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INTERMEDIATE BOSON PRODUCTION BY NEUTRINOS

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We consider the production, by neutrino's incident on nuclei, of the hypothetical intermediate bosons W of weak interactions. The treatment of Lee, Markstein, and Yang¹⁾ has been modified in two ways. For the coherent process we adopt more realistic nuclear form factors, and in the incoherent process the exclusion principle is allowed for. As a result both contributions to the cross section are substantially reduced. Always K will be taken to be zero (no an. mom.).

The coherent process.

The coherent cross section $\sigma_Z(\text{coh})$ depends on the assumed nuclear charge distribution. Lee et al used the exponential form

$$\rho(r) \propto e^{-\sqrt{12} r/a} \quad (1)$$

with root mean square radius

$$a = \sqrt{3/5} 1.3 \times 10^{-13} A^{1/3} \text{ cm}$$

To test the computer programme we repeated their calculations for Fe and obtained their results. Calculations with this distribution were then made for Cu^{63.5} and Al²⁷, which are of particular interest for the CERN experiment. Finally we adopted instead the fermi charge distribution²⁾

$$\rho(r) \propto \{1 + e^{-(r-R)/b}\}^{-1} \quad (2)$$

with

$$\begin{aligned} R &= 1.07 A^{1/3} 10^{-13} \text{ cm} \\ b &= .568 10^{-13} \text{ cm} \end{aligned}$$

The old and revised form factors, which are the fourier transforms of the charge distribution, are compared in fig 1. The fermi form factors fall off more rapidly with momentum transfer. The cross sections are decreased accordingly, as shown in fig 2 and fig 3 for boson masses m of 0.6 and 1.0 proton masses.

The incoherent process.

For the total non mesonic production cross section Lee et al. add to the coherent part $\sigma_z(\text{coh})$ an incoherent contribution

$$Z \{ \sigma_p - Z^{-2} \sigma_z(\text{coh}) \} \quad (3)$$

where σ_p is the cross section for a single proton. This formula can be derived on the assumption that the nucleons in the nucleus do not move and have uncorrelated positions. Here we still ignore the fermi motion of nucleons but allow for the correlation implied by the exclusion principle.

When the nucleus is treated as an ideal fermi gas, rather than an ideal classical gas, the formula (3) is replaced by

$$Z \sigma_p' \quad (4)$$

where σ_p' is obtained from σ_p by averaging over the fermi motion and excluding final states in which the recoiling proton momentum is within the occupied sphere. If the initial nucleon velocities are ignored the effect of the exclusion is just to introduce in the differential cross section a factor

$$\langle \Theta(K_f - |\vec{k} - \vec{q}|) \rangle_{\text{av}}$$

where Θ is the step function, $\vec{q}$ is the 3-momentum transfer, K_f the fermi momentum, and averaging is over all initial proton momenta $\vec{k}$. This gives the well-known reduction factor

$$\begin{aligned} R(|\vec{q}|^2) &= \frac{3}{2} \frac{|\vec{q}|}{2K_f} - \frac{1}{2} \left(\frac{|\vec{q}|}{2K_f} \right)^2 & \text{for } |\vec{q}| < 2K_f \\ &= 1 & \text{for } |\vec{q}| \geq 2K_f \end{aligned} \quad (5)$$

Now we have for nucleons initially at rest

$$|\vec{q}|^2 = q^2 + (q^2/2m_p)^2 \quad (6)$$

where m_p is proton mass and q^2 is the squared 4-momentum transfer. The second term is quite small, but has in fact been retained in the calculations. To allow for the exclusion principle then we merely modify as follows the proton form

factors³⁾ already occurring in the formulae:

$$F(Q^2) \rightarrow F(Q^2) \left\{ R(Q^2 + (Q^2/2m_p)^2) \right\}^{1/2} \quad (7)$$

Results are presented in fig 4, with $K_f = .284 m_p$. The effect of exclusion is substantial, the fractional reduction increasing with energy. Indeed there can be no effect at all until the kinematically permitted minimum momentum transfer is less than $2K_f$. For $E_\nu \gg m_W + m_\mu$

$$Q_{\min} \approx (m_W + m_\mu)^2 / 2E_\nu$$

Thus the exclusion principle can enter when

$$\frac{E}{m_W + m_\mu} \gtrsim \frac{m_W + m_\mu}{4K_f} \quad (8)$$

That the effect is so considerable when the energy is high shows the importance of small momentum transfers (fig 4).

We have ignored nucleon motion; this may not be very serious since in the gas model $|\vec{k}|/m_p \lesssim 3$. However eventually calculations must be made allowing for it. In the ideal gas model this would be in principle straightforward. We have not carried it out because the extra integration would strain the capacity of the available computer (IBM 709). One effect to be expected is a lowering of the threshold energy. In the region affected by exclusion we would guess that the cross section be further reduced. The momentum transfer to the nucleon has necessarily a forward component; the nucleons moving initially towards the neutrino, which would otherwise be the most effective, are then those most affected by exclusion. However the adequacy of the free gas model is questionable. It appears from our results that transitions to states of low excitation energy are important, and then a more detailed account of nuclear structure may be essential. For heavy nuclei we should at least introduce an effective mass, and perhaps allow for Coulomb effects⁴⁾. For these reasons we think it may be difficult to obtain quantitatively reliable results for the incoherent process.

Total cross section.

Finally in the table we compare total (non mesonic) cross sections computed as described above and with the original prescriptions of Lee et al.

TABLE OF TOTAL CROSS SECTIONS

E_ν	Al, 1 m _p		Cu, 1 m _p		Al, 0.6 m _p		Cu, 0.6 m _p	
	σ_{tot}	$\sigma_{\text{tot}}(\text{old})$	σ_{tot}	$\sigma_{\text{tot}}(\text{old})$	σ_{tot}	$\sigma_{\text{tot}}(\text{old})$	σ_{tot}	$\sigma_{\text{tot}}(\text{old})$
2	1.95		4.34		58.	107.3	118.4	211.
3	26.2	32.8	58.7	65.5	210.	364.	442.	795.
4	66.5	106.	144.	211.	451.	727.	1083.	1759.
5	121.	210.	251.	421.	762.	1214.	2027.	3138.
6	194.		396.		1118.		3203.	
7	285.		603.		1514.		4610.	
8	399.		880.		1950.		6233.	
9	529.		1230.		2445.		8100.	
10	680.		1660.		2984.		10250.	

E_ν = neutrino energy in GeV. All cross sections are in 10^{-38} cm² per nucleus.

FOOTNOTES AND REFERENCES

- 1) T.D. Lee, P. Markstein, and C.N. Yang, Phys. Rev. Letters 7 (1961) 429.
- 2) See for instance R. Herman and R. Hofstadter, High Energy Electron Scattering Tables, Stanford University Press 1960. In the calculations we took the fermi form factor to be zero for $Q > 500$ MeV/c.
- 3) Again, to check our programme, we first did the calculation with the same form factors as used in Ref.1, and obtained their results. Next we used more recent data (Proc. Int. Conf. on High Energy Physics, CERN 1962, p. 753) but no substantial difference resulted, presumably due to the fact that the changes below $Q = 600$ MeV/c are small.
- 4) For the coherent process some preliminary numerical integration of Coulomb effects based on the analysis given in a previous paper (M. Veltman, Physica 29 (1963) 161) has been made. The results indicate for low Z materials a decrease in the total cross section of the order of 0.5 Z percent, Z being the nuclear charge number, for neutrino energies around 6 GeV.

Fig 1.
Exponential and fermi form factors for Al^{27} and $\text{Cu}^{63.5}$.

6.

Fig 2

Coherent cross sections $\sigma_z(\text{coh})$ for Al^{27} and $\text{Cu}^{63,5}$ with boson mass $m_W = 0.6 m_p$.

Fig 3.

Coherent cross sections $\sigma_z(\text{coh})$ for Al^{27} and $\text{Cu}^{63.5}$ with boson mass $m_W = 1.0 m_p$.

cross section in 10^{-38} cm^2 per proton.

Fig 4.

Incoherent cross section per proton σ'_p for two boson masses, compared with the Lee et al. expression $\{\sigma_p - Z^{-2}\sigma_Z(\text{coh})\}$ for Al ($Z = 13$, exponential form factor) and with the cross section for a single proton σ_p .