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THE NEUTRON STRENGTH FUNCTION AND THE SHELL MODEL

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A B S T R A C T

The projection operator-equivalent potential method is employed for the study of neutron widths and the neutron strength function. Assuming that the residual potential can be written as a sum of two particle potentials it can be shown with the aid of another plausible assumption that the width can be written as a product of two factors. One of these is the square of the matrix element of V_R between the open channel wave function and a "shell model" two particle-one hole state. This factor which plays a crucial role measures the probability that in the first scattering of the incident particle a transition to the two particle-one hole state occurs. It contains the fluctuations in the width which accompany changes in the target nucleus. The second factor, giving the probability that the compound nucleus wave function contains the two particle-one hole state, decreases as the difference between the energy of the compound nucleus and the energy of the two particle-one hole state. This factor has a Lorentzian form with a half width Δ corresponding to the lifetime of the two particle-one hole state. These results are generalized to include the case where the energies of several two particle-one hole states fall close to each other. Here it is found that the giant resonance for the widths will now have a substructure which should be observable. The strength function is found to depend upon the average of the transition probabilities to the two particle-one hole states, the number of these states and Δ . The number is sharply limited by the requirements that angular momentum and parity be conserved in the transition and that energy be conserved to within an error equal to Δ . If the residual potential is expanded in multipoles, keeping terms up to and including the quadripole, it is found that for the most part in the region $40 \leq A \leq 64$ the only possible transitions involve the monopole, between $A = 68$ and 85 , both dipole and quadripole terms are effective as well while between 85 and 130 only the quadripole term can induce transitions. This behaviour enables one to obtain a fit to the data by means of three parameters: the average matrix elements squared for the monopole, dipole and quadripole parts of the residual potential. The resulting fit is excellent providing a description not only of the deviations of the strength function from the optical model result but also of the fluctuations in the strength function as the target nucleus is changed. Predictions of the dependence of the strength function for various isotopes become possible. The success of these calculations demonstrates the essential importance of the "first" collision which the incident particle can undergo.

I. Introduction

The complex potential of the optical model provides only a crude description of the interaction between a nucleon (or other projectile) and a target nucleus. In spite of the many successful applications of the model to elastic and inelastic scattering as well as to direct interactions, there are still many areas in which the model is inadequate even when augmented by the inclusion of nuclear deformations ¹⁾, coupling to excited states ²⁾, spin-orbit coupling ³⁾, surface absorption ⁴⁾, and non-local potentials ⁵⁾. An example is provided by the *s*-wave strength function. In Fig. 1 we show some of the presently available data and the fit obtained with a simple complex potential model ⁶⁾. Notice the anomalous dip in $\langle \sigma_s^0 \rangle / D$ near the mass number *A* equal to 115 and the less well-known rise near *A* equal to 70. Another notable feature of the experimental results is their fluctuation as we move from one target nucleus to another. The dip may be explained by concentrating the absorption on the surface ⁴⁾, but it is apparent that many fluctuations away from the "average" given by the optical model will remain. Sugie ⁷⁾ as well as Lane, Lynn, Melkanoff and Rae ⁸⁾ have suggested that this dip is a consequence of the actual details of the structure of the target nucleus and of the compound system formed in the reaction.

In order to obtain a greater systematic insight into this question it is necessary to develop our representation of the neutron-nucleus interaction beyond the first stage given by the optical model. Toward this end, we shall employ a qualitative picture of how the nucleon-nucleus interaction proceeds ⁹⁾, which as we shall see fits quite naturally into the projection operator reaction theory formalism ^{10), 11)}. This description amounts to a decomposition of the wave function for the compound system, consisting of target nucleus plus nucleon, into a hierarchy of more and more complex excitations of the target nucleus. The simplest part of this wave function consists of the target nucleus in the ground state plus the associated incident nucleon ^{*}). It is this substate which we

^{*}) For simplicity we consider only low energy incident projectiles so that elastic scattering is the only reaction which is energetically allowed.

shall refer to as the one-particle state upon which attention is focussed in 10), 11). As one can see from these references the interaction between nucleon and nucleus consists of two parts. The first part, the average interaction, has the important property of leaving unchanged the state of excitation of the target nucleus. This average interaction is, of course, very closely related to the shell model potential. The second part, usually referred to as the residual potential V_R , does give rise to excitations and is responsible for the generation of substates in which the target nucleus is excited. Thus, the one-particle state acquires a lifetime and a width, the latter being intimately related to the imaginary part of the complex optical potential. The nature of the excited states depends critically on V_R and we shall make the assumption that V_R is composed of two particle potentials ^{*)}. It is clear that successive applications of V_R to the one-particle state will produce two particle-one hole states, three particle-two hole states and so on to excitations of greater and greater complexity ^{**)}. Thus, the wave function for the compound system of target nucleus plus projectile may be decomposed into the one-particle state wave function plus wave functions describing these states of increasingly complex structure.

A compound nucleus is formed whenever the time spent by the system in the excited state configurations, without returning to the one-particle state, is of relatively long duration. In other words, there are particular combinations of excited states which make up an almost stationary state. In Refs. 10), 11) we show that if we make these states stationary by reducing their coupling to the one-particle state to zero, the resonances in the scattering will occur whenever the energy of the system is close to the eigen-energies of these stationary states. We shall call these states many-particle stationary states since in all their components more than one particle is excited. The strategic importance of the transition from the one-particle state to the two particle-one hole state should be noted. It is through this transition only that a compound nuclear state can be created or destroyed. If this transition probability is small,

*) The omission of collective terms restricts the present development to nuclei for whom the shell model description is particularly appropriate. Their inclusion would not change the qualitative nature of our conclusions. The extension to deformed nuclei will not be considered here.

**) More precisely three, four etc. quasi-particle states.

the width of the one-particle state will be correspondingly small, as will be the amplitude of the more complicated excitations which are generated from the two particle-one hole state. The probability of forming a compound nucleus will also be small and we may expect the above transition probability to be closely related to the resonance width, Γ , for particle absorption. In essence, the two particle-one hole states occupy the doorway to the compound state. It is only through these states that the residual potential can couple the compound state to the outgoing state of the system.

A first step in the analysis of the above picture of the wave function for the compound system was made in 10), 11) where particular attention was focussed on the one-particle state wave function and the effective Hamiltonian and Schroedinger equation which it satisfies was derived. Including the effects of transitions to excited states gives rise to resonances in the scattering which together with the direct interaction term (in the elastic scattering case under discussion, the potential scattering) make up the scattering amplitude. An expression for Γ is obtained in terms of a matrix element for the transition under the influence of V_R from the potential scattering wave function to the many-particle stationary state described above.

In this paper we take the next step in which our attention will now be focussed on the two particle-one hole state with zero coupling to the one-particle state. Again as in 10), 11) the technique involves the elimination of the more complex states and the effective Hamiltonian and Schroedinger equation are derived in Section II. The eigenfunctions of this Hamiltonian provide a representation of the many-particle stationary state. These solutions and the equation they obey are discussed, using the same kind of assumptions as were employed in 10), 11), in the discussion of resonances and potential scattering. We obtain an understanding of how they can give rise to the highly dense spectrum characteristic of the compound nucleus and simultaneously a shell model approximation for the strength function in spite of the many-particle nature of these

states. This discussion also provides a basis for the understanding of the giant resonance in the strength function. It suggests a number of additional phenomena which hopefully could be observed experimentally.

In Section III we derive an expression Eq. (III.5) for the strength function. It is possible to give a simple interpretation of this formula based on the "doorway" concept, an interpretation which shows that it has a validity which is more extensive than the special arguments employed in Section II might indicate. We shall present this argument now and then go on to derive a rough approximation to Eq. (III.5) which we then apply to the experimental data obtained for s neutrons for target nuclei $40 \leq A \leq 130$. A more detailed evaluation of the strength function for selected nuclei is made by Shakin¹²⁾ in the article which follows this one. In Section II, the general theory is developed; in Section III it is applied to the calculation of the strength function; Section IV contains some concluding remarks.

Formula (III.5) is

$$\langle \Gamma_s \rangle \cong 2\pi \sum_n \frac{D_n}{\Delta_n} \left| \langle \psi_n^{V_R} \chi^{(+)} \rangle \right|^2 \quad (\text{I.1})$$

Here $\langle \Gamma_s \rangle$ is the average width, ψ_n the wave function describing the two particle-one hole state, V_R the residual potential, $\chi^{(+)}$ the one particle no hole state, i.e., the wave function describing the motion of the incident particle in the average potential with the target nucleus. D_n is the average distance in energy between compound nuclear states which are created through the two particle-one hole state whose width (or inverse lifetime) is Δ_n .

Formula (I.1) can be understood on the basis of the following simple considerations. The absolute square of the matrix element $\langle \psi_n^{V_R} \chi^{(+)} \rangle$ can be interpreted as the transition probability from the two particle-one hole state ψ_n towards the "outside" which is the one particle-no hole state $\chi^{(+)}$.

In the latter state the particle is in fact "outside" the nucleus, except at the very beginning or at the very end, depending on in what direction one interprets $\chi^{(+)}$. Hence, $|\langle \psi_n^V \chi^{(+)} \rangle|^2$ is the neutron width of the state ψ_n . Let us for a moment assume that there is only one state ψ_n within an energy interval of the order Δ_n , that is, the sum in (I.1) has only one term. Then, we argue as follows: the average neutron width of a compound state must be the neutron width of the "doorway" state ψ_n , multiplied with the relative probability that the state ψ_n is realized in the compound state. Now, the "period of motion" in the compound state is $\hbar/D_n$ (see Blatt and Weisskopf, p. 387); this is the time needed to go through all configurations of the compound state. The lifetime of the configuration ψ_n is $\hbar/\Delta_n$ per definition of Δ_n . Hence, the relative probability of ψ_n in the compound state must be about D_n/Δ_n . If there is more than one "doorway" state available, the neutron width must, assuming the absence of interference, be the sum of the width for each "doorway", and we get expression (I.1).

Formula (I.1) implies the existence of the giant resonance in the strength function. Two conditions must be satisfied. First the one-particle wave function $\chi^{(+)}$ must be large inside the nucleus. Secondly there must exist a two-particle, one-hole state ψ_n which can be reached via the residual potential with approximate energy conservation. The square of the width of the giant resonance is given by the harmonic mean of the squares of the single particle widths associated with $\chi^{(+)}$, the one-particle state, and with the state ψ_n , the two particle-one hole state. Thus, the giant resonance width is less than either the single particle width or Δ_n . In principle a measurement of the deviation of the giant resonance width from the single particle width would yield the width of the two particle-one hole state.

Result (I.1), while moderately rigorous, is still too elaborate for comparison with experiment and we shall employ a further approximation:

$$\frac{\langle \Gamma_s \rangle}{D} \approx \frac{2\pi}{\Delta} \sum_n \left| \langle \psi_n V_R \chi^{(+)} \rangle \right|^2 \quad (\text{I.2})$$

where D is the average energy level spacing. The comparison between Eqs. (I.1) and (I.2) may be regarded as providing a definition of Δ . The pragmatic importance of (I.2) lies in its simpler representation of $\langle \Gamma_s \rangle$ but of course this may prove to be eventually inadequate and one will need to return to the more complex (I.1). Shakin's analysis¹²⁾ is based essentially on (I.2) in which he treats Δ as an empirical parameter and evaluates the matrix elements employing a quasi-particle picture of modern shell model theory. In this paper we shall, however, take further liberties with (I.2).

It is possible to extract the significant elements of Eq. (I.2) and so construct a very rough representation of it. The dependence of the matrix element on $\chi^{(+)}$ and on the width of the state ψ_n leads to the giant resonance behaviour of $\langle \Gamma_s \rangle/D$. More generally, we would expect the average behaviour, i.e., that part which does not depend on the detailed properties of ψ_n , to follow from these gross features so that the dependence of $\langle \Gamma_s \rangle/D$ on $\chi^{(+)}$ and the width of ψ_n must be contained in the strength function calculated from the optical model, $[\langle \Gamma_s \rangle/D]_{\text{OPT}}$. Another gross feature of (I.2) and a decisive one, as we shall see, is the number of two-particle, one-hole states ψ_n . These must, however, be classified according to type, and in doing this we take into account, to some extent, the properties of V_R .

Let us now make explicit use of the assumption that V_R can be represented as a superposition of two-body potentials. In addition, assume these potentials to be spin-independent so that V_R can be expanded in terms of purely spatial multipoles whose order we designate by L . Finally, we shall

also assume that we can ascribe orbitals to each of the particles and to the hole in the two particle-one hole state $\psi_n \cdot V_R$, then, has the effect of de-exciting the incident nucleon and exciting a nucleon in the target nucleus. For thermal neutron resonances upon which we will now center our attention, the incident neutron as well as the target nucleon, make transitions to bound orbitals of the compound system. With the above assumptions on V_R and ψ_n and for thermal neutrons whose orbital angular momentum, ℓ is zero, it follows that the matrix elements in $\langle \psi_n V_R \chi^{(+)} \rangle$ involving different values of L will not interfere. Thus expression (I.2) for the strength function can be decomposed into contributions, each involving a different multipole order. For each L there will be a number ν_L of different ψ_n possible. We can, of course, define for each L an average square matrix element averaged over the possible ψ_n . But in addition we now make the assumption that this average, once the optical model effect, discussed earlier, is factored out, is independent of the target nucleus. In other words, we assume that the major elements in the fluctuations of the strength function in going from one target nucleus to another are the number of two particle-one hole states which can be generated by a given multipole order and the multipole order itself, while the changes in the average square matrix element for a given multipole order are relatively minor. This seems to be borne out by Shakin's calculation¹²⁾ which shows that the matrix elements are random, the decisive factors being the density of two particle-one hole states. Finally, the expression of V_R in multipoles presumes convergence and therefore that only a finite number of multipoles need to be included in considering the experimental data. We have kept only a minimal number of terms, $L = 0, 1, 2$ and have found these few sufficient and self-consistent in the sense that the a^L decrease quite rapidly with L .

The expression (I.2) now becomes

$$\frac{\langle \Gamma_s \rangle}{D} \cong (\nu_0 a^0 + \nu_1 a^1 + \nu_2 a^2) \left[\frac{\langle \Gamma_s \rangle}{D} \right]_{\text{OPT}} \quad (\text{I.3})$$

where a^L gives 2π times the average square matrix element for multipole order L . This formula is the one we shall compare with experiment. The parameters Δ , a^0 , a^1 , and a^2 are determined empirically. With only these four parameters a good fit of the data is obtained from A equal to 40 to $A = 130$ where the non-sphericity of the target nuclei becomes an essential feature. It should be emphasized that not only is the average behaviour of $\langle \Gamma_s \rangle / D$ predicted, but also its fluctuations as the target nucleus is changed.

The reason for the success of the simple expression (I.3) can be inferred from Tables I and II in which ν_L is given for the cases in which the target nucleon making the transition is a neutron or a proton respectively. The shell model energy levels and associated quantum numbers employed were those of Green et al.¹⁷⁾ Only those transitions are considered for which angular momentum and parity are conserved. Those states, ψ_n , which have the same total angular momentum but where the angular momentum of the excited particle and that of the hole have combined to form differing intermediate angular momentum are counted as separate states. Finally, the energy must be conserved in the transition to within an error Δ . Our results are not sensitive to Δ , (we actually took $\Delta = 2$ MeV), unless it is varied over rather wide limits. From the results in Table I and II we see that for A between 40 and 64 the $L = 0$, the monopole is dominant for the most part; from A equal to 68 to A equal to 84 many $L = 1$ and $L = 2$ multipole transitions are possible. However, starting at $A = 88$ and on through $A = 130$ only $L = 2$ transitions are possible. The rough correlation with the structure in Fig. 1 is immediately clear. The peak around $A = 50$ is associated with the $L = 0$ monopole, the many large values of $\langle \Gamma_s \rangle / D$ in the region 70 to 80 are connected with the large number of $L = 1$ and $L = 2$ transitions possible while the dip stretching from 90 on through 130 is associated with the $L = 2$ multipole, indicating that a^2 must be considerably smaller than a^1 . This correlation is most remarkable and undoubtedly of great significance.

It is thus not surprising that a good fit can be obtained. The experimental values for $\langle \Gamma_s \rangle / D$ shown in Fig. 1 were divided by the optical model value as given by Campbell et al.⁶⁾. This ratio R is shown in Fig. 2. The constant a^0 was taken to be one since the optical model gives the experimental value at $A = 40$. The others, a^1 and a^2 , were chosen so as to give the best over-all fit from $A = 40$ to $A = 130$. The best fit was obtained for

$$a^0 = 1.0, \quad a^1 = 0.15, \quad a^2 = 0.05, \quad \Delta = 2 \text{ MeV} . \quad (\text{I.4})$$

The results are shown in Fig. 1 and Fig. 2. The excitation of both neutrons and protons in the target nucleus was included in the above, i.e., the residual potential for n-p and n-n interactions were taken to be of the same order of magnitude. Poor results were obtained, particularly for the maximum in R around $A = 110$, if the n-p interaction was assumed zero. After the parameters in (I.4) were fixed data on the tin isotopes was obtained from Oak Ridge¹⁸⁾. The comparison of theory and experiment is shown in Fig. 3. Shakin's levels were used. Agreement with experiment within this rather large experimental error is good.

The numerical values (I.4) are sensitive to the particular optical model employed so that they need not be directly meaningful. But the manner in which the behaviour of the experimental data is so closely reproduced is very significant. It indicates that the fluctuations in $\langle \Gamma_s \rangle / D$ as A and Z change follow from the concomitant changes in the shell model description of these nuclei. The quite good agreement obtained with the very crude formula Eq. (I.3) is very encouraging and its further application is now in order. But even more, the elaborate calculations required to evaluate Eq. (I.2) for a realistic shell model of a nucleus become definitely worthwhile. These have been carried out by Shakin¹²⁾ and are reported on in the following paper.

II. General Theory

It will be useful to review some of the material in 10), 11) in order to fix our notation and to give some of the pertinent results. The wave function for the total system Ψ is broken up in two parts, viz.

$$\Psi = P \Psi + Q \Psi \quad (\text{II.1})$$

$$Q \equiv 1 - P.$$

P is a projection operator which projects out of Ψ a part whose asymptotic dependence is identical with that of Ψ and thus contains all the experimentally observable effects. When the only possible reaction is elastic scattering, the asymptotic wave function is $A v(r_0) \psi_0(r_1, \dots, r_A)$ where A is the anti-symmetrization operator and ψ_0 is the wave function for the target in its ground state. As is emphasized in Ref. ¹¹⁾, P is not unique. Two possibilities were discussed there while Shakin ¹³⁾ has employed still another. In this paper, we shall not need to specify P , except for some general remarks to be given below. The Schroedinger equation for Ψ can be written then as a coupled equation for $P \Psi$ and $Q \Psi$ as follows

$$[E - H_{PP}](P \Psi) = H_{PQ}(Q \Psi) \quad (\text{II.2})$$

$$[E - H_{QQ}](Q \Psi) = H_{QP}(P \Psi) \quad (\text{II.3})$$

where

$$PHP \equiv H_{PP}, \quad PHQ \equiv H_{PQ}, \quad \text{etc.}$$

Since by choice $Q\Psi$ contains no outgoing wave solutions, we can solve (II.3) for $Q\Psi$:

$$Q\Psi = \frac{1}{E-H_{QQ}} H_{QP} P\Psi \quad (\text{II.4})$$

and now eliminate $Q\Psi$ from Eq. (II.2). In this way one obtains an effective Schroedinger equation for $P\Psi$

$$\left[E-H_{PP}-H_{PQ} \frac{1}{E-H_{QQ}} H_{QP} \right] (P\Psi) = 0 \quad (\text{II.5})$$

The narrow compound nuclear resonances arise from the term $H_{PQ}(E-H_{QQ})^{-1}H_{PP}$ in the effective Hamiltonian which varies rapidly with the energy. An isolated resonance occurs at an energy near an isolated singularity in the propagator $(E-H_{QQ})$, i.e., near an eigenvalue of H_{QQ} . Let

$$(\mathcal{E}_S - H_{QQ}) \bar{\phi}_S = 0 \quad (\text{II.6})$$

If $\bar{\phi}_S$ is a bound state and is normalized we may rewrite (II.5) as follows

$$(E-H') P\Psi = \frac{H_{PQ} \bar{\phi}_S \langle \bar{\phi}_S H_{QP} P\Psi \rangle}{E - \mathcal{E}_S} \quad (\text{II.7})$$

where

$$H' = H_{PP} + H_{PQ} \frac{\bar{\phi}_S}{E-H_{QQ}} H_{QP} \quad (\text{II.8})$$

and

$$\begin{aligned} Q_s \bar{\phi}_s &= 0 \\ Q_s \bar{\phi}_t &= \bar{\phi}_t \quad t \neq s. \end{aligned} \quad (\text{II.9})$$

H' varies slowly with the energy for E near $\mathcal{E}_s$; all the rapid variation of the effective Hamiltonian in (II.5) is now explicitly given by the term in the right-hand side of Eq. (II.7). Eq. (II.7) is "solved" in 10). The consequent transition amplitude $\mathcal{J}$ consists of two terms

$$\mathcal{J} = \mathcal{J}_P + \mathcal{J}_R. \quad (\text{II.10})$$

The first term gives the scattering, usually called the potential scattering, which follows from the solution of the Schroedinger equation

$$(E - H') \chi^{(+)} = 0. \quad (\text{II.11})$$

Empirically H' varies very slowly with the energy and it is a reasonable extrapolation, again borne out empirically, to equate it to the average potential for a single nucleon moving in the field of the target nucleus. The validity of this association depends critically upon the proper choice of the projection operator P . For our present purposes we need only assume that an appropriate operator exists; the operator P_I of Ref. 11) or the one employed in 13) should be close. The second term, $\mathcal{J}_R$, gives the resonance scattering amplitude. From it one can extract an expression for the

$$\Gamma_s = 2\pi \left| \langle \bar{\phi}_{sQP}^H \chi^{(+)} \rangle \right|^2 = 2\pi \left| \langle \chi^{(-)}_{HPQ} \bar{\phi}_s \rangle \right|^2 \quad (\text{II.12})$$

Since H' is the average potential and since H_{PQ} has the effect when operating on $\chi^{(+)}$ of generating states in which the target nucleus is excited (its expectation value with respect to the ground state is zero) it is reasonable in (II.12) to equate H_{PQ} and the residual potential V_R

$$\Gamma_s = 2\pi \left| \langle \bar{\phi}_s V_R \chi^{(+)} \rangle \right|^2. \quad (\text{II.13})$$

Recall that the $\bar{\phi}_s$ is an eigenstate of H_{QQ} with an eigenvalue $\mathcal{E}_s$ very close to the energy E of the system and the state $\chi^{(+)}$. This completes the review of the material in Refs. 10), 11) which we require here.

The next step in our analysis requires an investigation of the operator H_{QQ} and its associated bound state eigenfunctions $\bar{\phi}_s$ and their corresponding eigenvalue spectrum. As indicated in the introduction, our method for this problem involves a reapplication of the procedure above leading from (II.1) to (II.13). There we focussed our attention on the obviously most relevant sub-state of the system, the entrance channel in which the target nucleus is in its ground state. We found that the effective Hamiltonian can be broken up into two parts of which one H' , a slowly varying function of the energy near $\mathcal{E}_s$, gave the average interaction of a nucleon moving in the field of the target nucleus in the ground state. Note that H' differs from H_{PP} . H' includes H_{PP} as well as all the possible virtual transitions from the ground state and back to it except for those which are nearly on the energy shell. These latter give rise to the resonances.

The analogous dissection will now be performed for the states $\bar{\phi}_s$. The substates analogous to the entrance channel of the preceding paragraph are those excited states which are generated by the application of V_R to the entrance channel wave function. Taking V_R as consisting of a sum of pair

interactions each depending only upon the co-ordinates of two nucléons, we find that the states so generated are, in the language of many-body theory, two particle-one hole states or more accurately three quasi-particle states. This assumption on V_R is expected to be most valid near closed shells and for spherical undeformed nuclei. Note that these limitations on V_R can easily be lifted without affecting the arguments to be given below in an essential fashion.

We now isolate these states with the aid of the projection operator P_2 which is unity when operating on a two particle-one hole state and is zero for all other states, such as three particle-two hole states etc., which can be excited by repeated applications of V_R . We obtain the equation satisfied by $P_2 \bar{\phi}_s$, i.e., by that two particle-one hole component of the stationary state $\bar{\phi}_s$. It is

$$\left[\mathcal{E}_s - P_2 H_{QQ} P_2 - P_2 H_{QQ} Q_2 \frac{1}{\mathcal{E}_s - Q_2 H_{QQ} Q_2} Q_2 H_{QQ} P_2 \right] (P_2 \bar{\phi}_s) = 0 \quad (\text{II.14})$$

or

$$[\mathcal{E}_s - H_{\text{eff}}] P_2 \bar{\phi}_s = 0.$$

Note also that the matrix element (II.13), in virtue of our assumption on V_R becomes

$$\Gamma_s = 2\pi \left| \langle (P_2 \bar{\phi}_s)_{V_R} \chi^{(+)} \rangle \right|^2. \quad (\text{II.15})$$

Each of the bound states of (II.14) is associated with a compound nuclear level, and it is immediately clear that the large density of these levels

must come from the energy dependent part of the effective Hamiltonian in (II.14). Physically, this is to be expected since a considerable fraction of the wave function for a compound nuclear level should involve the more complex states such as the three particle-two hole state, etc. The effect of these terms is contained in the energy dependent part of the effective Hamiltonian. It is the many degrees of freedom which are thus available which is reflected in the dense spectrum of the levels of the compound nucleus.

We now show qualitatively that bound states of (II.14) will be associated with each of the singularities of the propagation factor $(E - Q_2 H_{QQ} Q_2)^{-1}$, when considered as a function of E . Between the poles of the propagation factor the associated potential will vary from $(+\infty)$ to $(-\infty)$, thus providing a complete range in potential strength and eigenvalues. To obtain the appropriate eigenvalue for Eq. (II.14) one needs to find those eigenvalues which are equal to E (or vice versa). This situation will recur between the next pair of poles of the propagation factor and in this way the compound nuclear spectrum is generated.

Further conclusions can be drawn if we separate off the terms in the effective Hamiltonian which are singular near $\mathcal{E}_{s*}$. We shall assume for simplicity that only the nearest pole is important. Let the eigenvalues and eigenfunctions of $Q_2 H_{QQ} Q_2$ be ϵ_α and ϕ_α respectively

$$Q_2 H_{QQ} Q_2 \phi_\alpha = \epsilon_\alpha \phi_\alpha \quad . \quad (\text{II.16})$$

Denote the eigenvalue closest to $\mathcal{E}_s$ by ϵ_σ and let p_σ and q_σ be the projection operators for the associated state ϕ_σ :

$$\begin{aligned} p_\sigma \phi_\tau &= \delta_{\sigma\tau} \phi_\tau \\ q_\sigma \phi_\tau &= (1 - \delta_{\sigma\tau}) \phi_\tau \end{aligned}$$

*) The effects of several poles can be readily included in the discussion. However, there is no appreciable change in our major conclusions so that there is no need to include this complication.

. Then Eq. (II.14) may be rewritten as follows

$$\left[\mathcal{E}_s - H'_{QQ} \right] P_2 \phi_s = \frac{P_2 H_{QQ} Q_2 \phi_\sigma \langle \phi_\sigma Q_2 H_{QQ} P_2 \phi_s \rangle}{\mathcal{E}_s - \epsilon_\sigma} \quad (\text{II.17})$$

where

$$H'_{QQ} = P_2 H_{QQ} P_2 + P_2 H_{QQ} Q_2 \frac{q_\sigma}{\mathcal{E}_s - Q_2 H_{QQ} Q_2} Q_2 H_{QQ} P_2. \quad (\text{II.18})$$

The most crucial assertion for this paper may now be made: namely that in the neighbourhood of $\mathcal{E}_s$, H'_{QQ} , to a good approximation, may be considered only a slowly varying function of $\mathcal{E}_s$. If subtracting off the contribution of one pole near $\mathcal{E}_s$ in H_{eff} is not sufficient to make the remainder energy insensitive, we can improve its behaviour by subtracting off the contributions of a few neighbouring poles as well. The important issue is not whether the approximate energy constancy of the remainder requires the subtraction of one or a few pole terms (see note p.15). The important issue is between the subtraction of a few and many terms. The results of this paper hold if only a few terms need be subtracted.

Once this assumption is made it follows that by piecing together the H'_{QQ} which is valid near each ϵ_σ one can construct an H'_{QQ} which varies slowly with energy for a larger domain of E than just the neighbourhood of one pole. It is important to realize that the same chain of reasoning when applied to the H' of Eq. (II.7) leads to the identity of the potential scattering between neighbouring pairs of resonances. In that case H' was identified with the average potential felt by a single nucleon moving in the field of the target nucleus. In the present case, we identify H'_{QQ} with the average Hamiltonian for a two particle-one hole state.

We can now go on to solve Eq. (II.17)

$$P_2 \bar{\phi}_s = \frac{1}{\mathcal{E}_s - H'_{QQ}} \frac{P_2 H_{QQ} Q_2 \phi_\sigma \langle \phi_\sigma Q_2 H_{QQ} P_2 \bar{\phi}_s \rangle}{\mathcal{E}_s - \epsilon_\sigma} \quad (\text{II.19})$$

By multiplying through on both sides by $\langle \phi_\sigma Q_2 H_{QQ} |$ we obtain an equation for $\mathcal{E}_s$

$$\mathcal{E}_s = \epsilon_\sigma + \left\langle \phi_\sigma Q_2 H_{QQ} \frac{P_2}{\mathcal{E}_s - H'_{QQ}} H_{QQ} Q_2 \phi_\sigma \right\rangle \quad (\text{II.20})$$

At this point we shall consider two cases. In the first, (i) the spectrum of H'_{QQ} , i.e., of the two particle-one hole states is such that there is only one such state whose energy is so close to $\mathcal{E}_s$ that it dominates the eigenfunction series in (II.20) for the Green's function $(\mathcal{E}_s - H'_{QQ})^{-1}$. In the second case, (ii) several such states are important. Let then

$$H'_{QQ} \psi_n = E_n^{(2)} \psi_n \quad (\text{II.21})$$

where ψ_n is a two particle-one hole state. Then in case (i)

$$\mathcal{E}_s = \epsilon_\sigma + \frac{|\langle \phi_\sigma Q_2 H_{QQ} P_2 \psi_n \rangle|^2}{\mathcal{E}_s - E_n^{(2)}} \quad (\text{II.22})$$

which may be readily solved for $\mathcal{E}_s$. The solution of interest is the one which reduces to ϵ_σ for vanishing $Q_2 H_{QQ} P_2$. We also note that for case (i)

$$P_2 \bar{\phi}_s = a_{sn} \psi_n \quad (\text{II.23})$$

where a_{sn} is a normalization factor. From Eq. (II.23) it follows that $P_2 \bar{\phi}_s$ has the identical form for all the resonances in the neighbourhood of $\mathcal{E}_s$, the only change in going from one level to another being given by the multiplicative constant a_{sn} . Before discussing the consequences of (II.23) it will be useful to obtain an expression for $|a_{sn}|^2$.

In this connection it is important to recall that $P_2 \bar{\phi}_s$ is not the complete wave function $\bar{\phi}_s$ and that in Eq. (II.15) it is $\bar{\phi}_s$ which is normalized to unity. $\bar{\phi}_s$ can be obtained from the equation

$$\bar{\phi}_s \equiv P_2 \bar{\phi}_s + Q_2 \bar{\phi}_s = \left[1 + \frac{1}{\mathcal{E}_s - Q_2^H Q_2} Q_2^H Q_2 \right] P_2 \bar{\phi}_s. \quad (\text{II.24})$$

Inserting Eq. (II.23), and separating off the strongly energy dependent term as in Eq. (II.17)

$$\bar{\phi}_s = a_{sn} \left[\psi_n + \frac{1}{\mathcal{E}_s - \epsilon_\sigma} \phi_\sigma \right] \langle \phi_\sigma Q_2^H Q_2 \psi_n \rangle + \frac{q_\sigma}{\mathcal{E}_s - Q_2^H Q_2} Q_2^H Q_2 \psi_n \quad (\text{II.25})$$

Note that these three terms are mutually orthogonal. From the normalization condition on $\bar{\phi}_s$ we obtain

$$|a_{sn}|^2 \left[1 + \frac{|\langle \phi_\sigma Q_2^H Q_2 \psi_n \rangle|^2}{(\mathcal{E}_s - \epsilon_\sigma)^2} + |W_n|^2 \right] = 1 \quad (\text{II.26})$$

where $|W_n|^2$ is the normalization integral for the last term on the right-hand side of Eq. (II.25). Although this integral depends upon both s and n , we make the assumption identical to that made on H_{QQ}^1 in the paragraph following

Eq. (II.18) that this s dependence is very weak and can be neglected.

Inserting Eq. (II.22) and solving for $|a_{sn}|^2$ we obtain

$$|a_{sn}|^2 = \frac{1}{1+|W_n|^2} \frac{[1+|W_n|^2] |\langle \phi_{\sigma}^{QH} P_2 \psi_n \rangle|^2}{(\mathcal{E}_s - E_n^{(2)})^2 + [1+|W_n|^2] |\langle \phi_{\sigma}^{QH} P_2 \psi_n \rangle|^2} \quad (\text{II.27})$$

We now return to Γ_s . From Eq. (II.15) and (II.23) we obtain

$$\Gamma_s = 2\pi |a_{sn}|^2 |\langle \psi_{nR}^V \chi^{(+)} \rangle|^2. \quad (\text{II.28})$$

Γ_s is thus broken up into two factors of which one is a "shell model" type term which is common to all resonances, s , while the second, $|a_{sn}|^2$, gives the dependence on s . The "shell model" type term gives the probability of a transition from the "one particle" open channel optical wave function $\chi^{(+)}$ to a two particle-one hole wave function ψ_n while $|a_{sn}|^2$ gives the probability of finding the system, in state s , in the simple state n . The functions $\chi^{(+)}$ and ψ_n are determined respectively by a one-particle potential and by its obvious analogue for the two particle-one hole situation.

We see that the strength of a compound nuclear resonance as given by its width Γ_s depends upon the existence of a simple two particle-one hole state, ψ_n , at an energy close to the resonant energy. Widths will be relatively small if there is a paucity of these levels in the energy region of interest or if, for reasons depending upon the character of ψ_n and the nature of the overlap of the wave functions, the matrix element itself is reduced in value. See Section I for further discussions of these points. Shakin, in the following paper, has evaluated these simple matrix elements with shell model wave functions and finds considerable cancellation.

It also follows from (II.28) that whenever $\chi^{(+)}$ has a single particle resonance, i.e., whenever its amplitude inside the nucleus is large, that Γ_s will on the average be larger and that a giant resonance will result, as is observed. In obtaining this conclusion we have assumed that $|a_{sn}|^2$ is random in its dependence on s with an average value which is fairly constant for $\mathcal{E}_s \cong E_n^{(2)}$ as is indicated by Eq. (II.27). The randomness is well known empirically¹⁴⁾ but it also follows from the randomness of the matrix elements occurring in (II.27) a consequence of the large cancellations which occur in their integration. As $\mathcal{E}_s$ begins to deviate more and more from $E_n^{(2)}$ we see from (II.27) that $|a_{sn}|^2$ on the average will decrease. There is an associated full width at half maximum Δ_{ns}

$$(\Delta_{ns}/2)^2 \simeq (1 + |W_n|^2) \left| \langle \phi_{\sigma} Q_2 H_{QQ} P_2 \psi_n \rangle \right|^2 \quad (\text{II.29})$$

which governs the rate at which $|a_{sn}|^2$ drops from its maximum value at least for the central portion of the Lorentzian form of Eq. (II.27). However, at the wings, i.e., when $|\mathcal{E}_s - E_n^{(2)}| > \Delta_{ns}$, the slow energy dependence of the factor $|W_n|^2$ should become significant. W_n measures the admixture to ϕ_s of states other than ψ_n and one can expect this factor to slowly increase as $\mathcal{E}_s$ departs more and more from $E_n^{(2)}$ and as another $E_n^{(2)}$ is approached. This leads us naturally to the discussion of case (ii).

Note that the width of the giant resonance is the harmonic average of the square of the single particle width associated with the resonance in $\chi^{(+)}$ and the average of the square of the width Δ_{ns} . This giant resonance width is thus smaller than either the single particle width or Δ_{ns} .

The connection of these considerations with the intermediate coupling model of Lane, Thomas and Wigner¹⁵⁾ is not clear. These authors infer from the giant resonance of the strength function that the stationary state which the open channel wave function joins at the surface of the nucleus is at that surface describable for the most part by one single particle wave function. The consequences of this assumption seem to us to be included to a great extent in the behaviour of the single particle function $\chi^{(+)}$. We have however added another condition for the existence of a giant resonance; namely that a simple two particle-one hole state exists at this energy. It is clear that more analysis is required before the relationship of the two models can be elucidated.

We consider now case (ii). Suppose then that there are several eigenvalues of H_{QQ}^1 , i.e., energies of the two particle-one hole states which are so close that it is no longer possible to consider the principal contribution to $\mathcal{E}_s$ and $P\bar{\mathcal{D}}_s$ to come from just one ψ_n . We must instead represent $P\bar{\mathcal{D}}_s$ by a linear combination of these ψ_n with closely spaced $E_n^{(2)*}$. Substituting this combination in Eq. (II.17) one obtains a secular equation for $\mathcal{E}_s$. This equation can, however, be obtained directly from Eq. (II.20)

$$\mathcal{E}_{sv} \approx \epsilon_\sigma + \sum_n \frac{|\langle \phi_{\sigma-Q_2} H_{QQ} P_2 \psi_n \rangle|^2}{\mathcal{E}_{sv} - E_n^{(2)}}$$

where ν numbers the different solutions of this equation. If we let X_ν be the corresponding normalized solution of Eq. (II.17), the expression for the width Γ_s , Eq. (II.15) becomes

$$\Gamma_s = 2\pi |a_{sv}|^2 \left| \langle X_\nu V_R \chi^{(+)} \rangle \right|^2. \quad (\text{II.30})$$

*) A rough measure of the appropriate energy scale is given by the average width $(\Delta_{ns}^2)^{\frac{1}{2}}$.

An expression for $|a_{s\nu}|^2$ analogous to Eq. (II.27) can be obtained from (II.24) and the normalization condition for $\bar{\phi}_s$. We obtain

$$|a_{s\nu}|^2 = \frac{1}{1+|W_\nu|^2} \frac{[1+|W_\nu|^2] |\langle \phi_\sigma Q_2^{H_{QQ}} P_2 X_\nu \rangle|^2}{[\mathcal{E}_s - \langle X_\nu, H_{QQ}' X_\nu \rangle]^2 + [1+|W_\nu|^2] |\langle \phi_\sigma Q_2^{H_{QQ}} P_2 X_\nu \rangle|^2} \quad (II.31)$$

Note that (II.30) and (II.31) reduce directly to (II.28) and (II.27) respectively if $X_\nu = \psi_n$. Explicit expressions for X_ν and the various matrix elements in (II.30) and (II.31) can be readily obtained from Eq. (II.17). Let

$$M_n^{(\sigma)} = \langle \phi_\sigma Q_2^{H_{QQ}} P_2 \psi_n \rangle.$$

Then

$$X_\nu = \frac{\sum_n \frac{M_n^{(\sigma)*}}{\mathcal{E}_{s\nu} - E_n^{(2)}} \psi_n}{\left[\sum_n \frac{|M_n^{(\sigma)}|^2}{(\mathcal{E}_{s\nu} - E_n^{(2)})^2} \right]^{\frac{1}{2}}} \quad (II.32)$$

The matrix elements of interest may now be easily evaluated

$$|\langle \phi_\sigma Q_2^{H_{QQ}} P_2 X_\nu \rangle|^2 = \frac{\left| \sum_n \frac{|M_n^{(\sigma)}|^2}{\mathcal{E}_{s\nu} - E_n^{(2)}} \right|^2}{\sum_n \frac{|M_n^{(\sigma)}|^2}{(\mathcal{E}_{s\nu} - E_n^{(2)})^2}} \quad (II.33)$$

$$\left| \langle \phi_{\sigma} Q_{2H} P_2 X_{\nu} \rangle \right|^2 \xrightarrow{|\mathcal{E}_s| \gg |E_n^{(2)}|} \sum_n |M_n^{(\sigma)}|^2 \quad (\text{II.34})$$

The expectation value of H'_{QQ} becomes

$$\langle X_{\nu} H'_{QQ} X_{\nu} \rangle = \frac{\sum \frac{|M_n^{(\sigma)}|^2 E_n^{(2)}}{(\mathcal{E}_{s\nu} - E_n^{(2)})^2}}{\sum \frac{|M_n^{(\sigma)}|^2}{(\mathcal{E}_{s\nu} - E_n^{(2)})^2}} \quad (\text{II.35})$$

$$\xrightarrow{|\mathcal{E}_{s\nu}| \gg E_n^{(2)}} \frac{\sum_n |M_n^{(\sigma)}|^2 E_n^{(2)}}{\sum_n |M_n^{(\sigma)}|^2} \quad (\text{II.36})$$

Finally the single particle matrix element in (II.30) is

$$\left| \langle X_{\nu} V_R \chi^{(+)} \rangle \right|^2 = \frac{\left| \sum_n \frac{M_n^{(\sigma)}}{(\mathcal{E}_{s\nu} - E_n^{(2)})} \langle \psi_{nV_R} \chi^{(+)} \rangle \right|^2}{\sum_n \frac{|M_n^{(\sigma)}|^2}{[\mathcal{E}_{s\nu} - E_n^{(2)}]^2}} \quad (\text{II.37})$$

$$\left| \langle X_{\nu} V_R \chi^{(+)} \rangle \right|^2 \xrightarrow{|E_s| \gg |E_n^{(2)}|} \frac{\left| \sum M_n^{(\sigma)} \langle \psi_{nR} V_R \chi^{(+)} \rangle \right|^2}{\sum |M_n^{(\sigma)}|^2}. \quad (\text{II.38})$$

It is important to bear in mind that the matrix elements $M_n^{(\sigma)}$ have large fluctuations in going from one state ϕ_{σ} to another and therefore that local deviations from the conclusions to be drawn below may be expected.

The dependence of $|a_{s\nu}|^2$ on E_s is for the most part very similar to that of $|a_{sn}|^2$ (Eq.(II.27)). There is a giant resonance structure in that $|a_{s\nu}|^2$ takes on its largest values in the central region in which the $E_n^{(2)}$ are grouped. At large distances from this region, $|a_{s\nu}|^2$ falls off with a width at half maximum given by (see Eq. (II.34))

$$(\Delta_{\nu s}/2)^2 \simeq (1 + |W_{\nu}|^2) \sum |M_n^{(\sigma)}|^2 \quad (\text{II.39})$$

with an apparent centre given by Eq. (II.36). We note that the width is greater simply in proportion to the root mean square of the individual widths $|M_n^{(\sigma)}|^2$ associated with each state ψ_n . The central region itself may show some structure. In the immediate neighbourhood of a particular $E_n^{(2)}$, the corresponding ψ_n dominates X_{ν} as can be seen from Eq. (II.32) so that we are reduced to the one state result of Eq. (II.27). Thus we may expect a local maximum in $|a_{s\nu}|^2$ near each $E_n^{(2)}$. There is some displacement of the maximum because of the influence of the other states. There will be some reduction in $|a_{s\nu}|^2$ as we move away from a given $E_n^{(2)}$.

A simple case, namely when there are just two $E_n^{(2)}$, $E_1^{(2)}$ and $E_2^{(2)}$, $E_1^{(2)} < E_2^{(2)}$ will illustrate this situation. Then

$$|\langle \phi_{\nu}^{\text{H}} Q_{\text{QQ}}^{\text{P}} X_{\nu} \rangle|^2 = \frac{[|M_1(\sigma)|^2 (\mathcal{E}_{s\nu} - E_2^{(2)}) + |M_2(\sigma)|^2 (\mathcal{E}_{s\nu} - E_1^{(2)})]^2}{|M_1(\sigma)|^2 (\mathcal{E}_{s\nu} - E_2^{(2)})^2 + |M_2(\sigma)|^2 (\mathcal{E}_{s\nu} - E_1^{(2)})^2}.$$

We see that in the region $E_1^{(2)} < \mathcal{E}_s < E_2^{(2)}$ the two terms in the numerator will have opposite phase and in fact the numerator would drop to zero if $|M_1(\sigma)|^2$ and $|M_2(\sigma)|^2$ were constants independent of σ . We may, however, expect that $|a_{s\nu}|^2$ will be lower in this energy region on the average. This effect will be present when more than two states ψ_n are involved and should on the average be more pronounced in the middle of the central region. In Γ_s this behaviour of $|a_{s\nu}|^2$ will be modified by the fluctuations of $|\langle X_{\nu} V_R \chi^{(+)} \rangle|^2$. Near a particular $E_n^{(2)}$ this factor reduces to its one state result, namely $|\langle \psi_n V_R \chi^{(+)} \rangle|^2$. In between these energy values $\langle X_{\nu} V_R \chi^{(+)} \rangle$ will be given by a linear combination of $\langle \psi_n V_R \chi^{(+)} \rangle$, the coefficients of each of these being proportional to $M_n(\sigma)$ and therefore quite random. In fact, if we assume that the phase of the matrix elements $M_n(\sigma)$ is random and average over σ we then obtain

$$[\langle X_{\nu} V_R \chi^{(+)} \rangle^2]_{\text{AV}} = \frac{\left[\sum \frac{|M_n(\sigma)|^2}{(\mathcal{E}_{s\nu} - E_n^{(2)})^2} |\langle \psi_n V_R \chi^{(+)} \rangle|^2 \right]}{\left[\sum \frac{|M_n(\sigma)|^2}{(\mathcal{E}_{s\nu} - E_n^{(2)})^2} \right]}_{\text{AV}} \quad \cdot \quad (\text{II.40})$$

We see that on the average $|\langle X, V_R \chi^{(+)} \rangle|^2$ is simply a weighted mean of the one-state matrix elements $|\langle \psi_n^V, \chi^{(+)} \rangle|^2$. The average (II.16) keeps the property of the exact matrix element in that it approaches $|\langle \psi_n^V, \chi^{(+)} \rangle|^2$ as E_s approaches $E_n^{(2)}$. In going from one value of $E_n^{(2)}$ to the next, $E_{n+1}^{(2)}$ the average moves smoothly from $|\langle \psi_n^V, \chi^{(+)} \rangle|^2$ to $|\langle \psi_{n+1}^V, \chi^{(+)} \rangle|^2$.

We may thus conclude that when more than one simple two particle-one hole state may contribute to the width Γ_s , that the giant resonance will show structure in its central region. If an average over compound nuclear levels is made ^{*)}, the giant resonance on Γ_s will approximately look as if only a single ψ_n contributes at $E_n^{(2)}$. But in between $E_n^{(2)}$ and $E_{n+1}^{(2)}$ because of the behaviour of $|a_{sv}|^2$ it can fall considerably below the value one would obtain if one simply superposed two giant resonances associated with ψ_n and ψ_{n+1} ^{**)} . This suggests that as the energy resolution is increased from very coarse to very fine one will find a series of substructures, starting with the giant resonance for one-particle states, going on to the two particle-one hole substructure suggested above and then on to substructures corresponding to three particle-two hole states until at the best resolution we observe the individual compound nuclear resonance. The experimental observation of these phenomena, if it exists, would provide important insights into the mechanism involved in generating a compound nuclear state. The sequence of substructures will eventually essentially terminate. It seems plausible to suppose that this termination will occur at structures of lower complexity when the system is near a closed shell situation or light nucleus and if the excitation energy of the compound system is relatively low. For example, the results obtained by Shakin ¹²⁾ in the following paper indicate that one need not go beyond the simple two particle-one hole state (i.e., take $|a_{sv}|^2 = 1$) for Pb^{209} target nucleus and probably for light nuclei.

*) Clearly the energy width of the average must be small compared to the spacing of the energies $E_n^{(2)}$ but large enough to include many compound nuclear levels.

**) The possibility of such a structure and the substructures to be now suggested has also been suggested by Kerman and Lemmer ¹⁶⁾ in connection with a related problem.

III. The Strength Function

In this Section we shall derive formula (I.1) which formed the basis of the comparison of the experimental data on the strength function with theory. Formula (I.1) was shown to predict the giant resonance in the neutron s state strength function. In addition, on the basis of (I.1) a semi-empirical formula for the strength function was developed; a formula which provided an excellent fit not only to the average behaviour of $\langle \Gamma_s \rangle / D$ but also to the fluctuations in $\langle \Gamma_s \rangle / D$ as the target nucleus is varied.

Consider first case (i) and sum Γ_s as given in Eq. (II.13) over all states ϕ_s . From the completeness of the latter it follows that

$$\sum_s \Gamma_s = 2\pi |\langle \psi_{nV_R} \chi^{(+)} \rangle|^2 \quad \text{case (i)} . \quad (\text{III.1})$$

According to Eqs. (II.27) and (II.28) only the levels within the giant resonance will contribute to this sum. Hence ^{*}) if N_n is the number of such levels the average of Γ_s is

$$\langle \Gamma_s \rangle \simeq \frac{1}{N_n} \sum_s \Gamma_s = \frac{2\pi}{N_n} |\langle \psi_{nV_R} \chi^{(+)} \rangle|^2 . \quad (\text{III.2})$$

Finally, defining an effective width Δ_n by the relation

$$\Delta_n \equiv D_n N_n \quad (\text{III.3})$$

where D_n is the level spacing for levels associated with ψ_n . Eq. (III.2) becomes

^{*}) It is possible to use (II.27) and by comparison with (III.2) obtain a relation between the density of levels $\underline{g}$, the effective average of $|M_n(\sigma)|^2$ and N .

$$\frac{\langle \Gamma_s \rangle}{D} \simeq 2\pi \frac{|\langle \psi_{nR}^V \chi^{(+)} \rangle|^2}{\Delta_n} . \quad (\text{III.4})$$

Δ_n is approximately equal to the root mean average of the width Δ_{ns} of Eq. (II.29). Note that the energy $E_n^{(2)}$ must agree with the energy of the system n within the width Δ_n . This important formula then relates the strength function to the probability of forming the two particle-one hole state ψ_n and the width of that state Δ_n which gives a rough measure of the probability of this state decaying into more complex configurations.

The extension of formula (III.4) to case (ii) is straightforward. Explicit use is made of (II.40) which shows that we can, after phase averaging write $\langle \Gamma_s \rangle$ as the sum of terms like (II.28). Employing again the completeness of $\underline{\phi}_s$ and definition (III.3) we obtain

$$\langle \Gamma_s \rangle \simeq 2\pi \sum_n \frac{D_n}{\Delta_n} |\langle \psi_{nR}^V \chi^{(+)} \rangle|^2 . \quad (\text{III.5})$$

This is the formula (I.1). For its applications to the experimental data see the discussion in Section I following (I.1).

IV. Some Concluding Remarks

The success of the approximate formula (I.3) and of the more precise expression (I.2) employed by Shakin ¹²⁾ demonstrates the crucial importance of the "first" collision. If this continues to be borne out by further calculations and more accurate $\langle \Gamma_s \rangle / D$ experiments, we may begin to employ (I.2) and (I.3) as tools for the investigation of nuclear structure. Eq. (I.3) particularly points to the sensitivity of $\langle \Gamma_s \rangle / D$ to the density of excited shell model levels of a particular type in the $(A+1)$ nucleus. The check with experiment indicates that the shell model level structure given by Green ¹⁷⁾ provides a good representation of this feature. Similarly Shakin's results may be considered as a verification of the quasi-particle picture for the isotopes of tin.

Additional experimental tests of the theory are suggested by the analysis in Section II which provides the basis from which (I.2) and (I.3) are derived. These all have to do with the structure of the giant resonance as discussed below Eq. (II.38). In experiments with moderate resolution, large enough to average over the fine structure resonances, but somewhat smaller than the giant resonance width Δ (ΔE of the order of $\Delta/10$ to $\Delta/100$) then the giant resonance should break up into a number ν of resonances whose average width is of the order of Δ/ν , each subsidiary resonance corresponding to a two particle-one hole state (or three quasi-particle state). By examining the width of these we could obtain a measure of Δ_n . Moreover, by measuring the height of the maximum as well the value of the average of the square of the matrix element $M_n^{(\sigma)}$ would be obtained. Thus, in principle one could determine the energy and width of these two particle-one hole state and of course their overlap with the compound nuclear state. Presumably as the energy resolution is increased data on the more complex states, e.g., three particle-two hole states, may also become obtainable.

It should also be mentioned that a measurement of the width Δ , of the giant resonance itself, would be useful. If Δ_{sp} is the single particle width then

$$\frac{1}{\Delta^2} \approx \frac{1}{\Delta_{sp}^2} + \left\langle \frac{1}{\Delta_{\nu_s}^2} \right\rangle$$

where Δ_{ν_s} is the width of the factor $|a_{\nu_s}|^2$ giving the relative fraction of the two particle-one hole states in the compound nuclear state (see Eq. (II.39)). Thus a measurement of Δ would yield information on $\left\langle 1/\Delta_{\nu_s}^2 \right\rangle$.

The fundamental nature of these experiments needs no emphasis. If they prove feasible we will have further conceptual insights into nuclear structure as well as new tools for its investigation.

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T A B L E ITOTAL NUMBER OF NEUTRON TRANSITIONS FOR EVEN-EVEN NUCLEI

A	N	Z	L0	L1	L2
40	20	20	1	0	0
44	24	20	1	0	0
50	26	24	1	0	0
54	28	26	1	0	0
54	30	24	1	3	0
58	30	28	1	1	0
60	32	28	1	0	0
64	36	28	1	0	0
68	38	30	1	4	5
70	40	30	1	5	4
74	42	32	1	6	3
78	44	34	1	5	3
80	44	36	1	8	5
84	48	36	1	8	5
88	50	38	0	0	2
90	50	40	0	0	2
94	54	40	0	0	6
98	56	42	0	0	3
100	56	44	0	0	4
104	60	44	0	0	8
108	62	46	0	0	7
110	62	48	0	0	7
114	66	48	0	0	2
118	68	50	0	0	0
120	70	50	0	0	3
130	76	54	0	0	3

T A B L E I I

TOTAL NUMBER OF PROTON TRANSITIONS FOR EVEN-EVEN NUCLEI

A	N	Z	L0	L1	L2
40	20	20	0	0	0
44	24	20	0	0	0
50	26	24	0	0	0
54	28	26	0	0	0
54	30	24	0	0	0
58	30	28	0	0	0
60	32	28	0	0	0
64	36	28	0	0	0
68	38	30	0	4	5
70	40	30	0	2	6
74	42	32	0	5	5
78	44	34	0	6	4
80	44	36	0	12	9
84	48	36	0	13	4
88	50	38	0	0	5
90	50	40	0	0	7
94	54	40	0	0	4
98	56	42	0	0	4
100	56	44	0	0	4
104	60	44	0	0	7
108	62	46	0	0	9
110	62	48	0	0	8
114	66	48	0	0	4
118	68	50	0	0	3
120	70	50	0	0	3
130	76	54	0	0	1

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FIGURE CAPTIONS

- Fig. 1 s-wave neutron strength function as a function of target atomic weight. The theoretical points are calculated with parameters $a^0 = 1$, $a^1 = 0.15$, $a^2 = 0.05$ and n-p force equal to the n-n force.
- Fig. 2 The ratio R of s-wave neutron strength functions to the corresponding optical model values. The theoretical points are calculated with the same parameters as for Fig. 1.
- Fig. 3 Reduced widths for tin isotopes.

FIG. 1. s-wave neutron strength function as a function of target atomic weight. The theoretical points are calculated with parameters $a^0 = 1$, $a^1 = .15$, $a^2 = .05$ and n-p force equal to the n-n force.

FIG. 2. The ratio R of s-wave neutron strength functions to the corresponding optical model values. The theoretical points are calculated with the same parameters as FIG. 1.

FIG. 3. Reduced widths for tin isotopes.