transfers the nucleus acts as Z charges e rather than one charge Ze and we lose one factor Z in the cross-section. This restriction of

the momentum transfer to the nucleus is, of course, to our disadvantage in a very high energy process but it will be found that the coherence requirement (1.3) will not weaken the energy dependence of the cross-section.

There will be an important gain in the cross-section for this process as compared to the $e^-e^- \rightarrow \mu^- + \mu^-$ or other possible second order weak processes as there will be a net gain in available phase space.

Before entering a discussion of possible diagrams that contribute to the S-operator for the process $\gamma + \bar{\nu} \rightarrow \mu^- + e^+ + \bar{\nu}$ we shall first discuss some important dynamical aspects of the problem.

2. Dynamical aspects of the weak pair creation $\gamma + \bar{\nu} \rightarrow \mu^- + e^+ + \bar{\nu}$

The weak pair production can be described by the following box diagram

Fig. 3

Box diagram for $\gamma + \bar{\nu} \rightarrow \mu^- + e^+ + \bar{\nu}$

We shall, in this paragraph, study the dynamical aspects of this process in the untrarelativistic limit when we impose the coherence requirement (1.3). The large mass difference of the final state μ -meson and electron will in a very significant way influence this process in spite of the fact that we are interested in the very high energy limit ($E_\gamma \gtrsim 100$ GeV).

6.

The dynamics of the process is essentially defined by three angles (θ, ψ, φ) defined in Fig. 4 below. The momentum transfer to the nucleus is given by

$$\vec{Q} = \vec{k} - \vec{q}_1 - \vec{q}_2 \quad (2.1)$$

and its square can now be written by means of these angles.

Fig. 4

$$|\vec{Q}|^2 = 2E^2 - 2E(\beta_1 E_1 \cos \theta + \beta_2 E_2 \cos \psi) - 2E_1 E_2 (1 - \beta_1 \beta_2 \cos \alpha) - m_e^2 - m_\mu^2$$

$$\cos \alpha = \cos \theta \cos \psi + \sin \theta \sin \psi \cos \varphi \quad (2.2)$$

$$\beta_1 = \frac{|\vec{q}_1|}{E_1} \quad \beta_2 = \frac{|\vec{q}_2|}{E_2}$$

To obtain (2.2) we have also made use of the fact that the Coulomb field cannot take up energy, that is

$$E = E_1 + E_2 \quad (2.3)$$

In the ultrarelativistic limit ($m \ll E$) we have

$$\beta = \sqrt{1 - \left(\frac{m}{E}\right)^2} = 1 - \frac{1}{2} \left(\frac{m}{E}\right)^2 - \frac{1}{8} \left(\frac{m}{E}\right)^4 - \dots \quad (2.4)$$

and further, the particles all come out within a narrow cone around the direction of the incoming photon ⁶⁾. We can then write (2.2)

$$|\vec{Q}|^2 = E_1^2 \theta^2 + E_2^2 \vartheta^2 + 2 E_1 E_2 \theta \vartheta \cos \varphi + \frac{1}{4} \left(\frac{m_e^2}{E_1} + \frac{m_\mu^2}{E_2} \right)^2 + \dots \quad (2.5)$$

and the coherence requirement (1.3) takes the form

$$E_1^2 \theta^2 + E_2^2 \vartheta^2 + 2 E_1 E_2 \theta \vartheta \cos \varphi + \frac{1}{4} \left(\frac{m_e^2}{E_1} + \frac{m_\mu^2}{E_2} \right)^2 \lesssim \frac{m_\pi^2}{A^{1/3}} \quad (2.6)$$

To see the consequence of (2.6) we shall anticipate a result from the calculations below. All the terms in the S-matrix element for this process contain one of the following factors in the denominator

$$\begin{aligned} & [m_\mu^2 (E - E_2) + m_e^2 E_2 + E E_2 \vartheta^2] \\ & [m_\mu^2 (E - E_1) + m_e^2 E_1 + E (E - E_1)^2 \theta^2] \end{aligned} \quad (2.7)$$

As $m_\mu \approx 207 m_e$ this factor in the denominator will very strongly favour the μ -meson to come out with near maximal energy. In the cross-section the denominator is quadratic in these factors, which makes the effect even more pronounced and it will be seen later that almost all electrons will be found in the energy interval $E_e \lesssim 50 m_e$.

It then follows that (2.6) essentially imposes restrictions on the phase space available for the μ -meson, as

$$\beta \lesssim \left(\frac{m_\mu}{E} \right) \cdot \frac{1}{A^{1/2}} \approx 10^{-4} \quad (2.8)$$

for $E = 300 \text{ GeV}$ and $A \approx 200$.

3. Calculation of the cross-section

The creation of a μe -pair in the field of a nucleus can be described by a large number of diagrams of which some typical examples are shown in Fig. 5. Altogether there are to this order in the perturbation expansion twelve diagrams.

Fig. 5

Some typical diagrams contributing to the process

$$\gamma + \text{Fe} \rightarrow \mu^- + e^+ + \text{Fe}$$

Some of these are negligible as they will have a factor Λ^2 replaced by a squared mass and we are assuming $\Lambda = 50$ GeV. However, there will still be eight diagrams which seem to give large contributions in a rough estimate. In view of the uncertainty in the cut-off and the obvious difficulties with an experiment of this type, it has been considered superfluous to calculate the cross-section based on all these diagrams. With the calculations of the $\mu \rightarrow e + \gamma$ ⁷⁾ as a base for estimates, we have chosen the two diagrams a) and b) above as representative for the process. It is expected that this will give the right energy behaviour and the right order of magnitude for the cross-section, and that a calculation of the remaining diagrams, therefore, would not add anything of interest at present.

In standard fashion we obtain the following S-matrix element from diagrams a) and b) in Fig. 5

$$S_{fi} = -i \frac{4e^3 G^2 \Lambda^4}{576 (2\pi)^{3/2}} \sqrt{\frac{m_e m_\mu}{2EE_1 E_2}} \delta(E-E_1-E_2) \frac{e_\mu^{(\lambda)}}{|\vec{Q}|^2} \{ A [q_{\delta\epsilon} q_{\delta\epsilon} + q_{\epsilon\epsilon} q_{\delta\delta}] - B q_{\delta\epsilon} q_{\nu\epsilon} \} \\ \left\{ \frac{(q_2 - k)_\mu}{(q_2 - k)^2 - m_e^2} \bar{w}^\mu(q_2) \gamma^\nu \gamma^\delta \gamma^\mu \gamma^\epsilon \gamma^\rho \gamma^\mu \gamma^\nu v^\nu(q_1) + \right. \\ \left. + \frac{(q_1 - k)_\mu}{(q_1 - k)^2 - m_\mu^2} \bar{w}^\mu(q_1) \gamma^\nu \gamma^\mu \gamma^\rho \gamma^\epsilon \gamma^\mu \gamma^\delta \gamma^\nu v^\nu(q_1) \right\} \quad (3.1)$$

where $\gamma^\mu = \gamma^\mu (1 + \gamma_5)$, $e_\mu^{(\lambda)}$ is the polarization vector for the incoming photon, A and B are numerical factors and the other notations are given in Fig. 3.

The complete expression for the cross-section based on (3.1) is complicated. Furthermore, it is only in the ultrarelativistic limit we can hope that it reaches a value that makes experiments possible. In this limit the cross-section simplifies a great deal and we get after the integration over the energy E_1 has been carried out.

$$\sigma = \frac{4C}{E} \int_{m_\mu}^{E-m_e} dE_2 \int d\Omega_\mu \int d\Omega_e \frac{|\bar{q}_1| |\bar{q}_2| E_2 (E-E_2)}{|\bar{Q}|^4} \cdot \left\{ \frac{m_\mu^2 (E-E_2)^2 + E^2 E_2^2 v^2}{[m_\mu^2 (E-E_2) + m_e^2 E_2 + E E_2^2 v^2]^2} + \frac{E_2 (E-E_2) \{ (E-E_2)^2 \theta^2 + E_2^2 v^2 - [E_2^2 (E-E_2)^2] \theta v \cos \varphi \}}{[m_\mu^2 (E-E_2) + m_e^2 E_2 + E E_2^2 v^2] [m_\mu^2 (E-E_2) + m_e^2 E_2 + E (E-E_2)^2 \theta^2]} + \frac{m_e^2 E_2^2 + E^2 (E-E_2)^2 \theta^2}{[m_\mu^2 (E-E_2) + m_e^2 E_2 + E (E-E_2)^2 \theta^2]^2} \right\} \quad (3.2)$$

$$C = \frac{\alpha (Z\alpha)^2 G^4 \Lambda^8}{144^2 (2\pi)^{11}} \cdot \{ B-5A \}^2; \quad \alpha = \frac{1}{137} \equiv \text{fine structure constant}$$

$$d\Omega_e = 2\pi \theta d\theta$$

$$d\Omega_\mu = v dv d\varphi$$

The angular integration can easily be done. Keeping in mind that they are restricted by the coherence requirement (2.6) we obtain for the leading term

$$\sigma \simeq \frac{4\pi^2 C}{E} \int_{m_\mu}^{E-m_e} dE_2 |\bar{q}_1| |\bar{q}_2| \frac{m_e^2 E_2}{(E-E_2) [m_\mu^2 (E-E_2) + m_e^2 E_2]^2} \ln \frac{4E_2^2 (E-E_2)^2}{E [m_\mu^2 (E-E_2) + m_e^2 E_2]} \quad (3.3)$$

and finally after integration over E_2

$$\sigma \simeq \frac{8\pi^2 C}{m_\mu^2} \ln \frac{E}{m_\mu} \quad (3.4)$$

or introducing C from above

$$\sigma \simeq 6.75 \times 10^{-2} \cdot \frac{\alpha (Z\alpha)^2 G^4 \Lambda^8}{(2\pi)^9 m_\mu^2} \ln \frac{E}{m_\mu} \quad (3.5)$$

We have introduced also here the numerical values for the constants A and B .

The expression (3.2) for the cross-section in the UR limit is completely symmetric in the μ -meson and the electron variables. This has been obtained by calculating the two diagrams a) and b) and is a characteristic feature of the first Born approximation in the external field problem. A more careful study of the process would lead to some asymmetry due to the difference in charge, etc. Corrections of this type are, however, completely negligible in comparison to other uncertainties.

4. Discussion

The differential cross-section (3.3) and the total cross-section (3.5) for the photoproduction of a μ e-pair in the field of a nucleus obtained in the preceding paragraph show some interesting features which we shall discuss somewhat more in detail.

We first observe that the total cross-section has a very weak energy dependence as compared to the $e^+e^- \rightarrow \mu^+\mu^-$ discussed in the introduction. It is interesting to note that the weak pair production shows exactly the same logarithmic energy dependence as ordinary pair production in the high energy limit. In spite of this weak energy dependence, however, the total cross-section for weak pair production is considerably larger than the usual cross-sections expected for forbidden (second order) weak interactions. For $Z = 80$, $E = 300$ GeV and $\mathcal{A} = 50$ GeV, we obtain from (3.5)

$$4.58 \times 10^{-41} \text{ cm}^2$$

This total cross-section may seem frustratingly small from the experimental point of view. However, the very peculiar energy distribution due to the large mass difference between the μ -meson and the electron makes this process easily distinguishable from other possible competing processes. It is clear from (3.4) the μ -meson energy spectrum is very strongly peaked for near maximal energy or equivalently, the electron spectrum is strongly peaked at a very low energy. In Fig. 6 (see figure leaflet at the end of this report) we have drawn the electron spectrum and it follows from this that nearly all the electrons come out with an energy less than $50m_e \approx 25$ MeV. This is experimentally of great importance as the methods of detection known at present do not distinguish, say, an electron from a μ -meson when the energy is so large that the masses are unimportant. A low energy outgoing electron can easily be identified, however. Furthermore, an event consisting of a low energy electron ($\lesssim 25$ MeV) and a very high energy charged particle (≈ 300 GeV) is almost surely a weak pair creation as other possible processes do not show this peculiar energy distribution. In ordinary pair creation, for example, the positron-electron pair will share the available energy almost equally as they have the same mass. The huge effect on the energy distribution of the μ -e mass difference, therefore, enhances strongly the chances to observe weak pair creation and compensates in part for the very small total cross-section.

In the present discussion on high intensity machines versus very high energy machines, the results obtained from the study of forbidden weak interactions strongly favour the former type of machines. The photoproduction of a μ e-pair is up till now the most promising possibility to study such forbidden weak interactions. Its weak energy dependence means that an energy increase by a factor 10^2 will only increase the total cross-section by a factor 5. Therefore, it seems more likely that the process will be made detectable by increasing the intensity rather than the energy for the incoming photons.

The field of forbidden weak interaction is, of course, by no means exhausted by the present study. There still remain other possible processes which fulfil the requirements we discussed in the introduction. Also a process

like $\mu + Z \rightarrow e + Z + n\pi$ ($n \leq 1$) which involves strongly interacting π -mesons could possibly yield information. Until some of the fundamental uncertainties in our approach are settled (the existence of one or two kinds of neutrinos, the existence of intermediate vector bosons, the existence of "quadratic" terms in the weak interaction Hamiltonian, etc.), we feel, however, that a more complete study of this field is not called for.

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