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NUCLEAR SCATTERING AT HIGH ENERGIES

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A B S T R A C T

A semi-empirical optical-type model is proposed for the elastic and quasi-elastic scattering of nucleons by nuclei in the GeV region where the interaction is mainly absorptive. The model is not based on a smooth nuclear potential but takes into account the granular nature of nuclear matter. It thus predicts at large scattering angles a nucleon-like tail to the differential cross-section which has been observed in preliminary experiments. It is made to conform to the optical theorem and to the requirement of consistency $\int d\sigma = \sigma_s$. The result for the differential cross-section is an especially simple formula containing nucleon and nuclear form factors. It is applied to some experimental results.

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I. Introduction

This paper comprises a theoretical discussion of the scattering of high energy nucleons (3 to 25 GeV) from nuclei. Some preliminary measurements using counters have been made at CERN ¹⁾ of the differential cross-section for the scattering of 25 GeV/c and 12.5 GeV/c protons from Beryllium. The scattering at a fixed angle for other element has been measured and compared to that of Be. For Be most of the scattering is confined to very small angles but a prominent feature of the result is the pronounced large angle tail resembling the diffraction pattern of a single nucleon. The scattering considered here is elastic or quasi-elastic in the sense the new particle creation is excluded while collisions which leave the target nucleus in an excited state or expell nuclear particles is included.

The only other published measurements of nuclear differential cross-sections are those of Preston, Wilson and Street ²⁾ at 3 GeV/c using emulsions to record the scattered protons after magnetic analysis. In this case the angular region explored was too small to indicate clearly the nucleon-like tail. Results for 25 GeV/c protons on C and Pb have also been obtained by Cvijanovich ³⁾ in emulsions without magnetic analysis. These, however, clearly show the tail.

On the theoretical side a detailed analysis of the scattering of 313 MeV protons on C has been given by Bethe ⁴⁾ and extended by Kerman, Mc Manus and Thaler ⁵⁾. Greider and Glassgold ⁶⁾ have treated scattering in the GeV range in terms of a smoothed out optical potential. Glauber ⁷⁾ has developed an approximate description of high energy collisions which may be taken as the basis for the present discussion.

Several points may be born in mind. It has been shown that in the GeV range p-p scattering is predominantly absorptive ^{8),9),10)}. This implies that the real part of the potential and hence of the

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scattering amplitude are very small and that spin dependent effects are also probably small ¹¹⁾. It would be expected that proton-nuclear scattering would show these characteristics at least to the same degree if not more so.

At 313 MeV Bethe ⁴⁾ was able to show that it is possible to construct the nuclear scattering amplitude from the measured p-p and n-p phase shifts. Here the main problem is to identify the Coulomb scattering, both pure and through the interference with the nuclear scattering. At 25 GeV, however, no phase shift solution to the p-p problem is likely as it would involve some one hundred partial waves. Furthermore, the Coulomb scattering is confined to very much smaller angles.

At 313 MeV it is possible to experimentally discriminate between pure elastic scattering and collision leading to other exit channels. (The nucleon-like tail represents a highly inelastic process from the nuclear point of view, i.e., ejection of a target nucleon.) At 25 GeV this is not possible since the momentum loss for scattering at 20 millirad. is of the order of 125 MeV/c or $\frac{1}{2}\%$ of the initial momentum and the effect of excited bound state of the target would be undetectable.

Placzek ¹²⁾ and Wick ¹³⁾ have analyzed the problem of slow neutron scattering on molecules or crystals. This problem has points of similarity to the one considered here. Placzek shows that the effect of an excited final state is of the order of $\Delta E/E$ where ΔE is the excitation energy and E is the energy of the bombarding particle. Placzek's analysis indicates how the present results might be extended to separate and treat nuclear inelastic scattering.

The aim of the present discussion is to develop a simple framework to describe high energy scattering from nuclei where absorption is the predominant effect. In doing so, we shall employ for guidance a

natural geometric model. It should be born in mind, however, that the model must not be pushed too far. Some of the geometric features are too idealized to apply to nuclear matter. Thus we will derive a relatively simple formula for the differential cross-section with a few parameters. While it may appear that the values of these parameters are fixed by the model it would be better to regard them as semi-empirical and look to the model only for guidance as to their magnitude. We will depend heavily on two important properties. Namely, the optical theorem ($\text{Im} f(0) = k \sigma_T / 4\pi$) and the requirement that the integral over the differential cross-section equal the scattering cross-section ($\int d\sigma = \sigma_s$).

In section II we outline the Glauber picture of high energy collisions. In section III the central problem of shadowing is treated and a result is obtained for the differential cross-section. In section IV there is a discussion of form factors and in section V the result is modified to account for the Fermi motion of the target nucleons. In section VI the result is applied to the experimental results that are available.

II. The high energy approximation

We shall first of all quote Glauber's result for high energy collisions and then make a few qualitative comments by way of justification. The derivation can be found in Reference ⁷⁾. The result is precisely that which would be obtained by taking seriously the analogy of physical optics. Thus Glauber's derivation may be regarded as justifying the optical analogy under the circumstances prescribed. The validity of the approximation requires these three conditions :

- a) $k \xi \gg 1$ where ξ is the range of the two nuclear scattering potential, k the bombarding wave vector,

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b) $|V|/E \ll 1$ where $|V|$ is the strength of the interaction and E the bombarding energy,

c) $R \ll k \delta^2$ where R is the radius within which the scattering system is confined.

These conditions are all well met by the system under consideration. The scattering is then all confined to very small angles, $\theta \simeq 4/k \delta$. Glauber points out that while a) and b) are required separately, no restriction is placed on the product $ka|V|/E = 2V a/\hbar v$. Thus it encompasses the regions of validity of both the Born approximation ($|V|a/\hbar v \ll 1$) and the W.K.B. approximation, ($|V|a/\hbar v \gg 1$).

The probability amplitude that a high energy particle will be scattered by a nucleus from a state having momentum $\vec{k}$ to one having momentum $\vec{k}'$ while the nucleus initially in a state i is left in a state j can be written

$$F_{ji}(\vec{k}, \vec{k}') = \langle j | f(\vec{k}, \vec{k}') | i \rangle \quad (2.1)$$

In our case where all scattered particles are detected (other than those from collisions creating new particles) the scattered intensity is given by

$$\frac{d\sigma}{d\Omega} = \sum_j F_{ij} F_{ji} \quad (2.2)$$

where the sum is over those final states accessible in the collision.

If it is assumed that those states not accessible make a negligible contribution to the sum, one can regard the states $|j\rangle$ as forming a complete set and that the closure property $\sum_j |j\rangle \langle j| = 1$ holds. This is discussed by Placzek¹²⁾ who evaluates this error of closure

in a particular case. Then

$$\frac{d\sigma}{d\Omega} = \langle i | f^*(\vec{k}'\vec{k}) f(\vec{k}\vec{k}') | i \rangle \quad (2.3)$$

One would expect $f(\vec{k}'\vec{k})$ to be an operator and in Glauber's formulation this is so. Part of the approximation consists of disregarding the operator character of f (see below) hence we regard $f(\vec{k}'\vec{k})$ as a number. We can then go to a co-ordinate representation, and Glauber's result is

$$\frac{d\sigma}{d\Omega} = \int |\psi_i(\vec{r}_1 \vec{r}_2 \dots \vec{r}_A)|^2 |f(\vec{k}\vec{k}')|^2 d\vec{r}_1 d\vec{r}_2 \dots d\vec{r}_A \quad (2.4)$$

where

$$f(\vec{k}\vec{k}') = \frac{ik}{2\pi} \int ds e^{i\vec{q} \cdot \vec{\rho}} [1 - e^{i\chi(\vec{\rho}, \vec{r}_1 \vec{r}_2 \dots \vec{r}_A)}] \quad (2.5)$$

and

$$\vec{q} = \vec{k} - \vec{k}'$$

is the momentum transfer taken to be perpendicular to $\vec{k}$ and $|\vec{q}| \simeq k\theta$.

ρ = impact parameter, $\vec{\rho}$ is perpendicular to $\vec{k}$ and has components ξ and η

$$ds = \rho d\rho d\varphi$$

$$\chi(\vec{\rho}, \vec{r}_1 \vec{r}_2 \dots \vec{r}_A) = \sum_j \chi_j(\vec{\rho} - \vec{s}_j)$$

$\vec{s}_j$ is the component of $\vec{r}_j$ perpendicular to $\vec{k}$ with component x_j and y_j .

$$\chi_j(\vec{\rho} - \vec{s}_j) = \frac{i}{\hbar v} \int_{-\infty}^{\infty} V(\xi - x_j, \eta - y_j, z_j) dz_j \quad (2.6)$$

V is the interaction between a target nucleon and the bombarding particle. V is assumed to be symmetrical and to have a range δ such that $\chi_j = 0$ for $|\vec{\rho} - \vec{s}_j| > \delta$.

We shall later take V to be purely imaginary. The integration is in a plane perpendicular to $\vec{k}$ (hence over all impact parameters) and over the region for which $e^{\chi} = 1$, that is, over the shadow or disc of the nucleus. For $\vec{\rho}$ outside the nucleon disc $e^{\chi} = 0$. It can be seen that the high energy approximation has transformed the three-dimensional integration of the Born approximation into a two-dimensional integration. The physical interpretation is that those particles which pass through the nucleus do so essentially in straight lines in accord with scattering at very small angles.

The expression, equation (2.5), for $f(\vec{k}'\vec{k})$ is identical to the optical result for Fraunhofer diffraction of light passing through and around an obstacle made up of a system of discs representing the nucleus. The second term of equation (2.5) (e^{χ}) is the integral prescribed by Huygen's principle over the radiation which is transmitted through the obstacle suffering thereby a phase shift and attenuation represented by e^{χ} . Where $e^{\chi} = 0$ the obstacle is black. The radiation passing nearby is diffracted. The first term in Eq. (2.5) is the Huygen's principle integral over all the space outside the nuclear disc which is transformed by Babinet's principle¹⁴⁾ into an integral over the corresponding aperture with phase change $e^{i\pi}$. The scattering amplitude results from the interference of these two terms. Equation (2.4), $d\sigma/d\Omega = \int |\psi|^2 |f|^2 d\tau$ then appears as an average over all configurations of the discs.

II.a Relationship to the partial wave treatment

It is of interest to relate the above expression to the partial wave treatment of scattering at high energies and we shall later need a familiar result which follows from it.

When the scattering potential is spherically symmetric the partial wave treatment gives

$$f(\theta) = \frac{i}{2k} \sum_l (2l+1) (1 - e^{2i\delta_l}) P_l(\cos \theta) \quad (2.7)$$

Strictly speaking $f(\theta)$ above refers only to the truly elastic scattering in which the nucleus is left in the ground state. However, for energies greater than one GeV no experimental discrimination is made between this and inelastic scattering except where particles are produced, and Eq. (2.7) is often regarded as including the inelastic scattering as well.

For high energies many partial waves are involved and the scattering is mainly in the forward direction. The following transformations are then permissible (see Bethe ⁴⁾ and Glauber ⁷⁾ for further discussions);

$$\sum_l (2l+1) f(l) \rightarrow 2 \int l dl f(l) \rightarrow 2k^2 \int p dp f(kp) \quad (2.8)$$

$$\begin{aligned} P_l(\cos \theta) &\rightarrow J_0(l\theta) \\ &\rightarrow J_0(kp\theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi e^{i\vec{q} \cdot \vec{p}} \end{aligned} \quad (2.9)$$

Hence

$$f(\theta) = \frac{ik}{\lambda} \int p dp e^{i\vec{q} \cdot \vec{P}} (1 - e^{2i\delta(p)}) \quad (2.10)$$

This is identical to Eq. (2.5) if the correspondence $2i\delta(p) \rightarrow \chi(p)$ is made. Note that the requirement of spherical symmetry is necessary for Eq. (2.10) otherwise the phase shift δ is a function of the azimuthal angle φ (as χ in fact is).

Equation (2.10) can be evaluated immediately for a sharp cut off model, i.e. one for which the phase shift (which may be complex) satisfies

$$\delta(p) = \delta, \quad \text{a constant,} \quad 0 \leq p < R$$

$$\delta(p) = 0 \quad p > R$$

Then

$$f(\theta) = \frac{ik}{4\pi} 2(1 - e^{2i\delta}) \pi R^2 g(kR\theta) \quad (2.11)$$

where the form factor

$$g(kR\theta) = 2J_1(kR\theta) / kR\theta \quad (2.12)$$

is defined in such a way ($g(0) = 1$) that the result of applying the optical theorem, $\lim_{\theta \rightarrow 0} f(\theta) = k \sigma_T / 4\pi$, is immediately apparent.

Let $e^{i2\delta} = e^{-2\delta_I} (\cos 2\delta_R + i \sin 2\delta_R)$ where δ_R and δ_I are both real, then

$$\sigma_T = 2 \left(1 - e^{-2\delta_I} \cos 2\delta_R \right) \pi R^2$$

This result incorporates both the black disc ($e^{-2\delta_I} = 0$) and the grey disc ($0 < e^{-2\delta_I} < 1$) cases and provides for possible potential scattering. The differential cross-section, which is

$$\frac{d\sigma}{d\Omega} \equiv \sigma(\theta) = |f|^2 = \left(\frac{kR^2}{2} \right)^2 \left| 1 - e^{-2i\delta} \right|^2 g^2(kR\theta) \quad (2.14)$$

has the angular dependence characteristic of the diffraction scattering by an absorbing (and dispersive) disc. In what follows we will assume that $\delta_R = 0$ and $e^{-2\delta_I} = a$. The parameter a is the fraction of the amplitude transmitted without change of phase. To find the scattering cross-section we may integrate Eq. (2.14) taking advantage of the fact that most of the scattering is in the forward direction by writing

$$\sigma_s = \int \sigma(\theta) d\Omega = 2\pi \int_0^\infty \theta d\theta \sigma(\theta)$$

since $\int_0^\infty \theta d\theta g^2(\theta) = 4\pi / (kR)^2$ we get

$$\sigma_s = \pi R^2 (1-a)^2 \quad (2.15)$$

The absorption cross-section $\sigma_a = \sigma_T = \sigma_s$ is

$$\sigma_a = \pi R^2 (1-a^2) \quad (2.16)$$

which accords with the definition of a , that $(1-a^2)$ is the fraction of the intensity transmitted by the disc. These results also follow from applying the limiting process indicated by Eq. (2.8) to the more familiar definitions, that is

$$\sigma_s = \lim_{k \rightarrow \infty} \frac{\pi}{k^2} \sum_{\ell} (2\ell+1) |1-a|^2 = \pi R^2 (1-a)^2$$

$$\sigma_a = \lim_{k \rightarrow \infty} \frac{\pi}{k^2} \sum_{\ell} (2\ell+1) (1-a^2) = \pi R^2 (1-a^2)$$

Again strictly speaking σ_s above should include only scattering in which the nucleus is left in the ground state although this distinction is sometimes neglected. If $f(\theta)$ Eq. (2.10) is regarded as referring to truly elastic scattering it cannot account for the nucleon-like tail which must be assumed to represent a highly inelastic process. If $f(\theta)$ is assumed to contain quasi-elastic scattering as well, it can still be shown that no spherically symmetric potential can account for the tail. Thus it seems that at high energies some account must be

taken of the granular nature of nuclear matter. This is not unreasonable since if a single nucleon is "seen" by of the order of one hundred partial waves the gaps between nucleons will be seen also.

II.b Placzek's problem

The connection between the present problem and that of the scattering of slow neutrons from molecules or crystals can now be stated. This discussion is introduced since the analysis of Placzek¹²⁾ and Wick¹³⁾ indicates how the static approximation is justified and how the present treatment might be extended to include the effects of final states.

We recall Eqs. (2.3) and (2.5)

$$\frac{d\sigma}{d\Omega} = \sum_j F_{ij} F_{ji} = (i | f^* f | i) \quad (2.17)$$

$$f = \frac{i}{\lambda} \int ds e^{i\vec{q} \cdot \vec{P}} \left[1 - e^{\chi(\vec{p}, \vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)} \right] \quad (2.18)$$

Here one would expect $f(\vec{k}, \vec{k}')$ to be an operator through the time dependence of the nucleon co-ordinates $r_j = r_j(t)$. In Glauber's formulation the time dependence is omitted as part of the approximation. However, if the final states of the scattering system (in our case the nucleons, in Placzek's case the nuclei of the lattice) differ in energy

by $\Delta E = E_j - E_i$ and $\Delta E/E_i$ is small. Plazcek's formulation shows how this time dependence is removed systematically. It contributes terms of order $\Delta E/E$ compared to the first term which is the static approximation, Eq. (2.17). These terms then reflect the effects of final states. The key point in this process is the short interaction time approximation¹³⁾. That is, the reasonable idea that the components of the scattering system move only slightly in their natural motion while the interaction takes place.

Consider now Eq. (2.18) and assume there is no overlapping or shadowing of the nucleons. We can then translate the origin of co-ordinates to the centre of each nucleon in turn and obtain

$$f = \frac{i}{\lambda} \sum_{j=1}^A e^{i\vec{q} \cdot \vec{r}_j} \int ds e^{i\vec{q} \cdot \vec{p}} (1 - e^{2i\delta^j(p)}) \quad (2.19)$$

where $\delta^j(p)$ is the phase shift function for a single nucleon. We recognize this as the high energy approximation for the expression

$$f = \frac{i}{2k} \sum_{j=1}^A e^{i\vec{q} \cdot \vec{r}_j} \sum_{\ell} (2\ell+1) [1 - e^{2i\delta_{\ell}^j}] P_{\ell}(\cos \theta) \quad (2.20)$$

For slow neutrons only the S wave phase shift enters, whence

$$f = \sum_j e^{i\vec{q} \cdot \vec{r}_j} a_j \quad (2.21)$$

where a_j is the scattering length

$$|a_j|^2 = \sin^2 \delta_0^j / k^2 = \sigma_s^j / 4\pi$$

Using Eq. (2.21) for $f(\vec{k}, \vec{k}')$ Eq. (2.4) is

$$\frac{d\sigma}{d\Omega} = \int |\psi_i|^2 |f|^2 d\vec{r}_1 d\vec{r}_2 \dots d\vec{r}_A$$

which is exactly the static approximation discussed by Placzek and applied to the scattering of X-rays and neutrons from crystals.

The obvious differences between the two systems are in the slow neutron case the lack of shadowing and S waves only. However, the assumptions underlying Wick's formalism, $\Delta E/E$ small and the short time of interaction, are the same in both cases. Thus it should be possible to translate Wick's formalism into terms of the present problem. However, the shadowing problem is such that it would appear to be unprofitable to go beyond the static approximation at this time.

III. The shadowing problem

In this section we consider the shadowing problem, that is to say, the fact that the absorption cross-section is not proportional to A but roughly proportional to $A^{2/3}$ (15). A fraction of the nucleons are screened off from the radiation or lie in the shadow of other nucleons.

Consider now the scattering amplitude $f(\vec{k}, \vec{k}')$ given by Eq. (2.5) in relation to a system of A nucleons. This we regard as a randomly arranged system of identical grey discs each of which has a given transparency a and a radius δ . The system is confined to a sphere of radius $R = r_0 A^{1/3}$ and $\delta < r_0$.

Equation (2.5) states

$$f(\vec{k}, \vec{k}') = \frac{i}{\lambda} \int d\vec{s} e^{i\vec{q} \cdot \vec{P}} \left(1 - e^{-\sum_{j=1}^A \chi(\vec{P} - \vec{s}_j)} \right) \quad (3.1)$$

The region of integration (the region for which $\sum_j \chi(\vec{P} - \vec{s}_j) \neq 0$) can now be divided into elements in a way that is illustrated by the simple example of three overlapping discs shown in Fig. 1.

For the vector $\vec{P}$ terminating

$$\begin{aligned} \text{in elements } 1, 2 \text{ or } 3, & \quad \sum_j \chi_j = a \\ \text{in elements } 4, 5 \text{ or } 6, & \quad \sum_j \chi_j = a^2 \\ \text{in element } 7, & \quad \sum_j \chi_j = a^3 \end{aligned}$$

Thus Eq. (3.1) becomes

$$f = \frac{i}{\lambda} \left[(1-a) \int_{123} d\vec{s} e^{i\vec{q} \cdot \vec{P}} + (1-a^2) \int_{456} d\vec{s} e^{i\vec{q} \cdot \vec{P}} + (1-a^3) \int_7 d\vec{s} e^{i\vec{q} \cdot \vec{P}} \right]$$

By adding and subtracting elements from each term this can be rewritten as

$$f = \frac{L}{\lambda} (1-a) \left[\int_{1234567} ds e^{i\vec{q} \cdot \vec{P}} + a \int_{4567} ds e^{i\vec{q} \cdot \vec{P}} + a^2 \int_7 ds e^{i\vec{q} \cdot \vec{P}} \right]$$

This can be generalized immediately to

$$f = \frac{L}{\lambda} (1-a) \left[\int_{S_0} ds e^{i\vec{q} \cdot \vec{P}} + a \int_{S_1} ds e^{i\vec{q} \cdot \vec{P}} + a^2 \int_{S_2} ds e^{i\vec{q} \cdot \vec{P}} \right] \quad (3.2)$$

where

S_0 = total area exposed to the radiation

S_1 = area shadowed once or more

S_2 = area shadowed twice or more

and so forth.

Now consider the forward direction $\vec{q} = 0$. Applying the optical theorem we get an expression for the total cross-section,

$$\sigma_T = 2(1-a) [S_0 + aS_1 + a^2S_2 + \dots] \quad (3.3)$$

By definition the absorption cross-section is given by

$$\begin{aligned}\sigma_a &= (S_0 - S_1)(1 - a^2) + (S_1 - S_2)(1 - a^4) + (S_2 - S_3)(1 - a^6) + \dots \\ &= (1 - a^2) [S_0 + a^2 S_1 + a^4 S_2 + \dots] \quad (3.4)\end{aligned}$$

These expressions converge fairly quickly since from the definition

$S_0 > S_1 > S_2 \dots$, and $a < 1$ (p-p scattering suggests $a \simeq \frac{1}{2}$).

σ_s and σ_a can now be estimated by geometric methods. The geometric treatment which is straightforward but somewhat tedious will not be given here but will appear elsewhere. Thus it can be shown that for large enough A , $A^{1/3} > 2$ say, σ_a can be represented to a very good approximation by

$$\begin{aligned}\sigma_a(A) &= \sigma_a(1) A^{\mathcal{N}} \\ \sigma_a(1) &= \pi \delta^2 (1 - \bar{a}^2)\end{aligned} \quad (3.5)$$

where $\bar{a}$ is an effective transparency somewhat smaller than a , and $2/3 < \mathcal{N} < 1$. By evaluating σ_T , Eq. (3.3), in the same way one obtains for the scattering cross-section, $\sigma_s = \sigma_T - \sigma_a$ the following

$$\begin{aligned}\sigma_s(A) &= \sigma_s(1) A^{\mathcal{N}} (1 + E(A^{1/3})) \\ \sigma_s(1) &= \pi \delta^2 (1 - \bar{a})^2\end{aligned}$$

The function $\epsilon (A^{1/3})$ is small and a slowly increasing function of $A^{1/3}$. We make the approximation of omitting ϵ . The cross-sections are now consistent if we write

$$\begin{aligned}\sigma_T(A) &= \sigma_T(1) A^{\bar{n}} = \sigma_s(A) + \sigma_a(A) \\ \sigma_T(1) &= 2(1-\bar{a}) \pi \delta^2\end{aligned}\tag{3.6}$$

noting that $2(1-\bar{a}) = (1-\bar{a}^2) + (1-\bar{a})^2$. It may be observed that for black discs, $\bar{a} = 0$, we have $\sigma_s = \sigma_a = \frac{1}{2} \sigma_T = \pi \delta^2 A^{\bar{n}}$.

The dependence on A can be viewed as follows. If δ were very much less than r_0 all the nucleons would be exposed, then $n = 1$, so that $\sigma_a \propto A$. On the other hand if $\delta = r_0$, $n = 2/3$, and $\sigma_a \propto A^{2/3}$ exactly. (For $\delta = r_0$ it is implied that the nucleons are deformed or squashed to conform to the volume available). The approximation made above implies that σ_s/σ_a is a constant independent of A . This is a doubtful point and should be regarded as a guide only, although it is not in conflict with experiment. It should be observed that the exact dependence of σ_T on A is not critically involved in the angular distribution for a given nuclide but only in its dependence on A . In what follows we will regard $A^{\bar{n}} \equiv N$ as the effective number of nucleons involved in the interaction.

Although the experimental data will be referred to again in section VI we mention here since it may be inferred (correctly) that the lines of the present analysis were suggested by them. The absorption cross-sections for a number of nuclei from Be to Pb have been measured at CERN at 24 GeV ¹⁵⁾ and elsewhere ¹⁶⁾. For the CERN results the dependence of σ_a on A for $A > 9$ is given to a high degree

of accuracy by $\sigma_a(A) = \sigma_a(1) A^{0.72}$ where $\sigma_a(1) = 38$ mb (see Fig. 3). σ has been measured for Be C and Al and for these $\sigma_s / \sigma_a \simeq \frac{1}{2}$ within experimental error, which gives $\bar{a} \simeq 1/3$. Combining these data one obtains $\delta = 1.17$ fermis. This implies that $\sigma_T(1) = 3/2 \sigma_a(1) = 57$ mb while for the free proton $\sigma_T = 40$ mb. Glauber⁷⁾ has pointed out that due to the Pauli principle the cross-section for a nucleon in nuclear matter may be larger than that for a free proton.

In what follows we shall regard $\bar{a}$ as a parameter to be determined by measuring σ_s / σ_a and not require it to be related to the transparency of a single nucleon. We thus drop the bar.

III.a The scattering amplitude

Let us now return to $f(\vec{k}, \vec{k}')$ Eq. (3.2), the scattering amplitude in an arbitrary direction. The optical theorem as employed above is satisfied if we write in place of Eq. (3.2)

$$f(\vec{k}, \vec{k}') = \frac{i(1-a)}{\lambda} \int ds e^{i\vec{q} \cdot \vec{P}} \quad (3.7)$$

where the integral is now over the area of $N \equiv A^{\nu}$, non-overlapping discs.

At this point we may inquire as to how the second requirement mentioned in section I, namely that $\int d\sigma = \sigma_s$, is satisfied. An estimate of this may be obtained by an approximate method used by Glauber⁷⁾. Referring to Eq. (2.3) we have

$$\begin{aligned}
 \int d\sigma &= (i | \int d\Omega |f|^2 | i) \\
 &= \frac{(1-a)^2}{\lambda^2} (i | \int ds \int ds' \int d\Omega e^{i\vec{q} \cdot (\vec{p} - \vec{p}')} | i) \quad (3.8)
 \end{aligned}$$

We now take the advantage of the fact that the scattering is mainly in the forward direction by replacing the integral over all angles of $\vec{k}'$ by an integration over the plane in k' space tangent to the sphere $k' = k$. Thus we replace the element of solid angle $dr = k'^2 \sin\theta d\theta d\varphi / k'^2$ by $k'_x k'_y \cos\theta / k'^2$ and integrate k'_x and k'_y from $-\infty$ to ∞ , putting $\cos\theta = 1$. This will over-estimate $\int d\sigma$. It follows that

$$\begin{aligned}
 \int d\Omega e^{i(\vec{k} - \vec{k}') \cdot (\vec{p} - \vec{p}')} &= \\
 &= \frac{1}{k'^2} \int_{-\infty}^{\infty} dk'_x e^{i(k'_x - k'_x)(\xi - \xi')} \int_{-\infty}^{\infty} dk'_y e^{i(k'_y - k'_y)(\eta - \eta')} \\
 &= \frac{4\pi^2}{k^2} \delta(\xi - \xi') \delta(\eta - \eta') \quad (3.9)
 \end{aligned}$$

and that the integral with respect to $\vec{p}'$ gives unity inside each of the N discs. Hence

$$\int ds \int ds' \int d\Omega e^{i\vec{q} \cdot (\vec{p} - \vec{p}')} = \lambda^2 \pi \delta^2 N \quad (3.10)$$

and finally

$$\int d\sigma = (1-a)^2 \langle i | \pi \delta^2 N | i \rangle = (1-a)^2 \pi \delta^2 N = \sigma_s \quad (3.11)$$

Returning to Eq. (3.7) the integration over each individual disc can be performed by translating the origin of co-ordinates to the centre of each disc in turn, with the result that

$$f(\vec{k}, \vec{k}') = \frac{i(1-a)}{\lambda} \pi \delta^2 g(\theta) \sum_{j=1}^N e^{i \vec{q} \cdot \vec{s}_j} \quad (3.12)$$

where $\vec{s}_j$ are the coordinates of the centers of the discs in the plane perpendicular to $\vec{k}$ and $g(\theta) = (2J_1(k \delta \theta) / k \theta \delta)$ is the nucleon form factor. Recalling Eq. (2.3) it follows that

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \langle i | |f|^2 | i \rangle \\ &= \left(\frac{k \sigma_T(i)}{4\pi} \right)^2 g^2 \langle i | \sum_{j,k=1}^N e^{i \vec{q} \cdot (\vec{s}_j - \vec{s}_k)} | i \rangle \end{aligned} \quad (3.13)$$

The terms in the double summation for which $j = k$, (the diagonal terms) add up to N . For ψ_i antisymmetric in the co-ordinates of all the nucleons each of the non-diagonal terms are the same and we may put

$$\sum_{j \neq k} \int |\Psi_i(\vec{r}_1 \vec{r}_2 \dots \vec{r}_A)|^2 d\vec{r}_1 d\vec{r}_2 \dots d\vec{r}_A e^{i\vec{q} \cdot (\vec{s}_j - \vec{s}_k)} \\ = \int P_2(\vec{r}_1 \vec{r}_2) d\vec{r}_1 d\vec{r}_2 e^{i\vec{q} \cdot (\vec{s}_1 - \vec{s}_2)} \sum_{j \neq k} 1 \quad (3.14)$$

where P_2 is the pair distribution function

$$P_2(\vec{r}_1 \vec{r}_2) = \int |\Psi_i(\vec{r}_1 \vec{r}_2 \dots \vec{r}_A)|^2 d\vec{r}_3 d\vec{r}_4 \dots d\vec{r}_A \quad (3.15)$$

and

$$\sum_{j \neq k} 1 = N^2 - N \quad (3.16)$$

We put

$$\int P_2(\vec{r}_1 \vec{r}_2) d\vec{r}_1 d\vec{r}_2 e^{i\vec{q} \cdot (\vec{s}_1 - \vec{s}_2)} = G^2(\theta) \quad (3.17)$$

We will specify the function $G^2(\theta)$ below and will regard it as the nuclear form factor, $G^2(0) = 1$. We have now completed the evaluation of $d\sigma/d\Omega$ (the exact form of G^2 being as yet unspecified).

Combining Eqs. (3.13), (3.14) and (3.17) it follows that

$$\frac{d\sigma}{d\Omega} = \left(\frac{k \sigma_T(A)}{4\pi} \right)^2 G^2(\theta) \left[(1 - G^2) \frac{1}{N} + G^2 \right] \quad (3.18)$$

where $\sigma_T(\Lambda) = \sigma_T(1) N$ and $N \equiv A^N$.

We now specify a form for G^2 . Noting that $P_2(\vec{r}_1, \vec{r}_2) = P_1(\vec{r}_1)P_1(\vec{r}_2)(1 + h(\vec{r}_1, \vec{r}_2))$ where $P_1(\vec{r}_1)$ is the probability distribution of a single particle and $h(\vec{r}_1, \vec{r}_2)$ is the correlation function imposed by the requirement of antisymmetry we put

$$P_1(\vec{r}) = P_1(\vec{r}_1) = P_1(r) = \left(\frac{2}{\pi}\right)^{3/2} \frac{1}{R^3} e^{-\frac{2r^2}{R^2}} \quad (3.19)$$

and omit $h(\vec{r}_1, \vec{r}_2)$. This gives

$$G^2(\theta) = e^{-\frac{(kR\theta)^2}{4}}$$

The probability distribution Eq. (3.19) falls to $1/e$ of its central value at $r = R/\sqrt{2}$ and not R . The exact form of $P_1(r)$ is not critical to the present discussion. As shown below $P_1(r)$ can be regarded as an approximation to other forms of probability distribution.

It may be observed here that Eq. (3.18) possesses a familiar optical property (Scmmerfeld¹⁴), namely that the light scattered from a random distribution of scattering objects (dust particles, raindrops, etc.) is proportional to the number of scattering objects. In Eq. (3.18) for angles greater than $\theta_c = 3.83/kR$, $G^2(\theta) \rightarrow 0$ and $d\sigma/d\Omega$ is in fact proportional to N .

In the optical case the usual reference is to an extended light source for which "the forward direction" does not have the same meaning. For Eq. (3.18) near the forward direction the first term (the incoherent part) becomes vanishingly small. This also accords with the optical analogy since in the forward direction a random distribution of scattering

centres can no longer impose destructively interfering phase differences. In this case $d\sigma/d\Omega$ is proportional to N^2 .

Having specified an analytical form for $d\sigma/d\Omega$ we must inquire again into the satisfaction of the requirement that $\int d\sigma = \sigma_s$.

With

$$2\pi \int_0^\infty g^2(\theta) \theta d\theta = 4\pi / (k\delta)^2$$

and

$$2\pi \int_0^\infty g^2(\theta) G^2(\theta) \theta d\theta = 4\pi / k^2 (S^2 + R^2)$$

we have from Eq. (3.18)

$$\int d\sigma = \sigma_s(A) \left[1 + \frac{S^2}{S^2 + R^2} (N-1) \right] \quad (3.20)$$

with $N \equiv A^n$ and $R = r_0 A^{1/3}$. Thus in the form Eq. (3.18) $\int d\sigma$ is greater than σ_s , in disagreement with our earlier result Eq. (3.11). The reason for this apparent conflict is in the nature of the approximation which makes

$$\int d\sigma e^{i(\vec{k}-\vec{k}') \cdot (\vec{p}-\vec{p}')} = \lambda^2 \delta(\xi - \xi') \delta(\eta - \eta')$$

Recall that in the integration of Eq. (3.9) $\cos \theta$ was replaced by 1 and that this must inevitably over-estimate the integral. Equation (3.20) shows that this approximation is good only so long as $(R/\delta) \gg 1$ which

is the case in the usual optical situation (extended source) but is excluded here by the requirement of section I. In fact, as can be seen from Eq. (3.20), this approximation precludes the possibility of coherence from the scattering centres. We shall remove this conflict by introducing a parameter. This must be done in such a way as to preserve the form of Eq. (3.18) and to preserve the optical theorem. This can only be done by modifying the first term of Eq. (3.18), the incoherent part, which vanishes in the forward direction. We therefore write in place of Eq. (3.18) the following

$$\frac{d\sigma}{d\Omega} = \left(\frac{k\sigma_T}{4\pi} \right)^2 \left[g^2(1-G^2) \frac{C}{N} + g^2 G^2 \right] \quad (3.21)$$

where C is this parameter. C is now determined by demanding that $\int d\sigma = \sigma_S$ whence

$$C = \left(1 + \frac{S^2}{r_0^2 A^{2/3}} \right) \left(1 - \frac{A^n}{1 + A^{2/3} r_0^2 / S^2} \right) \quad (3.22)$$

The parameter C may be regarded as a correction for the defects of those approximations already made.

IV. Form factors

In section II the sharply cut-off opacity function for a single disc was discussed and the following results obtained

$$\frac{d\sigma}{d\Omega} = \left(\frac{k\sigma_T}{4\pi} \right)^2 \left(\frac{2J_1(k\delta\theta)}{k\delta\theta} \right)^2$$

$$\sigma_s = (1-a)^2 \pi \delta^2 \quad (4.1)$$

$$\sigma_a = (1-a^2) \pi \delta^2$$

Regnier ¹⁷⁾ has discussed extensively opacity functions and the corresponding form factors and derives an equivalence condition which shows that the form factor is rather insensitive to the exact shape of the opacity function. For our purposes it can be shown that for a variety of forms for the opacity function $(1 - a(\rho))$ there exists a sharp cut-off model which has the same σ_s and σ_a and for angles $\theta < 0.80 \theta_0$, where $\theta_0 = 3.83 / k\delta$ is the first zero of J_1 , has a form factor which differs negligibly from the Bessel function form.

In particular, an opacity function defined by

$$(1 - a(\rho)) = b e^{-\left(\frac{\rho}{d}\right)^2}, \quad b \leq 1$$

leads to

$$\frac{d\sigma}{dn} = \left(\frac{k}{4\pi} 2\pi d^2 b \right)^2 e^{-\frac{(k\delta d)^2}{2}}$$

$$\sigma_s = \pi d^2 b \quad (4.2)$$

$$\sigma_a = \frac{1}{2} \pi d^2 b (4-b)$$

By comparing Eqs. (4.1) and (4.2) it can be seen that we can specify a Gaussian distribution which is equivalent to the sharp cut-off model so far as σ_s and σ_a are concerned provided we put $b = 2(1-a)$ and $d^2 = \delta^2/2$. This suggests that we compare the form factors $(2J_1(k\delta\theta)/k\delta\theta)^2$ and $e^{-\frac{(k\delta\theta)^2}{4}}$, noting that for small $k\delta\theta$ both functions approach $(1 - (k\delta\theta)^2/4 + \dots)$ and that

$$\int_0^\infty \left(\frac{2J_1(k\delta\theta)}{k\delta\theta} \right)^2 \theta d\theta = \int_0^\infty e^{-\frac{(k\delta\theta)^2}{4}} d\theta = \frac{2}{(k\delta)^2}$$

Such a comparison shows that these do in fact differ negligibly in the region of $\theta \lesssim 0.8 \theta_0$ while the Gaussian of course shows no diffraction minima.

However, it can be seen that this correspondence breaks down for $a < \frac{1}{2}$. In this case we can find an equivalent smoothed off model in the following way. Assume that the absorbing potential is proportional to the matter density and that this density is given by the Gaussian

$$\beta(r) = (d\pi)^{-\frac{3}{2}} e^{-\frac{r^2}{d^2}}$$

where $d = \sqrt{2/3} \, r_{\text{rms}}$, r_{rms} being the root mean square radius of the distribution. The absorbing potential is then $V(r) = -iV_0 e^{-\frac{r^2}{d^2}}$. The corresponding opacity function (see Bethe ⁴) is

$$(1 - a(p)) = \left(1 - \exp\left\{-\gamma e^{-\frac{p^2}{d^2}}\right\}\right)$$

where $\gamma = \sqrt{\pi} \, d V_0 / \hbar v$. Note that the first term in an expansion of this for small γ is

$$(1 - a(p)) = \gamma e^{-\frac{p^2}{d^2}}$$

which corresponds to the Born expansion and is widely used as an approximation to obtain the form factor.

The scattering amplitude and differential cross-section can be integrated and yield

$$\frac{d\sigma}{d\Omega} = \left(\frac{\kappa \sigma_T}{4\pi}\right)^2 q^2(\theta)$$

$$\sigma_s = \pi d^2 [2X(\gamma) - X(2\gamma)]$$

$$\sigma_a = \pi d^2 X(2\gamma) \quad (4.3)$$

where $X(\gamma) = \ln \gamma + 0.577 - E_1(-\gamma)$
 $E_1(-\gamma) =$ the integral exponential

and $g^2(\theta)$ is a power series in γ , see Reference 4). Thus so far as σ_s and σ_a are concerned we can specify a sharp cut-off model which is equivalent provided $a = (X(2\gamma)/X(\gamma)) - 1$ and $d^2 = \delta^2/2X(\gamma)$. The form factor computed from the power series with the appropriate value for γ lies between that for the sharp cut-off form and the Gaussian form and shows diffraction minima of less prominence.

In what follows we shall use the form factor $g^2 = e^{-\frac{(k\delta\theta)^2}{4}}$ characteristic of a Gaussian for which $d = \delta/\sqrt{2}$ for the nucleon and regard it as an approximation to either a sharp cut-off opacity or the opacity which arises from a Gaussian potential.

For the same reasons we shall use for the nuclear form factor $G^2 = e^{-\frac{(kR\theta)^2}{4}}$ and regard the transparency as an empirical parameter.

V. Fermi motion

In this section some account will be taken of the motion of the target nucleons. In doing so the target will be regarded as a Fermi gas and the process treated by pure kinematics. However, it should be born in mind that the real motion of the nucleons would enter through the terms of order $\Delta E/E$ and through the effect of the ground state wave function on $G(\Theta)$. In fact, of course, the diffraction picture is complementary to the ballistics picture and to that extent is in conflict with it. Diffraction scattering arises from the interference of the primary wave with the "hole" in the wave front. On the other hand one might expect a purely kinematic effect on the nucleon-like tail for the following reason. For target particles in isotropic motion there must be a net probability that a bombarding particle is scattered through a larger angle than if they were stationary. The angular spread of the tail should be increased. It might be hoped that this could be represented by using a slightly smaller nucleon radius in the form factor and we shall find that this is so.

Consider now a high-energy particle with momentum $\vec{k}$ in the z direction striking a nucleon at rest and being scattered into $\vec{k}'$ at a small angle Θ (laboratory system) in the xy plane. Introduce a new set of co-ordinate axes $x'y'z'$ with z' parallel to $\vec{k}'$ and x' in the xz plane, Fig. 2a. Now assume the target is in motion with momentum $\vec{p}$ ($\hbar$ units) having polar angles β and φ in $x'y'z'$. We transform from the frame of reference in which the target is at rest to one in which the target is moving. The bombarding particle is now scattered through an additional angle χ into $\vec{k}''$ such that $\vec{k}'' = \vec{k}' + \vec{p}$ where $p_{\perp} = p \sin \beta$ and $\vec{p}_{\perp}$ is perpendicular to $\vec{k}'$. It is assumed that $p \ll k$ and hence $k'' \simeq k' \simeq k$ in which case if q' is the new transverse momentum transfer $q' \cos \psi = q + p_{\perp} \cos \varphi$, Fig. 2b.

If we confine ourselves to angles large enough that p is substantially less than q then $\cos \psi \simeq 1$ and $q' \simeq q + p \sin \beta \cos \varphi$. Dividing this by k we have $\theta^* \simeq \theta + \frac{p}{k} \sin \beta \cos \varphi$ where $\theta^* \simeq q'/k$ is the new scattering angle, Fig. 2a. The nucleon tail is in fact observed at angles for which $q \gtrsim 500 \text{ MeV}/c$.

Now the probability $d\sigma(\theta^*)$ of a particle being scattered at an angle θ^* is given by the probability, $\sigma_0(\theta)$, of being scattered into the direction θ times the probability $W(\vec{p})d^3\vec{p}$ that $\vec{p}$ is in such a direction as to redirect it into θ^* . That is

$$d\sigma(\theta^*) = \sigma_0(\theta) W(\vec{p}) d^3\vec{p} \quad (5.1)$$

where $\theta = \theta^* - p \sin \beta \cos \varphi / k$. If $W(\vec{p})$ is assumed to be spherically symmetric, $W(\vec{p}) = w(p) / 4\pi$, then

$$\begin{aligned} \sigma(\theta^*) &= \int d\sigma(\theta^*) \\ &= \frac{1}{4\pi} \int d^3\vec{p} w(p) \sigma_0(\theta^* + \alpha(\vec{p})) \end{aligned} \quad (5.2)$$

where $\alpha(\vec{p}) = -\frac{p}{k} \sin \beta \cos \varphi$ and $d^3\vec{p} = p^2 dp \sin \beta d\varphi$. For a degenerate Fermi gas

$$w(p) = 3/p_F^3 = \text{a constant}, \quad p < p_F$$

$$w(p) = 0, \quad p > p_F$$

Noting that $\alpha(\vec{p}) \ll \theta^*$, we may develop $\sigma_0(\theta^* + \alpha)$ in a Taylor series around $\alpha = 0$ and drop the star on θ^* .

$$\sigma_0(\theta + \alpha) = \sigma_0(\theta) + \sum_m \frac{\alpha^m}{m!} \left(\frac{d^m \sigma_0(\theta + \alpha)}{d\alpha^m} \right)_0 \quad (5.3)$$

Equation (5.2) can now be integrated. All odd powers of α vanish.

The first two non-vanishing terms contain

$$\begin{aligned} \overline{\alpha^2} &= \frac{1}{4\pi} \int \alpha(\vec{p}) \omega(p) p^2 dp \sin\beta d\beta d\varphi \\ &= \frac{1}{5} \frac{p_F^2}{k^2} \end{aligned} \quad (5.4)$$

$$\overline{\alpha^4} = \frac{3}{35} \frac{(p_F)^4}{k^4} \quad (5.5)$$

We now introduce for $\sigma_0(\theta)$ the diffraction form

$\sigma_0(\theta + \alpha) = \exp - \left\{ k^2 \delta^2 (\theta + \alpha)^2 / 4 \right\}$ discussed above and find

$$\left(\frac{d^2 \sigma_0}{d\alpha^2} \right)_0 = \sigma_0(\theta) \frac{k^4 \delta^4 \theta^2}{4} \left(1 - \frac{2}{(k\delta\theta)^2} \right) \quad (5.6)$$

$$\left(\frac{d^4 \sigma_0}{d\alpha^4} \right)_0 = \sigma_0(\theta) \frac{k^8 \delta^8 \theta^4}{16} \left(1 - \frac{12}{(k\delta\theta)^2} + \frac{12}{(k\delta\theta)^4} \right) \quad (5.7)$$

These are substituted in

$$\sigma(\theta) = \sigma_0(\theta) + \frac{1}{2} \overline{\alpha^2} \left(\frac{d^2 \sigma_0}{d\alpha^2} \right)_0 + \frac{1}{24} \overline{\alpha^4} \left(\frac{d^4 \sigma_0}{d\alpha^4} \right)_0 + \dots$$

to obtain

$$\sigma(\theta) = \sigma_0(\theta) \left\{ 1 + \frac{(k\delta\theta)^2}{4} \frac{(p_F\delta)^2}{10} \left(1 - \frac{2}{(k\delta\theta)^2} \right) + \frac{1}{2} \frac{(k\delta\theta)^4}{16} \frac{(p_F\delta)^4}{140} \left(1 - \frac{12}{(k\delta\theta)^2} + \frac{12}{(k\delta\theta)^4} \right) + \dots \right\} \quad (5.8)$$

For sufficiently large angles we can neglect all but the first term in each of the parentheses and compare this expression with

$$e^{\frac{(k\delta\theta)^2}{4} \frac{(p_F\delta)^2}{10}} = 1 + \frac{(k\delta\theta)^2}{4} \frac{(p_F\delta)^2}{10} + \frac{1}{2} \frac{(k\delta\theta)^4}{16} \frac{(p_F\delta)^4}{100} + \dots \quad (5.9)$$

We thus write

$$\sigma(\theta) = D e^{-\frac{(k\delta^*\theta)^2}{4}} = D \sigma_0(k\delta^*\theta)$$

where,

$$\delta^{*2} = \delta^2 (1 - (p_F \delta)^2 / 10) \quad (5.10)$$

and the constant D is introduced so that $\int \sigma(k, \theta) d\Omega = \int D \sigma_0(k, \theta) d\Omega$. It will later be absorbed. We expect this to be a reasonable representation at least at large angles. The parameter δ^* is an effective nucleon radius to be used in the form factor for the incoherent part and is a purely kinematic effect. Our main interest in the above discussion has been to show that indeed the effect of Fermi motion can be represented by a reduced nucleon radius δ^* . Equation (5.10) should be regarded only as a guide to the extent of the effect.

Finally the differential cross-section as now modified by this effect reads

$$\frac{d\sigma}{d\Omega} = \left(\frac{k \sigma_T}{4\pi} \right)^2 \left[q^2(\delta^*) (1 - G^2) \frac{C}{N} + q^2(\delta) G^2 \right] \quad (5.11)$$

where now C is given by

$$C = \left(\frac{\delta^*}{\delta} \right)^2 \left(1 + \frac{\delta^{*2}}{r_0^2 A^{2/3}} \right) \left(1 - \frac{A^m}{1 + A^{2/3} r_0^2 / \delta^2} \right) \quad (5.12)$$

Equation (5.11) with C given by Eq. (5.12) is the final result of this analysis and will be applied in the next section to some experimental results. In view of the approximations made it seems hardly

necessary to repeat the comment made earlier that Eq. (5.11) should be regarded as a guide, especially in its dependence on A , to predict only the prominent features. The parameters especially δ^* and C may be regarded as semi-empirical to be adjusted to fit the experiments. On the other hand it contains built-in consistency checks that should be useful for the analysis of experimental data.

VI. An application

We have already referred in section III to the experiments to which this analysis will be applied. Figure 3 shows the measured values of $\sigma_a^{15)}$ and $\sigma_T^{1)}$. The relevant parameters are: $n = 0.72$, $\sigma_a(1) = 38 \text{ mb}$, $a \simeq 1/3$ and $\delta = 1.17 \text{ f}$.

To account for the Fermi motion through Eq. (5.10) we use a Fermi energy of 70 MeV hence $p_F = 1.87 \times 10^{13} \text{ cm}^{-1}$. Equation (5.10) then gives $\delta^* = 0.88 \text{ f}$. For Be we take $R = 1.40 A^{1/3} = 2.92 \text{ f}$ and $\sigma_T = 270 \text{ mb}$. All the parameters of Eq. (5.11) are now fixed. The result is plotted in Fig. 4 along with the preliminary data of Ashmore et al. ¹⁾ at 25 GeV/c and 12.5 GeV/c. The author is much indebted to these workers for permission to cite these unpublished data here.

We now turn to the dependence of $d\sigma/d\Omega$ on A . The relative intensities of scattering per nucleon at a fixed angle ($\simeq 8 \text{ millirad.}$) compared to that for Be have been measured for Li C Al Cu and Pb. These are shown in Fig. 5 together with the dependence predicted by Eq. (5.11). For the theoretical curve in Fig. 5 we have arbitrarily put the parameter $\eta = 0.200 =$ a constant, the Be value.

Now Eq. (5.12) predicts the dependence of C on A . In fact, for the parameters given above Eq. (5.12) makes $C \rightarrow 0$ for $A > 200$. The magnitude of C Eq. (5.12) depends strongly on the assumption that $\sigma_s / \sigma_a = \text{a constant}$ through the requirement that $\int d\sigma = \sigma_s$. This is especially true for large A . It is however premature to speculate on this apparent defect.

It is felt that the degree of agreement found is sufficient to justify the model as providing a useful framework with which to analyze experimental data and a point of departure for farther theoretical developments. It is also evident from the considerations that have entered the analysis that high-energy scattering of nucleons or other strongly interacting particles will yield information on nuclear structures not otherwise obtainable.

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FIGURE CAPTIONS

- Fig. 1 - None.
- Fig. 2 (a and b) - None.
- Fig. 3 - Dependence of σ_T and σ_a on A .
- Fig. 4 - Differential for protons on Be. The solid line is the prediction of Eq. (5.11). The broken lines show the coherent and incoherent contributions.
- Fig. 5 - Dependence of $A^{-1} d\sigma/d\Omega$ (at 8 millirad) on A (predicted by Eq. (5.11)) with $A^{-1} d\sigma/d\Omega$ for Be taken as unity, and $C = a$ constant. The broken line shows the incoherent contribution.

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3

Fig. 4

Fig. 5