Gamma-ray bursts from internal shocks in a relativistic wind: temporal and spectral properties

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ABSTRACT
We construct models for gamma-ray bursts where the emission comes from internal shocks in a relativistic wind with a highly non uniform distribution of the Lorentz factor. We follow the evolution of the wind using a very simplified approach where a large number of layers interact by direct collisions but where all pressure waves have been suppressed. We suppose that the magnetic field and the electron Lorentz factor reach large equipartition values in the shocks. Synchrotron photons emitted by the relativistic electrons have a typical energy in the gamma-ray range in the observer frame. Synthetic bursts are constructed as the sum of the contributions from all the internal elementary shocks and their temporal and spectral properties are compared to the observations. We reproduce the diversity of burst profiles, the “FRED” shape of individual pulses and the short time scale variability. Synthetic bursts also satisfy the duration-hardness relation and individual pulses are found to be narrower at high energy, in agreement with the observations. These results suggest that internal shocks in a relativistic wind may indeed be at the origin of gamma-ray bursts. A potential problem is however the relatively low efficiency of the dissipation process. If the relativistic wind is powered by accretion from a disc to a stellar mass black hole it implies that a substantial fraction of the available energy is injected into the wind.

Key words: gamma-rays: bursts; radiation mechanisms: non thermal; shock waves; accretion discs.

1 INTRODUCTION
Since 1991 the BATSE experiment on board the Compton GRO satellite has observed more than 1800 gamma-ray bursts (hereafter GRBs). The burst distribution is isotropic over the sky but non homogeneous in distance (Fishman and Meegan, 1995 and references therein) which has been regarded as a strong indication that GRBs lie at cosmological distances (Paczynski 1991). However the possibility that GRBs belong to a large galactic halo (Hartmann et al. 1994) could not be excluded a priori. This long standing controversy (see Nemiroff et al. 1995) about the burst distance scale may finally be solved by the recent observations of transient optical counterparts for two GRBs. In the case of GRB 970228 (van Paradijs et al. 1997; Sahu et al. 1997a) the point-like counterpart appears to be associated to an extended source which is probably a distant galaxy. The case of GRB 970508 is even more spectacular since the spectrum of the counterpart shows Fe II and Mg II lines at a redshift \( z = 0.835 \) (Metzger et al. 1997). If the association of the visible source to the burst is confirmed, GRB 970508 must be a very distant object at \( z \geq 0.835 \). If GRBs are placed at cosmological distances the \( \log N - \log P \) (peak flux) curve and the value of \( <V/V_{\text{max}} > \) can be naturally interpreted in terms of cosmological effects and indicate that GRBs have typical redshifts in the range 0.3 – 1 (Piran 1992; Mao & Paczyński 1992; Fenimore et al. 1993).

Modelling GRBs is a difficult task due to the extreme diversity of burst profiles, the non thermal spectra and the lack of clear signature for the emission processes involved. Most cosmological models however share some common characteristics. The source, which must be able to release (between 10 keV and 10 MeV) an energy \( E_{\gamma} \gtrsim 10^{51} \text{erg} \cdot \text{sr}^{-1} \) on a time scale of seconds, is generally supposed to be a stellar mass black hole accreting material from a disc. Such a configuration can result from the coalescence of two neutron stars (Eichler et al. 1989; Paczyński 1991; Narayan, Paczyński & Piran 1992), the disruption of the neutron star in a neutron star – black hole binary (Narayan Paczyński & Piran 1992; Mochkovitch et al. 1993) or the collapse of a massive star (Woosley 1993). The power emitted by cosmological GRBs is orders of magnitude larger than the Eddington luminosity and cannot come directly from the disc surface. The released energy instead drives a wind which has to become relativistic both to produce gamma-rays and to avoid photon-photon annihilation along the line of sight (Baring 1995; Sari & Piran 1997a). The Lorentz factor \( \Gamma \) must reach values of \( 10^2 - 10^3 \) which limits the allowed amount of baryonic pol-
lution in the flow to a very low level. A few mechanisms which could possibly achieve such a severe constraint have been proposed: (i) magnetically driven outflow originating from the disc or powered by the Blandford-Znajek (1977) effect (Thompson 1994; Mészáros & Rees 1997a), (ii) reconnection of magnetic field lines in the disc corona (Narayan, Paczyński & Piran 1992) or (iii) neutrino-antineutrino annihilation in a funnel along the rotation axis of the system (Mészáros & Rees 1992; Mochkovitch et al. 1993, 1995). It is supposed in (i) and (ii) that the field has reached huge values $B \gtrsim 10^{15}$ G. Concerning (iii) Ruffert et al. (1997) have shown recently that $\nu\bar{\nu}$ annihilation does not provide enough energy to account for cosmological GRBs except may be for very massive discs, as those which could result from the collapse of a massive star.

The energy initially stored in kinetic form within the relativistic wind must then be converted into gamma-rays. This can be done during the deceleration of the wind resulting from its interaction with the interstellar medium (Rees & Mészáros 1992; Mészáros & Rees 1993) or a dense radiation field (Shemi 1994). In the first case the emission comes from electrons accelerated in the forward and reverse shocks which then radiate synchrotron and inverse Compton photons in a magnetic field frozen in the wind or which has come to equipartition with the shocked material (Mészáros, Laguna & Rees 1993). In the second case, photons of the ambient radiation field (in the central regions of globular clusters or active galactic nuclei) interact with the electrons (Shemi 1994; Shaviv & Dar 1995) or ions (Shaviv & Dar 1996) of the wind and undergo a boost in energy by a factor $\Gamma$. With $\Gamma \lesssim 10^{3}$, optical photons can be shifted to the gamma-ray range. In these two models the duration of the burst is $t_b \sim \frac{2r_{dec}}{c}$ where $r_{dec}$ is the deceleration radius of the wind. Another possibility suggested by Rees & Mészáros (1994) consists to suppose that the Lorentz factor in the wind is variable so that successive shells can have large relative velocities leading to the formation of internal shocks. The energy which is dissipated in these shocks can then be radiated as gamma-rays via the production of pions in proton-proton collisions (Paczynski & Xu 1994) or by synchrotron (or inverse Compton) emission of accelerated electrons. There are several important differences between the deceleration and internal shock models. In deceleration models, the duration and time profile of the burst depend on $\Gamma$, on the value of the deceleration radius and on the structure of the emitting shell. In internal shock models, the duration of the burst is directly related to the duration of energy injection at the source and the time profile is essentially determined by the variations of the Lorentz factor (Sari & Piran 1997a).

Burst profiles and spectra have already been obtained (sometimes in a rather detailed manner) in the case of deceleration models (Fenimore, Madras & Nayakshin 1997; Panaitescu et al. 1997; Panaitescu & Mészáros 1997) and the purpose of this work is to present the same kind of quantitative analysis for internal shock models. In Sect. 2 we describe our method to follow the evolution of the relativistic wind and we discuss the emission processes; Sect. 3 and 4 respectively deal with the temporal and spectral properties of our synthetic burst models and Sect. 5 is the conclusion.

## 2 A SIMPLE MODEL OF THE RELATIVISTIC WIND

### 2.1 Description of the model

We do not discuss in this paper the nature of the source (coalescence of two neutron stars, neutron star – black hole binary, collapse of a massive star or something else) which is initially responsible for the energy release. We also suppose that a relativistic wind carrying the energy has emerged from the source, with an average Lorentz factor $\Gamma$ of a few hundreds.

To study the evolution of the relativistic wind we have developed a very simple model where a succession of layers are emitted every 2 ms with a varying Lorentz factor, during a total time $t_{bw}$. The mass of the layers is proportional to $1/\Gamma$ so that the energy injection rate is constant. We follow the layers as the wind expands and when a rapid layer (of mass $m_1$ and Lorentz factor $\Gamma_1$) catches up with a slower one ($m_2$, $\Gamma_2 < \Gamma_1$) they collide and merge to form a single shell of resulting Lorentz factor $\Gamma_r$. If the dissipated energy $\epsilon = m_1 c^2 \Gamma_1 + m_2 c^2 \Gamma_2 - (m_1 + m_2) c^2 \Gamma_r$, (1)

can be radiated in the gamma-ray range on a time scale shorter than the shell expansion time (in the comoving frame)

$$t_{ex} \simeq \frac{r}{\Gamma c},$$

where $r$ is the shell radius, the burst profile will be made by a succession of elementary contributions of duration ($in the observer frame$)

$$\Delta t \simeq \frac{r}{2c\Gamma r}.$$ (3)

We estimate $\Gamma_r$ by considering that most of the energy available in the collision has been already released when the less massive of the two layers has swept up a mass comparable to its own mass in the other layer (internal shocks being only mildly relativistic). Then

$$\Gamma_r \simeq \sqrt{\Gamma_1 \Gamma_2},$$

and

$$\epsilon = m c^2 (\Gamma_1 + \Gamma_2 - 2\Gamma_r), \quad m = \min(m_1, m_2).$$ (5)

After the complete redistribution of momentum and energy the Lorentz factor of the merged layer finally becomes

$$\Gamma_r = \sqrt{\Gamma_1 \Gamma_2 \frac{m_1 \Gamma_1 + m_2 \Gamma_2}{m_1 \Gamma_1 + m_2 \Gamma_2}}.$$ (6)

Two more conditions have to be satisfied for an elementary shock to be produced and observed: i) the two layers must collide at a relative velocity larger than the local sound speed and ii) the wind must be transparent to the emitted photons. The relative velocity of the two layers is given by

$$\frac{v_{rel}}{c} \simeq \frac{\Gamma_1^2 - \Gamma_2^2}{\Gamma_1^2 + \Gamma_2^2},$$ (7)

with $\Gamma_1 > \Gamma_2$ and $\Gamma_1 > 1$. We have adopted a sound speed $v_s/c = 0.1$ but we have checked that other choices make little differences in the results since the main contribution to the burst comes from shocks corresponding to large differences of the Lorentz factor $\frac{\Gamma_1}{\Gamma_2} \gtrsim 2$. 


The transparency of the wind to the emitted photons has been computed in the following way: in each collision between layers a “photon shell” is generated which then catches up with all the layers ahead of it. It travels through a total optical depth

$$\tau = \kappa \tau \sum_{i > \tau} \frac{m_i}{4\pi r_i^2}$$,  \hspace{1cm} (8)$$

where $\kappa$ is the Thomson opacity and $r_i$ the radius of layer $i$ when it is reached by the photon shell. The sum is over all indices $i$ larger than $i_\text{m}$, which corresponds to the two merged layers. For the average Lorentz factor $\bar{\Gamma} \geq 100$ used here it appears that the wind is transparent to the emitted photons except for a few early collisions.

The arrival time of each of the elementary contributions from internal shocks is calculated relatively to a signal which would have travelled at the speed of light from the source to the observer. It is given by

$$t_a = t_e - \frac{r}{c}$$,  \hspace{1cm} (9)$$

where $t_e$ is the emission time and $r$ the distance to the source of the merged layers. The evolution of the system is followed until all the layers are ordered with $\Gamma$ decreasing from the front to the back of the wind. The efficiency of the dissipation process can then be obtained as

$$\epsilon_d = \frac{\sum_i \epsilon_i}{\sum_i m_i c^2 \Gamma_i}$$,  \hspace{1cm} (10)$$

where the $\epsilon_i$ are the energies released in each of the internal elementary shocks and the $m_i, \Gamma_i$ are the initial masses and Lorentz factors of the layers.

This very simple approach is naturally very crude because it neglects all pressure waves propagating throughout the layers. Nevertheless, we expect that it can still capture the basic features of the real process. As a first example we have represented in Fig.1 the evolution of the Lorentz factor at different times when the initial distribution of $\Gamma$ consists of 5000 layers (i.e. $t_e = 10$ s) with $\Gamma(n) = 400$ for $n = 1$ to 4000 and $\Gamma(n) = 100$ for $n = 4001$ to 5000 ($n = 1$ for the last emitted layer). Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer. Such a wind is then divided between a “slow” part which is emitted first and a rapid part for the last emitted layer.
where most of the collisions take place is $B_{eq} \sim (10^2 - 10^3) \, \text{G}$ depending on the ratio $\Gamma_1/\Gamma_2$.

Synchrotron emission by the accelerated electrons in the magnetic field occurs at a typical energy (in the observer frame)

$$E_{\text{syn}} = 50 \left( \frac{\Gamma_\nu}{300} \right) \left( \frac{B}{1000 \, \text{G}} \right) \left( \frac{\Gamma_\nu}{100} \right)^2 \, \text{eV} ,$$

which corresponds to the UV range for $\Gamma_\nu = 100$. Gamma-rays can be produced by inverse Compton scattering on the synchrotron photons. Then

$$E_{\text{IC}} \simeq E_{\text{syn}} \Gamma_\nu^2 = 500 \left( \frac{\Gamma_\nu}{300} \right) \left( \frac{B}{1000 \, \text{G}} \right) \left( \frac{\Gamma_\nu}{100} \right)^4 \, \text{keV} ,$$

and the fraction of the total power which is radiated by the inverse Compton process is

$$\alpha_{\text{IC}} = \frac{\tau_\nu \Gamma_\nu^2}{1 + \tau_\nu \Gamma_\nu^2} ,$$

where

$$\tau_\nu = \frac{\kappa_T M_\nu}{4\pi r_\nu^2} ,$$

is the optical depth of the shell of mass $M_\nu$ and radius $r_\nu$ which contains the relativistic electrons. An estimate of $M_\nu$ is

$$M_\nu = \frac{t_{\text{syn}}}{1 + \tau_\nu \Gamma_\nu^2} M_{\text{shock}} ,$$

where

$$t_{\text{syn}} = 6 \left( \frac{\Gamma_\nu}{100} \right)^{-1} \left( \frac{B}{1000 \, \text{G}} \right)^{-2} \, \text{s} ,$$

is the synchrotron time of the relativistic electrons and $M_{\text{shock}}$ is the mass flow rate across the shock, both in the comoving frame of the shocked material. Since the shock moves with a Lorentz factor $\Gamma_s \simeq \bar{\Gamma}$, $M_{\text{shock}}$ can be approximated by

$$M_{\text{shock}} \simeq \frac{M}{\bar{\Gamma}} .$$

From (18), (19) and (21) we obtain an implicit expression for $\tau_\nu \Gamma_\nu^2$

$$\tau_\nu \Gamma_\nu^2 = \frac{\kappa_T M_{\text{shock}} t_{\text{syn}} \Gamma_\nu^2}{4\pi r_\nu^2} .$$

With $r_\nu \sim c t_{\text{var}} \Gamma_\nu^2$, relations (20) and (21) for $t_{\text{syn}}$ and $M_{\text{shock}}$, (13) and (14) for $B_{eq}$ and $n$, Eq. 22 gives (for $\kappa_T = 0.2 \, \text{cm}^2 \, \text{g}^{-1}$)

$$\tau_\nu \Gamma_\nu^2 \simeq 0.3 \frac{\alpha_\nu}{\alpha_B} ,$$

which, for $\alpha_\nu \sim \alpha_B$ yields $\tau_\nu \Gamma_\nu^2 \simeq 0.24$ and $\alpha_{\text{IC}} \simeq 0.19$. A larger fraction of the dissipated energy can be converted to gamma-rays if the magnetic field does not reach equipartition. With for example $\frac{2 \alpha_\nu}{\alpha_B} = 100$, $\Gamma_\nu^2$ rises to 5 and $\alpha_{\text{IC}}$ to 0.83. Smaller values of $\alpha_B$ could still increase $\alpha_{\text{IC}}$ but the energy of the inverse Compton photons would then become too small.

Conversely, if the electron Lorentz factor is large enough gamma-rays can be directly produced by synchrotron emission. This will be the case if, instead of (12) one uses the expression of $\Gamma_\nu$ given by Bykov & Mészáros (1996) who consider the scattering of electrons by turbulent magnetic field fluctuations. They get

$$\Gamma_\nu \simeq \left[ \frac{\alpha m_\nu}{\zeta} \left( \frac{\epsilon}{m_e c^2} \right) \right]^{1/(3 - \mu)} ,$$

where $\alpha m_\nu$ is the fraction of the dissipated energy which goes into magnetic fluctuations, $\zeta$ the fraction of the electrons which are accelerated and $\mu$ the index of the fluctuation spectrum. With $1.5 \leq \mu \leq 2$, $\alpha m_\nu = 0.1 - 1$, $\zeta \sim 10^{-3}$ and $\epsilon/m_e c^2 \simeq 500$ (for $\Gamma_1/\Gamma_2 = 4$) values of $\Gamma_\nu$ in the range $10^3 - 10^4$ can be obtained.

A fraction of the synchrotron photons will be shifted to even higher energy by inverse Compton scattering which now occurs in the limit where

$$w = \frac{\Gamma_\nu E_{\text{syn}}}{m_e c^2} \simeq 33 \left( \frac{\Gamma_\nu}{10^2} \right)^3 ,$$

is large, $E_{\text{syn}} = E_{\nu} \Gamma_\nu$ being the synchrotron energy in the comoving frame. The inverse Compton photons have an energy

$$E_{\text{IC}} \simeq \Gamma_\nu m_e c^2 = 5 \left( \frac{\Gamma_\nu}{10^2} \right) \, \text{GeV} ,$$

in the comoving frame and carry a fraction

$$\alpha_{\text{IC}} = \frac{\Gamma_\nu^2}{w} ,$$

of the dissipated energy. The optical depth $\tau_\nu$ has to be computed with the Klein-Nishina cross-section which, in the limit $w \gg 1$, gives

$$\tau_\nu \simeq \left[ \frac{\kappa_T M_{\text{shock}} t_{\text{syn}} \Gamma_\nu^2}{4\pi r_\nu^2 (1 + \tau_\nu \Gamma_\nu^2/w)} \right] \left( \frac{3}{8\pi} \right) [1 + \ln(2w)] ,$$

and therefore

$$\frac{\tau_\nu \Gamma_\nu^2}{w} \left( 1 + \tau_\nu \Gamma_\nu^2/w \right) \simeq 8 \times 10^{-4} [1 + \ln(2w)] \left( \frac{\dot{E}}{10^{32} \, \text{erg} \, \text{s}^{-1}} \right) \times \left( \frac{t_{\text{var}}}{1 \, \text{s}} \right)^{-2} \left( \frac{\Gamma_\nu}{300} \right)^{-6} \left( \frac{B}{1000 \, \text{G}} \right)^{-4} \left( \frac{\Gamma_\nu}{10^2} \right)^{-5} .$$

The very large exponents which appears in (29) show that a small variation of the parameters can induce large changes in the relative importance of the synchrotron and inverse Compton processes. In practice, we find that $\alpha_{\text{IC}}$ is generally small in the early part of a burst but increases at later times essentially due to the reduction of the equipartition magnetic field in shocks at large distances from the source (see Sect. 3.1 below).

To be efficient the emission process must also occur on a time scale $t_{\text{syn}}$ shorter than the shell expansion time $t_{\text{ex}}$ (Eq. 2). Using $t \sim c t_{\text{var}} \Gamma_\nu^2$, this condition becomes

$$\frac{t_{\text{syn}}}{1 + Q_{\text{IC}}} < \frac{1}{\Gamma_{\nu \text{var}}} ,$$

where $Q_{\text{IC}} = \tau_\nu \Gamma_\nu^2$ (resp. $\tau_\nu \Gamma_\nu^2/w$) for $w \ll 1$ (resp. $w \gg 1$). With expressions (11), (13), (14) and (20) above, (30) can be written

$$2 \times 10^{-4} \alpha_B^{-1} (\Gamma_{\text{int}} - 1)^{-1} \left( \frac{\dot{E}}{10^{32} \, \text{erg} \, \text{s}^{-1}} \right)^{-1} \times \left( \frac{t_{\text{var}}}{1 \, \text{s}} \right)^5 \left( \frac{\Gamma_\nu}{300} \right)^{-1} < 1 + Q_{\text{IC}} .$$
It is more easily satisfied when gamma-rays directly come from synchrotron emission since then $\Gamma_1 \sim 10^3$. However, even for $\Gamma_1 \sim 10^5$ the emission time remains smaller than the expansion time as long as the shocks are sufficiently strong ($\Gamma_1/\Gamma_2 \gtrsim 2$).

Some results of a model where GRBs are produced by the inverse Compton process have been already presented elsewhere (Mochkovitch & Fuchs, 1996). In this study we limit ourselves to synchrotron emission models except when we discuss the optical properties which strongly differ between the two cases. In synchrotron emission models the total efficiency for the conversion of wind kinetic energy to gamma-rays below a few MeV is given by

$$f_{\text{tot}} = f_d \times \alpha \left(1 - \alpha_{\text{IC}}\right).$$  \hspace{1cm} (32)

where $\alpha$ is the fraction of the dissipated energy which is transferred to the electrons. According to Bykov & Mészáros (1996) $\alpha$ is comparable to the fraction $\alpha_M \sim 0.1 - 1$ of the energy which is initially injected into magnetic fluctuations. The total efficiency therefore does not exceed a few percent which imposes severe constraints on the energy source or/and system geometry.

The energy available from disc accretion to a black hole

$$E \approx \frac{1}{6} M_d c^2 = 3 \times 10^{53} \left(\frac{M_d}{M_\odot}\right) \text{ erg},$$  \hspace{1cm} (33)

where $M_d$ is the disc mass, can be less than $10^{53}$ erg for the coalescence of two neutron stars since numerical simulations indicate that $M_\sim \sim 0.2 - 0.3 M_\odot$ (Rasio & Shapiro 1994; Davies et al. 1994; Ruffert, Janka & Schaefer 1996). The conversion of disc gravitational energy into wind kinetic energy should then be very efficient in order to account for cosmological GRBs. Naturally, if the wind is beamed in a solid angle $\delta \Omega$ along the system axis the energy requirement is smaller by a factor $\delta \Omega / 2\pi$ but one has now to face a statistical problem since even optimistic estimates of the neutron star merging rate are not considerably larger than the burst rate (Phinney 1991; Narayan, Piran & Shemi 1991; Tutukov & Yungelson 1993; Lipunov et al. 1995). The situation is less critical if the disc results from the disruption of a neutron star by a black hole or the collapse of a massive star because the mass of the disc can be $M_d \sim 1 M_\odot$ (and even $M_d \sim 10 M_\odot$).

### 3 Temporal Properties

#### 3.1 Burst profiles

We first study the temporal properties of our burst models when the Lorentz factor in the wind has the simple shape shown in Fig.1. We inject an energy $E = \frac{10^{52}}{t_4}$ erg.s$^{-1}$sr$^{-1}$ which, for an efficiency of conversion into gamma-rays of a few percent, yields a total energy $E = \frac{10^{51}}{t_4}$ erg.sr$^{-1}$ for a burst lasting a few seconds. We obtain the burst profiles (in number of photons per second between 50 and 300 keV, which corresponds to BATSE bands 2 and 3) by adding the contributions of all the internal elementary shocks which occur during the expansion of the wind as explained in Sect. 2. For each shock we compute $\alpha_{\text{syn}} = 1 - \alpha_{\text{IC}}$ (Eq. 27) where $\alpha_{\text{IC}}$ is the fraction of the energy which goes to inverse Compton photons, the equipartition magnetic field $B_{\text{eq}}$ (Eq. 31) with $\alpha = \frac{2\delta}{\Gamma_1}$ (Eq. 27) and $\alpha_{\text{IC}}$ which corresponds to BATSE bands 2 and 3) by adding the thin lines to $\Gamma_e$ given by Eq. 24 and the thick lines to $\Gamma_e$ given by Eq. 24.

**Figure 3.** Physical parameters governing the emission mechanism: (a) fraction $\alpha_{\text{syn}} = 1 - \alpha_{\text{IC}}$ of the energy which is radiated by the synchrotron process; (b) Lorentz factor $\Gamma_e$ of the relativistic electrons; (c) equipartition magnetic field $B_{\text{eq}}$; (d) synchrotron energy $E_{\text{syn}}$. In (a), (b) and (d) the thick lines correspond to $\Gamma_e$ given by Eq. 24 and the thin lines to $\Gamma_e = 10^4$.

**Figure 4.** Burst profiles for the initial distribution of the Lorentz factor shown in Fig.1. The count rate $C_{2+3}$ (in arbitrary unit) is given in the interval 50 – 300 keV, corresponding to BATSE bands 2 and 3; (a) profile obtained with Eq. 24 for the electron Lorentz factor; (b) same as (a) with $C_{2+3}$ in logarithmic scale which illustrates the exponential decay after maximum; (c) profile obtained with a constant $\Gamma_e = 10^4$; (d) same as (c) with $C_{2+3}$ in logarithmic scale.

13 with $\alpha_{\text{MN}} = 1/3)$, the electron Lorentz factor $\Gamma_e$ (Eq. 24 with $2\delta = 1000$ and $\mu = 1.75$) and the synchrotron energy $E_{\text{syn}}$ (Eq. 15). These quantities are represented in Fig.3 as a function of arrival time $t_\alpha$. As in Fig.2 there are two branches corresponding to the forward and reverse shocks. Typical values are $10^2 - 10^4$ G for the magnetic field, $2 - 20 \times 10^3$ for the electron Lorentz factor and 10 keV – 1 MeV for the synchrotron energy.

The photons produced in the elementary shocks are distributed according to a synchrotron spectrum

$$\frac{dn(E)}{dE} \propto \frac{e}{E_{\text{syn}}} \left(\frac{E}{E_{\text{syn}}}\right)^{-\varepsilon},$$  \hspace{1cm} (34)

with $x = 2/3$ for $E < E_{\text{syn}}$, $2 < x < 3$ for $E > E_{\text{syn}}$ and where $\varepsilon$ is the energy which is dissipated in the shock (Eq. 5). The resulting burst profile is shown in Fig.4a for a high-energy index $x = 2.5$. Cosmological effects (time dilation and redshift) have been taken into account, assuming that the burst is located at $z = 0.5$. The duration $t_{90}$ which, according to the definition used for the BATSE data is the time during which 90% of the total fluence is received (excluding the first and last 5%) is 10.04 s, very similar to the duration of wind emission $t_\alpha$. The decay after maximum is close to an exponential as can be seen in the plot of the logarithm of the count rate versus time in Fig.4b where there is a quasi linear decline after maximum between $t \approx 12$ s and $t \approx 20$ s. However the rise time in our profile is not much shorter than the decay time and the burst is therefore not very dissymmetric if we except the low intensity exponential tail after $t = 15$ s. We found that a profile much closer to the characteristic FRED (Fast Rise Exponential Decay) shape observed in many bursts (or in individual pulses inside a complex burst) can be obtained by assuming that the electron Lorentz factor varies more slowly with $\varepsilon$ (the dissipated energy per proton) than a power-law of index $1/(3 - \mu)$ (Eq. 24). This would be the case if the fraction $\zeta$ of accelerated electrons increased with $\varepsilon$. Such a behavior is observed in simulations of collisionless non relativistic shocks (Bykov, private communication) and we have supposed that it remains valid in the relativistic limit. Moreover, we have made the simple choice $\zeta \propto \varepsilon$ which leads to a characteristic Lorentz factor for the electrons which is independent of $\varepsilon$. The synchrotron energy in the elementary shocks has been represented in Fig.3d for a constant $\Gamma_e = 10^4$ and the corresponding burst profile is shown in Fig.4c. It has now a typical FRED shape with a
the initial distribution of the Lorentz factor is made of a rapid component (with an average value of $\Gamma$ of a few hundreds) and of some slower layers (with $\Gamma \simeq 100$). The total mass in the slow layers has to be comparable to the mass in the rapid component in order to keep the efficiency at a reasonable level. A few examples of synthetic profiles are presented in Fig.6. It can be seen that a great diversity of burst shapes can be obtained if the distribution of the Lorentz factor in the wind varies from one event to the other.

3.2 Short timescale variability
The profiles shown in Fig.4 and 6 have a satisfactory general shape but do not exhibit any variability on a short time scale. Rapid temporal variations cannot result from small irregularities of the emitting surface since photons coming from many different regions in space and time are received at the same time by the observer which leads to a loss of coherence of the temporal variations (Woods & Loeb 1995; Sari & Piran 1997b,c). Instead, one has to consider again that the Lorentz factor itself can fluctuate at the millisecond level. This is not unrealistic if the flow at the origin of the wind is very irregular and turbulent since one millisecond corresponds to the typical dynamical time scale of a disc orbiting a stellar mass black hole. Nothing being known about the temporal spectrum of the fluctuations we have simply added a random fluctuation to the Lorentz factor of each of the layers initially injected (every 2 ms) in the wind. The adopted amplitude for these fluctuations is 10 – 20% of the average value of $\Gamma$. The resulting profiles represented in Fig.7 now show the rapid temporal variations which are seen in most observed bursts.

4 SPECTRAL PROPERTIES
4.1 Burst spectrum
The overall burst spectrum is the sum of all the elementary contributions (Eq. 34) from the internal shocks. The spectrum corresponding to the profile shown in Fig.7a is represented in Fig.8 (again a cosmological redshift $z = 0.5$ has been assumed). Its shape can be easily understood: let $E_{\text{syn}}^\text{min}$ and $E_{\text{syn}}^\text{max}$ be the minimum and maximum of the synchrotron energy for the whole set of elementary shocks. As long as $E$ is smaller (resp. larger) than $E_{\text{syn}}^\text{min}$ (resp. $E_{\text{syn}}^\text{max}$) the number of photons $n(E)$ per unit energy interval is a power-law of index $-2/3$ (resp. $-2.5$) and in the intermediate region the current index evolves from $-2/3$ to $-2.5$ as $E$ goes beyond the value of $E_{\text{syn}}$ for a growing number of elementary shocks. In the four BATSE bands, between 20 keV and a few MeV, the spectrum can be well described with Band's formula (Band et al. 1993)

$$n(E) = A \left( \frac{E}{100 \text{ keV}} \right)^{\alpha} \exp \left( -\frac{E}{E_0} \right) \text{ for } (\alpha - \beta)E_0 \geq E,$$

$$n(E) = A \left( \frac{(\alpha - \beta)_0}{100 \text{ keV}} \right)^{\alpha - \beta} \exp((\beta - \alpha) \left( \frac{E}{100 \text{ keV}} \right)^\beta) \text{ for } (\alpha - \beta)_0 \leq E.$$
Figure 8. Spectrum of the burst corresponding to the profile of Fig.7a. The number of photons per energy interval $n(E)$ and the product $E^2 n(E)$ are shown in arbitrary units. The dashed line is a fit of the spectrum with Band’s formula in the interval 10 keV – 10 MeV. The product $E^2 n(E)$ is maximum at the peak energy $E_p = 365$ keV.

with typical values found in observed bursts. The product $E^2 n(E)$ is also shown in Fig.8. It is maximum at the peak energy $E_p = 365$ keV where the bulk of the emission takes place.

4.2 Duration-hardness ratio relation

Shorter bursts are expected to be harder in our model. Internal shocks are formed at an approximate radius $r \sim c t_{\text{var}} \Gamma^2$ and are therefore closer to the source if the burst evolves on a short time scale. Assuming that the injected power $\dot{E}$ is independent of the duration of wind injection $t_w$ the equipartition magnetic field is stronger and the synchrotron energy is larger in the dissipation region for shorter bursts. To obtain the duration-hardness relation we compute the hardness ratio $HR_{32}$ (defined as the ratio of the number of photons received in BATSE band 3 to that in band 2) for bursts with homothetic initial distributions of the Lorentz factor but different values of $t_w$. For a given duration $t_0$ the hardness ratio is a function of the high energy index of the elementary spectrum (Eq. 34) (the bursts becoming softer when $x$ increases from 2 to 3), the detailed history of the Lorentz factor and of the cosmological redshift $z$. Complex bursts tend to be harder because the distribution of the Lorentz factor varies on a shorter time scale at given $t_w$ and dissipation therefore begins earlier than in more regular bursts of same duration. Finally cosmological effects shift bursts in the $HR_{32} - t_0$ diagram downward (redshift) and to the right (time dilation). However the shift is nearly colinear to the intensity instead of decreasing like $z$ for a single (i.e. dominated by one main pulse or made of just a few pulses) while in complex bursts with many pulses the correlation between hardness and count rate is clearly visible. Also, the hardness of successive pulses remains essentially correlated to the intensity instead of decreasing like in 50% of the Ford et al. (1995) sample.

Figure 9. Duration-hardness ratio ($HR_{32}$) relations for a simple (one pulse) burst and a complex burst with five pulses. The relations are shown for three values of the high energy index $x = 2, 2.5$ and 3 of the elementary synchrotron spectrum and two redshifts $z = 0.3$ and 1. It can be seen that the effect of the redshift is negligible.

Figure 10. Evolution of hardness with time: the instantaneous value of the peak energy $E_p$ (thick line) is represented together with the burst profile (thin line). Upper panel: for the simple one pulse burst of Fig.4c. Lower panel: for the more complex burst of Fig.6a.

4.3 Spectral evolution

4.3.1 Instantaneous hardness

The spectral evolution of GRBs shows a few trends which are followed by a majority of bursts but also suffer some exceptions and are therefore not universal (Bhat et al. 1994; Ford et al. 1995). First, spectral hardness and count rate appear to be correlated. Within intensity pulses both increase and decrease together, the hardness usually preceding the count rate. Another trend is a global hard-to-soft evolution over the course of the burst outside intensity pulses. Finally, later pulses tend to be softer than earlier pulses, even if they have a greater intensity.

We have compared the spectral evolution of our synthetic burst models to these observational results. The hardness can be obtained as a function of time through the estimation of the instantaneous value of $E_p$, the energy of the peak of $E^2 n(E)$. We first considered the simple burst of Fig.4c which has a characteristic FRED shape and a duration $t_0 = 10.23$ s. The hardness and count rate evolve similarly (see Fig.10) but their maxima are separated by a time lag of 0.89 s, the maximum of $E_p$ occurring before that of the count rate. In complex bursts the hardness increases during intensity pulses and also precedes the count rate (Fig.10).

The correlation between spectral hardness and count rate is therefore correctly reproduced in our models. We encountered more difficulties with the global hard-to-soft evolution which was observed in 70% of the sample of bright long bursts studied by Ford et al. (1995). In synthetic bursts (Fig.10) it is present as long as the profiles remain relatively simple (i.e. dominated by one main pulse or made of just a few pulses) while in complex bursts with many pulses only the correlation between hardness and count rate is clearly visible. Also, the hardness of successive pulses remains essentially correlated to the intensity instead of decreasing like in 50% of the Ford et al. (1995) sample.

4.3.2 Pulse shape as a function of energy

When observed in spectral bands of increasing energy, pulses become narrower as shown by Norris et al. (1996) who analyzed a large number of pulses in the four BATSE bands. They found that their (half maximum) width can be well represented by a power-law

$$W(E) \propto E^{-0.4}.$$  

(39)

The same relation was obtained by Fenimore et al. (1995) who used the autocorrelation of averaged burst profiles instead of individual pulses. In our synthetic burst models the pulse width also decreases at high energy. Figure 11 represents a single pulse burst in the four BATSE bands. The width can be fitted by power-laws such (39) but not with a unique exponent $p = -0.4$ for all the pulses. We indeed get $p \approx -0.4$ for pulses of $2 - 10$ s but for pulses of $0.1 - 1$ s, $p \approx 0.2$. Norris et al. (1996) obtained $p = -0.4$ as an average on a collection of pulses with duration ranging from 0.1 to 10 s but do not provide the value of p for the shorter and longer pulses separately which does not allow a detailed comparison with our theoretical results.
4.4 The $E_p$ - fluence relation

Liang & Kargatis (1996) discovered in a sample of 37 BATSE bursts an exponential dependence of the peak energy $E_p$ on the photon fluence $F_{\text{ph}}$ during the declining part of intensity pulses i.e.

$$E_p \propto \exp(-a F_{\text{ph}}).$$

The photon fluence is defined as the integral of the photon flux from the beginning of the burst and is the slope of the Log$E_p$ - $F_{\text{ph}}$ relation. In complex bursts the slope stays approximately constant from one pulse to another. Synthetic bursts also follow a relation such (40) but the slope in successive pulses can somewhat vary, especially if they have very different intensity or duration (see Fig.12).

4.5 X-ray and optical counterparts

The X-ray and optical counterparts recently discovered in two GRBs have been interpreted in the context of cosmological models as the emission coming from a relativistic shell expanding in the interstellar medium (Wijers, Rees & Mészáros 1997; Vietri 1997; Waxman, 1997a,b). If however GRBs are produced by internal shocks in a wind, X-ray to optical photons should also be emitted together with the gamma-rays before the afterglow resulting from the interaction with the interstellar medium.

To compute these early counterparts we consider a typical burst where an energy $E = \frac{2 \times 10^{52}}{\Gamma^2} \text{erg} \cdot \text{sr}^{-1}$ has been injected into the wind with a 5% efficiency for the conversion into gamma-rays between 50 keV and 300 keV. For a GRB located at 2 Gpc ($z \sim 0.5$) the observed fluence in BATSE bands 2 and 3 will be $F_x \approx 2 \times 10^{-6} \text{erg} \cdot \text{cm}^{-2}$. This value of $F_x$ is then used to normalize a synthetic spectrum from which the expected flux in the X-rays and visible can be finally obtained. In practice these fluxes are highly variable (as the gamma-rays) and have been averaged over the duration $t_{90}$ of the burst.

We did not consider in this paper inverse Compton emission models but we now briefly discuss their optical properties since they greatly differ from synchrotron emission models. When the gamma-rays come from inverse Compton scattering, a fraction $\alpha_{\text{syn}} \sim 1 - \alpha_C$ of the total power is emitted at a typical synchrotron energy $E_p = \frac{F_x}{\Gamma^2} \approx 10 - 100$ eV where $E_p$ is the peak energy of the gamma-ray spectrum. The fraction $\alpha_{\text{syn}}$ is fixed by the ratio $\alpha_C/\alpha_B$ through Eq. (17) and (23). Preliminary results indicate that for $\alpha_C/\alpha_B = 1$ the emission in the visible of a burst of fluence $F_x$ and duration $t_{90} = 10$ s could be as bright as $V = 5 - 6$ which is already excluded by the limit set by the ETC and GROCE instruments (Krimm, Vanderspek & Ricker 1996; Lee et al. 1997). With $\alpha_C = 100$ the predicted magnitude becomes $V = 8 - 9$ still within the reach of ETC and GROCE.

In synchrotron emission models the optical counterpart is much weaker. Taking for example the spectrum
being obtained from the observations. For GRB 970508 a fit of the data also provides \( F_{\text{max}} \sim 6 \times 10^{-28} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \) (Sahu et al. 1997b). With the gamma-ray fluence measured for this burst and assuming a 5\% efficiency for the conversion of wind kinetic energy to gamma-rays we obtain

\[
\sqrt{\Omega \alpha_{\text{IR}}} \sim 3 \times 10^{-2}.
\]

If we adopt this value as typical we find that the afterglow of the burst (given in example above) with \( E_{\text{BD}} = 2 \) and \( D_{\text{OBD}} = 2 \) has a V magnitude of 18.9 before the phase of power-law decline. In inverse Compton emission models the initial optical counterparts produced by internal shocks is considerably brighter than the afterglow but in synchrotron emission models the two contributions have comparable brightness. However, the optical signal will present a short interruption if the duration of the burst is smaller than the deceleration time. Conversely, if \( t_w \gg t_{\text{dec}} \), the two contributions overlap and form a continuous signal. In any case, a detailed photometric follow-up in the optical range beginning before the end of the gamma-ray burst would certainly provide crucial informations about the emission mechanisms.

5 CONCLUSIONS

We have developed a simple model to compute the temporal and spectral properties of GRBs under the assumption that they originate from internal shocks in a relativistic wind. We have not discussed the critical point of how such a wind could form but the recent observations of optical counterparts for GRB 970228 and GRB 970508 seem to indicate that a relativistic shell was indeed present in these objects. The distribution of the Lorentz factor in the wind has no reason to be uniform and variations on several timescales (down to one millisecond which corresponds to the dynamical timescale of a relativistic disc orbiting a stellar mass black hole) can be expected. Layers of different velocities will collide and form internal shocks within the relativistic wind and the energy dissipipated in these shocks can be emitted in the form of gamma-rays. We have followed the evolution of the relativistic wind using an approach where all pressure waves are suppressed so that layers only interact through direct shocks. We assume that the magnetic field reach equipartition in these shocks and that the electron Lorentz factor remains close to a constant value \( \Gamma \sim 10^4 \). This could be the case if, according to Bykov & Mészáros (1996) a small fraction \( \zeta \) of the electrons is accelerated in the shocks and if \( \zeta \) is also approximately proportional to the dissipated energy. Gamma-rays are then directly produced by synchrotron emission from the relativistic electrons.

This procedure allows us to construct synthetic bursts whose temporal and spectral properties are compared to the observations. We obtain a series of encouraging results:

1) It is possible to generate a great diversity of profiles simply by playing with the initial distribution of the Lorentz factor in the wind.
2) The profile of individual pulses is asymmetric, close to a “FRED” shape.
3) The short time scale variability of the profiles can be explained if the Lorentz factor itself varies at the millisecond level.
4) Synthetic spectra can be fitted with Band’s formula with parameters \( \alpha, \beta \) and \( E_0 \) comparable to those of observed bursts.
5) The duration-hardness relation is a natural consequence of the model since in short bursts dissipation occurs closer to the source where the magnetic field and the synchrotron energy are larger.
6) Spectral hardness and count rate are correlated during burst evolution, the hardness generally preceding the count rate.
7) The pulse width decreases with increasing energy following a power law \( W(E) \propto E^{2-p} \) with \( p \sim 0.4 \) for pulses of duration 2–10 s.
8) In the declining part of intensity pulses the peak energy \( E_p \) decreases exponentially with photon fluence.

Some other properties of observed GRBs are however not so well reproduced by our model:

1) The shortest pulses tend to decay faster than they rise instead of being symmetric.
2) Synthetic bursts do not show a global hard to soft evolution as frequently as real bursts.
3) The slope of the \( E_p \) – fluence relation can differ among pulses inside the same burst.

It is not yet clear whether these difficulties represent real problems for our model or are simply a consequence of some of the crude assumptions we have made. In particular, the evolution of the wind should be followed with a detailed hydrodynamical code (Daigne and Mochkovitch, in preparation) rather than with the simple method used here. A more fundamental issue may be the rather low efficiency for the conversion of wind kinetic energy to gamma-rays. If GRBs result from the coalescence of neutron stars the wind cannot be strongly beamed since the merging rate is not very much greater than the burst rate. A large fraction of the energy released in the coalescence should therefore be injected into the wind.

We believe that the present work has shown that if a relativistic wind carrying enough energy can be produced, the internal shock model appears as a convincing candidate to explain GRBs. Ways to both reach a high efficiency in the generation of the wind and to avoid at the same time baryonic pollution are among the difficult problems which remain to be solved.

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