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HIGHER SYMMETRIES AND POMERANCHUK CONDITIONS

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A B S T R A C T

A general and rather simple method is given which allows to find the number of independent total cross-sections at high energies in the framework of a group of symmetry when Pommeranchuk conditions are used.

Recently, several symmetries for strong interactions have been proposed which were expected to hold - at least in first approximation - at high energies. Moreover, the Pomeranchuk¹⁾ theorem relates high-energy cross-sections (diagonal absorptive amplitudes) reducing possibly the number of independent cross-sections allowed by the symmetries. We want to propose here a simple method for obtaining the number of independent cross-sections in a given symmetry scheme, once the Pomeranchuk theorem has been used ; the straightforward way of finding it is, in general, complicated.

We are well aware of the fact that this problem is rather an academic one, especially because there is not much hope that any higher symmetric group could apply to total cross-sections. In any case, we believe that the mathematical structure of the problem is worthwhile to be investigated.

The Pomeranchuk theorem states that, at high energies, the total cross-section for AB collision is equal to that for $\bar{A}\bar{B}$ collision. Any symmetry groups particles in some multiplets. A and B can belong to the same multiplet or to different ones.

Referring to the absorptive forward amplitude, we rewrite the Pomeranchuk theorem as follows :

$$\langle AB | T | AB \rangle = \langle \bar{A}\bar{B} | T | \bar{A}\bar{B} \rangle \quad (1)$$

We must first discuss what the Pomeranchuk conditions mean when applied to higher representation of symmetries and, even first of all, the meaning of total cross-sections. Total cross-sections are just forward diagonal (elastic) absorptive amplitudes. In the problems we are accustomed to treat all the amplitudes between irreducible representations are needed in order to express the diagonal amplitudes $\langle A_i B_j | T | A_i B_j \rangle$ where the indices specify the position of the particles in the multiplets. This, however, may not be

the case for sufficiently complicated groups or representations. In such cases, the independent amplitudes necessary to express $\langle A_i B_j | T | A_i B_j \rangle$ can be different from the ones needed for $\langle \sum c_{il} A_l B_j | T | \sum c_{il} A_l B_j \rangle$. Both amplitudes are, in some way, related to total cross-sections: they refer to total cross-sections for initial states differently prepared. The amplitudes $\langle AB | T | AB \rangle$ - of which we are speaking and whose number we are looking for - refer to any state A , formed by any linear combination of members of a multiplet incident on any state B , giving rise again to A and B . In this sense, the relation (1) is sometimes called generalized Pomeranchuk theorem whose validity, however, is as well founded as the usual relations between well-defined incident states.

Let us now distinguish two different cases :

- a) A and $\bar{A}$ (and/or B and $\bar{B}$) belong to the same multiplet,
- b) A and $\bar{A}$ as well as B and $\bar{B}$ do not belong to the same multiplet.

In the case a), Eq. (1) shall give a relation between AB scattering amplitudes which could reduce the number of independent total cross-sections allowed by the symmetry. In the case b), Eq. (1) relates different amplitudes (AB and $\bar{A}\bar{B}$ scattering) giving, therefore, no reduction in the number of AB cross-sections. We shall thus concentrate our interest on the case a).

The main point of the method proposed is to consider the amplitude

$$\langle A\bar{A} | T | B\bar{B} \rangle \quad (2)$$

instead of

$$\langle AB | T | AB \rangle \quad (3)$$

Group theory ensures us that the numbers of independent amplitudes for processes (2) and (3) are the same (see Appendix) : this simply means that the crossing matrix is square. We stress that the study of the number of independent amplitudes allowed by any symmetry (including eventually the Pomeranchuk relations) can be done, as well, by using amplitudes (2) or (3) : this is a group theoretical property and has nothing to do with analytic continuation. For the amplitudes (2) the Pomeranchuk relation reads

$$\langle \bar{A}\bar{A} | T | B\bar{B} \rangle = \langle \bar{A}\bar{A} | T | B\bar{B} \rangle \quad (4)$$

or

$$\langle \bar{A}\bar{A} - \bar{A}\bar{A} | T | B\bar{B} \rangle = 0 \quad (5)$$

In order to deduce the consequences of Eq. (5), we propose the following simple procedure. Find first the irreducible representation of the group under consideration for the direct products $A \otimes \bar{A}$ and $B \otimes \bar{B}$ separately. It is then clear that the number N_p of independent cross-sections allowed by a particular group of symmetry and the Pomeranchuk conditions, is given by the number of transitions which can take place between symmetric irreducible representations of $A \otimes \bar{A}$ and irreducible representations of $B \otimes \bar{B}$.

Let us now obtain N_p for some particular symmetries.

I. THE ISOSPIN GROUP

Let us call a and b the isospins of multiplets A and B (to have A and $\bar{A}$ in the same multiplet, a has to be an integer). The irreducible representations characterized by the total isospin I contained in the products $a \otimes \bar{a}$ and $b \otimes \bar{b}$ are simply

$$\begin{aligned}
 a \otimes \bar{a} &= 0 \oplus 1 \oplus 2 \oplus \dots \oplus 2a \\
 b \otimes \bar{b} &= 0 \oplus 1 \oplus 2 \oplus \dots \oplus 2b
 \end{aligned}
 \tag{6}$$

Because the representations for $a \otimes \bar{a}$ with even (odd) I are symmetric (antisymmetric) and the transitions conserve I , N_p is given by the number of states with even I that appear simultaneously in $a \otimes \bar{a}$ and $b \otimes \bar{b}$. Let us call c the smaller of the two numbers a and b . Then N_p is given by the number of even integers which lie between 0 and $2c$. Therefore,

$$N_p = \begin{cases} c + 1 & \text{if } c \text{ is even}^*) \\ c + \frac{1}{2} & \text{if } c \text{ is odd.} \end{cases}
 \tag{7}$$

The isospin group we have discussed is particularly easy to treat because of the simplicity of the multiplication law, the simple correlation between the symmetry and the dimension of the irreducible representations appearing in the product decomposition of $A \otimes \bar{A}$, and the fact that in the product decomposition a representation of a given dimension enters only once.

Higher symmetries do not have such general simple properties and consequently we were unable to obtain a general formula analogous to (7). In the higher symmetries we shall study (SU_3 and G_2); we shall, therefore, analyze only the simpler representations which, by the way, are those which eventually could have physical meaning.

*) This formula has already been obtained with a different method by C.N. Yang (to be published).

II. THE SU₃ GROUP

In the framework of this group, two models were proposed.

i) The Sakata model ²⁾

Here (N, P, Λ) belong to the representation of dimension 3; mesons (π , K, $\bar{K}$, η^0) belong to the representation of dimension 8. The product decomposition is :

$$3 \otimes \bar{3} = 1 \oplus 8 \quad (8)$$

$$8 \otimes \bar{8} = 1 \oplus 8 \oplus 8' \oplus 10 \oplus \bar{10} \oplus 27 \quad (9)$$

As it is well known, representations 1, 8 and 27 are symmetric, while 8', 10 and $\bar{10}$ are antisymmetric. Therefore, in the case of meson-baryon (πp) cross-sections $N_p = 2$ ($1_B \rightarrow 1_M$, $8_B \rightarrow 8_M$ where B stands for baryons and M for mesons) instead of 3 if the Pomerenchuk conditions are not used $\int 1_B \rightarrow 1_M$, $8_B \rightarrow 8_M$, $8_B \rightarrow 8'_M$ that correspond in the AB channel to $3 \otimes 8 = 3 \oplus 6 \oplus 15$.

For meson-meson scattering we will have 3 independent amplitudes ($1 \rightarrow 1$, $8 \rightarrow 8$, $27 \rightarrow 27$ instead of 6 amplitudes, the transition $8 \rightarrow 8'$ in this case being forbidden by charge conjugation).

ii) The octet model ³⁾

The mesons, as in the Sakata model, are in the representation 8. Baryons (N $\wedge$ Σ Ξ) belong here also to the representation 8. The rule of multiplication is given by (9). Remembering, however, that for baryons we have no restriction to the symmetry of the representations, for meson-baryon scattering we find $N_p = 4$ corresponding to the transitions $1_B \rightarrow 1_M$, $8_B \rightarrow 8_M$, $8'_B \rightarrow 8_M$, $27_B \rightarrow 27_M$ instead of

7 amplitudes ($1_B \rightarrow 1_M$, $8_B \rightarrow 8_M$, $8_B \rightarrow 8'_M = 8'_B \rightarrow 8_M$, $8'_B \rightarrow 8'_M$, $10_B \rightarrow 10_M$, $10_B \rightarrow \bar{10}_M$, $27_B \rightarrow 27_M$) which would be present if the Pomeranchuk relations are not used.

Of course, the case of meson-meson scattering in the octet model is identical with the corresponding case of the Sakata model discussed in i). In a similar way, the representations of higher dimensions of SU_3 group could be analyzed.

III. THE G_2 GROUP

In this group ⁴⁾, mesons and baryons are usually assigned to the representation $7 \left[\begin{smallmatrix} \bar{1} & 1 \\ 0 & 0 \end{smallmatrix} \right] (\bar{1} K \bar{K})$ for mesons and $(N \Sigma H)$ for baryons, the law of multiplication being

$$7 \otimes 7 = 1 \oplus 7 \oplus 14 \oplus 27.$$

In the product decompositions 1 and 27 are symmetric while 7 and 14 are antisymmetric. This means that both for meson-meson and meson-baryon scattering $N_p = 2 \left[\begin{smallmatrix} 1 \rightarrow 1, 27 \rightarrow 27 \end{smallmatrix} \right]$ instead of 4 independent cross-sections which would occur if the Pomeranchuk conditions are not used $\left[\begin{smallmatrix} 1 \rightarrow 1, 7 \rightarrow 7, 14 \rightarrow 14, 27 \rightarrow 27 \end{smallmatrix} \right]$.

The results obtained for higher symmetries are summarized in Table 1 while for the isospin group N_p is generally given by Eq. (7).

g r o u p		Meson - Meson		Meson - Baryon	
		N	N _p	N	N _p
SU ₃	Sakata (3, 8)	6	4	3	2
	octet (8, 8)	6	4	7	4
G ₂	(7, 7)	4	2	4	2

T a b l e I

In column 1, the groups under discussion are listed while, in column 2, one finds the dimension of the representations to which baryons and mesons belong respectively. N represents the number of independent amplitudes allowed by the group, while N_p is the number of independent total cross-sections when also the Pomeranchuk conditions are used.

Finally, we want to stress that the clue of our method, indeed rather simple, was to use Eq. (5) for the Pomeranchuk conditions and work, therefore, on the representations $A \otimes \bar{A}$, $B \otimes \bar{B}$ instead of $A \otimes B$, $\bar{A} \otimes \bar{B}$. In order to calculate N_p, it was sufficient to know that the crossing matrix (i.e., the matrix which relates the two product representations) is square. This is ensured by group theory and, therefore, we did not need any detailed property of the crossing matrix to obtain the number of independent cross-sections.

A seminar given by Professor C.N. Yang drew our attention to this problem. We are also grateful to him for a useful discussion. It is a pleasure for us to thank Dr. H. Epstein for his interest in our work and for an illuminating conversation.

A P P E N D I X

Let us consider 4 particles a_1, a_2, a_3, a_4 and reactions among them of the form

$$\begin{aligned} a_1 + a_2 &\rightarrow \bar{a}_3 + \bar{a}_4 \\ a_1 + a_3 &\rightarrow \bar{a}_2 + \bar{a}_4 \\ a_1 &\rightarrow \bar{a}_2 + \bar{a}_3 + \bar{a}_4 \end{aligned} \quad \text{etc.} \quad (A1)$$

We shall show that the number of independent amplitudes N allowed by any (compact) group is the same for all reactions considered above. In particular

$$N_{a_1 a_2 \rightarrow \bar{a}_3 \bar{a}_4} = N_{a_1 a_3 \rightarrow \bar{a}_2 \bar{a}_4} \quad (A2)$$

which is the result used in the text (crossing matrix is square).

Let us use the following notations :

- $\mathcal{D}_i$ - the irreducible representation to which a_i belong
- χ_r - the character of any irreducible representation $\mathcal{D}_r$
- χ - the character of the product representation $\mathcal{D}_1 \otimes \mathcal{D}_2$
- χ' - the character of the product representation $\mathcal{D}_3 \otimes \mathcal{D}_4$.

$N_{a_1 a_2 \rightarrow \bar{a}_3 \bar{a}_4}$ shall be clearly given by

$$N_{a_1 a_2 \rightarrow \bar{a}_3 \bar{a}_4} = \sum_r n_r^r \quad (A3)$$

where n^r and m^r are the numbers of equivalent irreducible representations $\mathcal{D}^r$ which enter in the decomposition products $\mathcal{D}_1 \otimes \mathcal{D}_2$ and $\mathcal{D}_3 \otimes \mathcal{D}_4$ respectively. From the theory of characters

$$\begin{aligned} n_r &= (\chi, \chi_r) = \int \bar{\chi}(g) \chi_r(g) dg \\ m_r &= (\bar{\chi}', \chi_r) \end{aligned} \quad (A4)$$

where g indicates the element of the group.

From (A3) and (A4) and using the completeness of the characters, we get

$$\begin{aligned} N_{a_1 a_2} \rightarrow \bar{a}_3 \bar{a}_4 &= \sum_r (\chi, \chi_r) (\chi_r, \chi') = (\chi, \bar{\chi}') = \\ &= \int \bar{\chi}(g) \chi'(g) dg = \int \bar{\chi}_1(g) \bar{\chi}_2(g) \bar{\chi}_3(g) \bar{\chi}_4(g) dg. \end{aligned} \quad (A5)$$

Manifestly, the last term of (A5) is invariant under the permutations of indices necessary to study the set of reactions in (A1). Therefore, all N for different reactions (A1) shall be the same and equal to

$$N = \int \bar{\chi}_1(g) \bar{\chi}_2(g) \bar{\chi}_3(g) \bar{\chi}_4(g) dg. \quad (A6)$$

The result obtained here is valid even for a more general class of groups than the class considered here, as was pointed out to us by Dr. H. Epstein.

R E F E R E N C E S

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