Diffraction scattering in the multiperipheral model
Amati, Daniele ; Cini, M ; Stanghellini, A

Published in Nuovo Cimento 30 (1963) 193-204

**This article has been scanned with the kind permission of the
Società Italiana di Fisica.**

From: Angela Oleandri <oleandri@sif.it>
Date: October 12, 2007 4:07:47 PM GMT+02:00
To: Salvatore Mele <Salvatore.Mele@cern.ch>
Subject: Re: Request for permission to reproduce published material.

Dear Dr Mele,

further to your request, sent in July to Carmen Vasini (herewith attached)
I'm pleased to inform you that the Italian Physical Society is happy to
grant you permission of reproduction of the listed material for your
archival purposes at no charges.

Diffraction Scattering in the Multiperipheral Model (*).

D. AMATI (**), M. CINI (***) and A. STANGHELLINI

CERN - Geneva

(ricevuto il 13 Aprile 1963)

Summary. — The diffraction scattering due to the shadow of the absorption as given by the multiperipheral model for high-energy inelastic collisions, is calculated. The Regge pole that would appear in the elastic amplitude if only inelastic intermediate states are considered in the unitarity condition, turns out to dominate only in the forward direction. For $t \neq 0$ multiple elastic scattering effects become important. Explicit expressions for the diffraction pattern are obtained which show features which differ from those of the Pomerančuk-Regge pole. Particularly the shrinking is predicted to decrease with energy and to be larger for $N\bar{N}$ than for πN scattering.

1. — Introduction.

A model for high-energy production processes, the multiperipheral model, has recently been proposed ⁽¹⁾. Its predictions about the average properties of inelastic high-energy collisions are very simple and seem to be consistent with the present experimental information. The properties of elastic diffraction scattering have also been discussed in the framework of the model, by introducing the inelastic amplitudes in the unitarity condition

$$(1.1) \quad \text{Im } T_{el} \equiv A(s, t) = \frac{1}{2}(T_{in}^+ T_{in} + T_{el}^+ T_{el})$$

(*) A brief discussion of the results obtained in this work has been presented in *Phys. Lett.* (in press).

(**) Istituto di Fisica dell'Università, Palermo.

(***) On leave from Istituto di Fisica dell'Università, Roma.

⁽¹⁾ D. AMATI, S. FUBINI and A. STANGHELLINI: *Nuovo Cimento*, 26, 896 (1962); hereafter referred to as ASF.

Under the assumption that the inelastic amplitudes dominate in eq. (1.1), a first approximation to $A(s, t)$ was found to be, in the limit of large s ,

$$(1.2) \quad A_0(s, t) = c(t)s^{\alpha(t)}$$

which shows the well-known Regge pole behaviour. The next approximation, which consists in introducing $iA_0(s, t)$ for T_{el} in the right-hand side of eq. (1.1) was found to be a continuous superposition of powers of s : in the language of Regge this has been described as a cut in the angular momentum complex plane. In AFS attention was called to the fact that the effect of this correction, and of the higher-order ones which could be obtained by further iteration of eq. (1.1), should be important particularly when $\alpha(0) = 1$ (*). Some speculations about the possibility that, by virtue of cancellations, all the cuts might disappear in the exact amplitude, have recently been advanced. Although this seems to us rather unlikely to happen, also in view of the fact that all the attempts to find the simple Regge pole behaviour in field theory have not been successful up to now, we do not want to enter into this question, but we prefer to take a more phenomenological, and perhaps more constructive, attitude. Namely, we will take the multiperipheral model as a reasonable theory of high-energy inelastic collisions, and will work out quantitatively the solution of the unitarity condition (1.1); in other words, we shall obtain the complete diffraction scattering which is due to the absorption given by the multiperipheral model.

In the limit of large s , eq. (1.1) becomes, by using (1.2),

$$(1.3) \quad A(s, t) = A_0(s, t) + \frac{2}{(8\pi)^2} \frac{1}{s} \int_{-\infty}^0 \int_{-\infty}^0 dt' dt'' T_{el}^+(s, t') T_{el}(s, t'') \tau(t, t', t''),$$

where

$$(1.4) \quad \tau(t, t', t'') = \frac{\theta[-t^2 - t'^2 - t''^2 + 2tt' + 2tt'' + 2t't'']}{\sqrt{-t^2 - t'^2 - t''^2 + 2tt' + 2tt'' + 2t't''}}.$$

Equation (1.3) links, for any fixed value of s , all the negative values of t , and therefore, in order to solve it, one has to know the explicit t -dependence of $c(t)$ and $\alpha(t)$.

(*) Indeed for $\alpha(0) < 1$ the correction would be depressed by a power $s^{\alpha(0)-1}$ as compared to the inelastic contribution and could therefore be disregarded in the asymptotic limit. For $\alpha(0) > 1$ the elastic shadow would blow up with energy more than any power of s , as expected by the Froissart theorem.

We shall use the form (*)

$$(1.5) \quad c(t) s^{\alpha(t)} \simeq a s \left(\frac{s}{s_0} \right)^{\dot{\alpha}(0)t} \equiv a s \exp [b(s)t],$$

where

$$(1.6) \quad b(s) = \dot{\alpha}(0) \ln \frac{s}{s_0},$$

and a is just the asymptotic value of the inelastic cross-section, s_0 a suitable constant. The form of eq. (1.5) is plausible for small values of t (of the order of the masses of the particles) in which we will be interested. In eq. (1.3), however, also large values of t occur. We will use the expressions (1.5) and (1.6) in the whole range of integration due to the fact that for small values of t the integral shall be dominated by small values of t' and t'' due to the exponential behavior of (1.5).

Besides the imaginary part $A(s, t)$, also the real part $D(s, t)$ of T_{el} enters in eq. (1.3). The complete determination of the elastic amplitude requires therefore the use of the fixed- t dispersion relation connecting $D(s, t)$ to $A(s, t)$ (**):

$$(1.7) \quad D(s, t) = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{A(s', t)}{s' - s} ds' + \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{A(s', t)}{s' + s + t - \sum_i m_i^2} ds'.$$

In solving (1.3), however, we shall make the physical assumption that at high energy the elastic amplitude is essentially imaginary (***) so that from eq. (1.3) and eq. (1.4) we write

$$(1.8) \quad A(s, t) = a s \exp [b(s)t] + \frac{2}{(8\pi)^2 s} \int_{-\infty}^0 \int_{-\infty}^0 dt' dt'' A(s, t') A(s, t'') \tau(t, t', t'').$$

We shall check at the end that our solution for A and D obtained through (1.8) and (1.7) indeed satisfies this assumption ensuring therefore the consistency

(*) We have assumed $\alpha(0) = 1$ because of the observed constancy of the inelastic cross-sections. For the reasons recalled above, this is just the case in which the shadow of the elastic channel could be of importance.

(**) Since we are interested in the elastic diffraction scattering the + sign is taken for the crossed channel contribution (amplitude with positive signature).

(***) The fact that the elastic amplitude is mainly imaginary at high energy is physically meaningful. Any single channel is expected to have a vanishing small contribution. The elastic one, however, must have an imaginary part due to the shadow of all the open channels.

of our results. Equation (1.8) is a nonlinear integral equation in which the inhomogeneous term is just the Pomeron-Regge pole as given in eq. (1.5). This is the only trajectory which enters in (1.5) because we are interested in the asymptotic behaviour of A , *i.e.*, the dominant power of s . We want to make clear that we shall not be authorized to disregard the lowest power of $\log s$ as compared to the highest ones: in other words, our asymptotic limit is a limit in powers of s and not in powers of the logarithm of s . This fact is of importance because—as we shall see—our solution shall not be expressed as a simple power of s .

2. — Solution of the unitarity equation.

By defining

$$(2.1) \quad F_s(t) = \frac{A(s, t)}{as},$$

eq. (1.8) reduces to the form

$$(2.2) \quad F_s(t) = \exp[bt] + \frac{\lambda}{4\pi} \int_{-\infty}^0 \int_{-\infty}^0 F_s(t') F_s(t'') b \tau(t, t', t'') dt' dt'',$$

with

$$(2.3) \quad \lambda = \frac{a}{8\pi b},$$

where b has been taken inside the integral to make it nondimensional. The iterative solution of (2.2) will have the form of a power series in λ

$$(2.4) \quad F_s(t) = \sum_{n=0}^{\infty} \lambda^n F_n(t) \quad F_0(t) = \exp[bt].$$

The $(n+1)$ -th term is easily seen to be given by the following recurrence relation:

$$(2.5) \quad F_{n+1}(t) = \frac{1}{4\pi} \sum_{m=0}^n \int_{-\infty}^0 \int_{-\infty}^0 F_m(t') F_{n-m}(t'') b \tau(t, t', t'') dt' dt''.$$

Now if we make the « ansatz »

$$(2.6) \quad F_n(t) = \frac{c_n}{n+1} \exp\left[\frac{bt}{n+1}\right].$$

Equation (2.5) is satisfied provided the coefficients c_n satisfy the recurrence relation

$$(2.7) \quad c_{n+1} = \frac{1}{4} \sum_{m=0}^n c_m c_{n-m}, \quad c_0 = 1,$$

because (2.6) introduced in (2.5) gives

$$(2.8) \quad \int_{-\infty}^0 \int_{-\infty}^0 F_m(t') F_{n-m}(t'') b \tau(t, t', t'') dt' dt'' = \frac{\pi}{n+2} \exp \left[\frac{bt}{n+2} \right] c_m c_{n-m}.$$

The relation (2.7) is the same that one gets when the second-degree equation

$$(2.9) \quad c = 1 + \frac{\lambda}{4} c^2$$

is solved by iteration with

$$c = \sum_{n=0}^{\infty} \lambda^n c_n.$$

Therefore the expansion in powers of λ of the solution (*)

$$(2.10) \quad c(\lambda) = 2 \frac{1 - \sqrt{1 - \lambda}}{\lambda}$$

gives the coefficients c_n , namely,

$$(2.11) \quad c_n = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{4 \cdot 6 \cdot 8 \dots (2n+2)} = \frac{(2n)!}{4^n (n!)^2 (n+1)}, \quad n \geq 1.$$

For $n \gg 1$ we shall use Stirling's formula to obtain

$$(2.12) \quad c_n \sim \frac{1}{\sqrt{\pi}} \frac{1}{n^{\frac{3}{2}}}.$$

The solution of eq. (1.8) turns out therefore to be

$$(2.13) \quad A(s, t) = as F_s(t) = as \sum_{n=0}^{\infty} \frac{\lambda^n}{n+1} c_n \exp \frac{bt}{n+1}.$$

Since we are looking for real coefficients, eq. (2.7) gives the limitation

$$(2.14) \quad \lambda \leq 1.$$

This limitation is connected with the unitarity condition; in fact, in our model

(*) We choose the minus sign in front of $\sqrt{1-\lambda}$ because we want the total amplitude $A(s, t)$ to vanish when a goes to zero.

the parameter λ is simply the S -wave inelastic cross-section

$$\lambda = 1 - |\exp[2i\delta_0]|^2$$

as can be seen by making directly the partial wave analysis of A_0 . Therefore as soon as λ overcomes 1, the S -wave begins to violate unitarity.

Equation (2.13) is the solution of our integral equation. Because of the limitation (2.14) and of the fact that, for large n , $c_n \sim 1/n^3$, the series is indeed rapidly convergent for any value of t . This shows the legitimacy of our expansion and furthermore suggests that reliable numerical results should be obtained by taking into account a limited number of terms.

In order to understand better the physical implications of this solution, however, we shall obtain closed expressions for A both for $t=0$ and for values of bt large compared to 1.

3. - Diffraction in the forward direction.

Let us begin to study the case $t=0$. The function A and its derivatives with respect to t in the forward direction ($t=0$) are

$$(3.1) \quad \left\{ \begin{aligned} \frac{A(s, 0)}{s} &= a \sum_{n=0}^{\infty} \frac{\lambda^n c_n}{n+1}, \\ \frac{A'(s, 0)}{s} &= a \sum_{n=0}^{\infty} \frac{b}{n+1} \frac{\lambda^n c_n}{n+1}, \\ &\dots \dots \dots \\ \frac{A^r(s, 0)}{s} &= a \sum_{n=0}^{\infty} \left(\frac{b}{n+1} \right)^r \frac{\lambda^n c_n}{n+1}, \end{aligned} \right.$$

from which it is easy to obtain

$$(3.2) \quad \left\{ \begin{aligned} \frac{A(s, 0)}{s} &= \frac{a}{\lambda} \int_0^\lambda d\lambda' c(\lambda'), \\ A'(s, 0) &= \frac{b}{\lambda} \int_0^\lambda d\lambda' A(s, 0; \lambda'), \\ &\dots \dots \dots \\ A^r(s, 0) &= \frac{b}{\lambda} \int_0^\lambda d\lambda' A^{r-1}(s, 0; \lambda'), \end{aligned} \right.$$

where $c(\lambda)$ is given by (2.10). The explicit expression for $A(s, 0)$ is

$$(3.3) \quad \frac{A(s, 0)}{s} = \frac{4a}{\lambda} \left[\log \frac{(1 + \sqrt{1-\lambda})}{2} - \sqrt{1-\lambda} + 1 \right].$$

Let us discuss some physical consequences of (3.2) and (3.3). As the energy increases λ tends to zero as $1/\log s$. Therefore at very high energy the forward direction will be dominated by the pole; the maximum correction will be for $\lambda=1$, where

$$(3.4) \quad \frac{A(s, 0)}{s} \simeq 4a[1 - \log 2] \sim 1.23a,$$

which means that the elastic cross-section is at most about 20% of the inelastic one, disappearing afterwards with the energy.

The same arguments can be repeated for the t -derivatives of A : the pole term is there even more important due to the better convergence of the series in terms of which the t -derivatives are given.

We can conclude that the forward amplitude (and therefore the total cross-section) and its derivatives with respect to t at sufficiently high-energy will be dominated by the inelastic processes (Pomerančuk-Regge pole in our approach) (*).

4. - The diffractive amplitude for $t \neq 0$.

Let us now investigate eq. (2.13) in general for $t \neq 0$. As a function of n , each term in the sum is a product of two competing factors: one ($\lambda^n c_n/(n+1)$) which tends to depress the high values of n , the other one ($\exp[bt/(n+1)]$) which, the larger being bt , the stronger depresses the small values of n . It is clear that for a given value of λ and bt , the bigger contribution to the sum will come from an intermediate value of n ($\bar{n}(bt)$), which increases for increasing values of bt . Let us find an explicit expression for $\bar{n}$ for large bt . In such a case the small values of n do not contribute appreciably to the sum (2.9) and therefore we can use the approximate expression of c_n for large n

$$(4.1) \quad \frac{A(s, t)}{s} \simeq \frac{a}{\lambda\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{1}{n^{\frac{3}{2}}} \exp \left[-n\varrho + \frac{bt}{n} \right],$$

(*) Similar conclusions were reached by VAN HOVE by solving the unitarity condition for each partial wave and summing afterwards over l . We thank Professor VAN HOVE for having shown to us his unpublished results.

where

$$\varrho = \ln \frac{1}{\lambda} = \ln \frac{8\pi b}{a}.$$

It is easy to see from (4.1) that the value of n for which the contribution to the sum is the largest is given by

$$(4.2) \quad \bar{n} = \frac{-\frac{5}{4} + \sqrt{25/16 - \varrho b t}}{\varrho}.$$

Therefore, in order to have $\bar{n} \gg 1$,

$$(4.3) \quad -bt \gg \varrho + \frac{5}{2}.$$

The half-width Δn is of the order of $\sqrt{\bar{n}}$, so that if the condition (4.3) is satisfied, we are entitled to write (4.1) as

$$(4.4) \quad \frac{A(s, t)}{s} \simeq \frac{a}{2\sqrt{\pi}} \int_0^\infty \frac{dn}{n^{\frac{3}{2}}} \exp \left[-n\varrho + \frac{bt}{n} \right] = \frac{8\pi}{-t} \left(\frac{1}{2\sqrt{-bt}} + \sqrt{\varrho} \right) \exp[-2\sqrt{-b\varrho t}].$$

We want to stress that, because b increases as $\log s$, while ϱ increases as $\log \log s$ only, for any fixed value of t , the condition (4.3) is satisfied for sufficiently large energies. From the preceding result, we see that while the Regge pole dominates at $t=0$, for $t \neq 0$ the successive iterations turn out to be of importance so that—in the asymptotic limit—the diffractive amplitude shows a completely different behaviour as compared to that of the Pomeron-Regge pole. In fact the expression for A given in eq. (4.4) does not have the form of a power of s .

For any fixed (finite) energy, in the forward direction, the Regge pole is a reasonable good approximation of the amplitude and for $-t \gg (\varrho + \frac{5}{2})/b$ the amplitude is given by (4.4). In the intermediate region of t the amplitude will be well represented by a small number of terms which can be computed directly.

Before going over to the analysis of the physical implication of our results, let us discuss the real part of the amplitude.

5. — The real part of the amplitude.

In order to write a dispersion relation for $D(s, t)$, we note that because the asymptotic behaviour of $A(s, t)$ is bounded by s , we need to perform one

subtraction on the dispersion relation (1.7). We obtain for the elastic amplitude

$$(5.1) \quad D(s, t) = f(t) + \frac{s}{\pi} \left(P \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{A(s', t)}{s' - s} - P \int_{s_{th}}^{\infty} ds' \frac{A(s', t)}{(s' + t - \sum_i m_i^2)(s' + s + t - \sum_i m_i^2)} \right).$$

From this expression one sees that $f(t)$ and the low- s region of integration contribute to the asymptotic behaviour of $D(s, t)$ with an s^0 term, which—as we will show—shall be negligible in the asymptotic limit. Therefore asymptotically

$$(5.2) \quad D(s, t) \simeq \frac{2s^2}{\pi} P \int_0^{\infty} \frac{A(s', t)}{s'(s'^2 - s^2)} ds',$$

where A is given by eq. (2.13). The n -th term of (5.2) will contribute to the integral (5.2) with

$$(5.3) \quad \begin{aligned} D_n(s, t) &= \frac{2a}{\pi} s^2 \frac{c_n}{n+1} P \int_0^{\infty} \frac{\lambda^n \exp[bt/(n+1)]}{s'^2 - s^2} ds' = \\ &= \frac{2a}{\pi} s^2 \frac{c_n}{n+1} \left(\frac{a}{8\pi\alpha} \right)^n \left(\frac{s}{s_0} \right)^{\alpha t/(n+1)} \int_0^{\infty} \frac{x^{\alpha t/(n+1)} dx}{(\ln x + \ln s/s_0)^n (x^2 - 1)} = \\ &= -as \frac{c_n \lambda^n}{n+1} \exp \left[\frac{bt}{n+1} \right] \left[\operatorname{ctg} \frac{\pi}{2} \left(1 + \frac{\alpha t}{n+1} \right) \left[1 + O \left(\frac{n}{\ln s/s_0} \right) \right] \right]. \end{aligned}$$

We shall restrict ourselves to $\alpha t < 1$ in order to avoid the «ghost» that, as is well known, appears if $\alpha(t)$ vanishes.

Let us disregard, for the moment, the $O(n/(\log s/s_0))$ term. From eq. (5.3) we see that the higher is n , the smaller is D_n with respect to A_n due to the cotangent that vanishes as $\alpha t/n$. For bt of the order of unity or smaller, the cotangent vanishes as α/b , so that at high energy $D/A \sim 0$ (in the same way as it was expected for the Regge pole at small t). For large values of bt we know that the values of n which contribute to the sum are of the order of $\bar{n} \sim \sqrt{bt}$, therefore

$$(5.4) \quad \frac{D}{A} \sim \frac{\alpha t}{\bar{n}} \leq \frac{1}{\sqrt{bt}}.$$

This result can be confirmed analytically. Indeed one can obtain for all values

of t summing (5.3)

$$(5.5) \quad \frac{D(s, t)}{A(s, t)} \sim \frac{d \ln A(s, t)}{dt} \frac{\pi \alpha t}{2b},$$

which contains the results previously discussed, if one uses for A the expressions (3.2) for small t and (4.4) for large bt . The terms $O(n/(\log s/s_0))$ in eq. (5.3) would give contributions of the order $1/\sqrt{\log s/s_0}$ with respect to the one calculated before.

Summarizing our preceding discussion, in our model asymptotically $D/A \sim 0$ for all values of t and therefore we have justified our procedure. For a Regge pole, instead D/A is energy-independent and vanishes for $t=0$ only.

6. - Physical consequences of the model.

We will discuss the behaviour of the diffraction scattering on the basis of the preceding results.

We recall at first that the amplitude as well as its derivatives with respect to t , at $t=0$, were similar to the corresponding expressions for the Regge pole A_0 and tend to them in the asymptotic limit of infinite energy. We point out, however, that at machine energies the elastic cross-section is a nonnegligible fraction of the inelastic one ($\sim \frac{1}{5}$): from our formula (3.2) λ turns out to be of order one. This is consistent with the value of λ obtained from (2.3) with the values $a \approx 40$ mb and $b \approx 4 \text{ GeV}^{-2}$, which are suggested by experimental data around 10 GeV.

For $|t|$ not too small, so that condition (4.3) is satisfied, the amplitude is given by eq. (4.4). From it we can derive the following quantities:

$$(6.1a) \quad S \equiv \frac{d \ln(A/s)}{d \ln s} = -\frac{\alpha}{2b} + \frac{2\alpha t(1+\varrho)}{1+2\sqrt{-\varrho b t}} \underset{b \rightarrow \infty}{\sim} -\alpha \sqrt{-\frac{(\varrho+1)^2 t}{\varrho b}},$$

$$(6.1b) \quad \frac{d \ln(A/s)}{dt} = -\frac{3}{2t} + \frac{2\varrho b}{1+2\sqrt{-\varrho b t}} \underset{b \rightarrow \infty}{\sim} \sqrt{-\frac{\varrho b}{t}},$$

$$(6.1c) \quad \frac{dS}{dt} \underset{b \rightarrow \infty}{\sim} \frac{\alpha(1+\varrho)}{2\sqrt{-\varrho b t}},$$

$$(6.1d) \quad \frac{dS}{d \ln s} \underset{b \rightarrow \infty}{\sim} \frac{\alpha^2 \sqrt{-t}}{(\varrho b)^{\frac{3}{2}}} (1+\varrho^2).$$

Before discussing them let us write the corresponding expressions for the Po-

merančuk-Regge pole $A_0(s, t)$:

$$(6.2a) \quad S = t\alpha,$$

$$(6.2b) \quad \frac{d \ln(A/s)}{dt} = b,$$

$$(6.2c) \quad \frac{dS}{dt} = \dot{\alpha},$$

$$(6.2d) \quad \frac{dS}{d \ln s} = 0.$$

Now S represents the shrinking of the diffraction pattern as a function of energy. From (6.1a) we see that it is expected to decrease with energy and to increase with t less than the linear dependence on t assumed for $\alpha(t)$. The variation of the shrinking with energy is given by (6.1d), it decreases with rate $\sqrt{(\ln \ln s)/(\ln s)^3}$ while it does not vary for the Regge pole. Although we do not want to take seriously any numerical evaluation of our asymptotic formulae at the energies of present experiments, a very rough evaluation of the magnitude of the predicted effects may be of some interest. For NN scattering at 10 GeV, we can take $\lambda \simeq 0.9$ from the ratio between elastic and inelastic cross-sections quoted above. We can also assume tentatively $b \sim 4 \text{ GeV}^{-2}$. Then formulae (1.6) and (2.3) give at 30 GeV $\lambda = 0.6$ and $b = 6 \text{ GeV}^{-2}$. For these values of b it is clear that the condition (4.3) for the validity of our asymptotic formulae is not very well satisfied at $-t = 1 \text{ GeV}^2$. Nevertheless if we introduce these values for bt and λ it is not unsatisfactory to find that the shrinkage at 30 GeV is 1.5 times smaller than the shrinkage at 10 GeV. Even without attaching too much meaning to these figures, the effect is sizeable.

Another interesting property of our model is that the shrinking, as well as the other physical quantities, depends on the value of the total cross-section for the process in question even if $\alpha(t)$ is always the same (*), while this was not the case for the Regge pole, eqs. (6.2). This is easily understood if one considers the nonlinearity of our integral eq. (1.8), which makes the normalization of the input to play an important role. It is easy to see from (6.1a) that

$$(6.3) \quad \frac{dS^2}{da} \sim -\frac{1-\rho^2}{\rho^2} \frac{\dot{\alpha}^2}{ba} t.$$

(*) In the expression (1.5) for the Regge pole, α is bound to be the same for every process, while a —which is equal to the inelastic cross-section for the process we consider—changes from process to process; s_0 is another parameter that is not bound to be the same for any process.

The sign of this derivative, and therefore the dependence of the shrinking on the inelastic cross-section of the process considered depends on the value of ρ . From the values quoted above we see immediately that at machine energies $\rho < 1$ [$\rho > 1$ would be incompatible with the relatively large elastic cross-section found experimentally] and therefore we expect

$$\frac{dS^2}{da} > 0.$$

This means that at these energies the shrinking would be smaller for processes with smaller cross-sections, *i.e.*, smaller for πN than $N N$ scattering.

All the qualitative trends of the amplitude previously discussed seem to be in rather good agreement with the properties of πN and $N N$ diffraction scattering experimentally found recently ⁽²⁾. Considering the still rather limited amount of experimental data and the uncertain validity of our asymptotic formulae in the energy region attained up to now, we shall not take the risk of pushing the comparison to a more quantitative level.

* * *

We wish to thank Professors S. FUBINI and L. VAN HOVE for interesting discussions.

⁽²⁾ (a) Post-deadline paper at the A.P.S. Meeting, January 1963, presented by S. J. LINDENBAUM (unpublished); (b) BOLOGNA, CERN, LIVERPOOL and MICHIGAN COLLABORATION: private communication by G. GIACOMELLI; (c) S. BRANDT *et al.*: CERN and ECOLE POLYTECHNIQUE COLLABORATION (CERN preprint).

RIASSUNTO

Si calcola lo scattering di diffrazione dovuto all'ombra dei processi anelastici usando per questi processi ad energia elevata le previsioni del modello multiperiferico. Il polo di Regge, che apparirebbe nell'ampiezza elastica quando si considerino soltanto processi anelastici come stati intermedi nelle condizioni di unitarietà, domina soltanto in avanti. Per $t \neq 0$ gli effetti di scattering elastico multiplo diventano importanti. Si ottengono espressioni esplicite per l'ampiezza di diffrazione che differiscono notevolmente da quella prevista dal polo di Pomerančuk-Regge. In particolare si prevede che il restringimento del picco diminuisca con l'energia e che esso sia maggiore per l'urto $N-N$ che per l'urto $\pi-N$.