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TEST OF THE CONSERVED VECTOR CURRENT HYPOTHESISIN $\Sigma^{\pm} \rightarrow \Lambda^0$ LEPTONIC DECAYS

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It has been proposed ^{1), 2)} that the decay processes

$$\Sigma^- \rightarrow \Lambda^0 + e^- + \bar{\nu} \quad (1a)$$

$$\Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu \quad (1b)$$

could provide a test of the conserved vector current (C.V.C.) hypothesis ³⁾.

In the present work we show how such a test can be performed through the combined measurement of the branching ratio for the above decays, the average Λ^0 polarization from unpolarized Σ 's ⁴⁾ and the Λ^0 hyperon spectrum. The matrix element for process (1a) can be written as

$$2\pi i G (\bar{e} \gamma_\mu \frac{1+\gamma_5}{2} \nu) \langle \Lambda^0 | J_\mu^A(0) + J_\mu^V(0) | \Sigma^- \rangle \delta^4(p_\Sigma - p_\Lambda - p_\nu - p_e) \quad (2)$$

where G is the Fermi coupling constant, J^A and J^V the strangeness conserving axial and vector currents. From Lorentz invariance we can express the matrix elements of J^V and J^A in terms of six form factors; for the case of even $\Sigma \Lambda$ parity we have

$$(2\pi)^3 \langle \Lambda^0 | J_\mu^V | \Sigma^- \rangle = \bar{u} \Lambda^0 [a(q^2) \gamma_\mu + b(q^2) \sigma_{\mu\nu} q_\nu + b'(q^2) q_\mu] u \Sigma^- \quad (3)$$

$$(2\pi)^3 \langle \Lambda^0 | J_\mu^A | \Sigma^- \rangle = \bar{u} \Lambda^0 [c(q^2) \gamma_\mu \gamma_5 + d(q^2) \sigma_{\mu\nu} q_\nu \gamma_5 + d'(q^2) q_\mu \gamma_5] u \Sigma^- \quad (4)$$

$$q_\mu = p_e^\mu + p_\nu^\mu$$

It is convenient to classify contributions according to forbiddenness : the terms with δ_μ and $\gamma_\mu \gamma_5$ give allowed contributions if $a(0)$ and respectively $c(0)$ are different from zero. The other terms give at most first forbidden contributions ⁵⁾. Terms with b' and d' being of the order of M_e/M_Λ will be neglected in the following.

Under the C.V.C. hypothesis one finds ¹⁾ that ⁶⁾

$$a(0) = 0 \quad (5)$$

$$b(0) = \mu_x / (M_\Lambda + M_\Sigma) = \left(\frac{8\pi}{e^2 \tau_{\Sigma^0}} \right)^{1/2} \left(\frac{M_\Sigma}{M_\Sigma^2 - M_\Lambda^2} \right)^{3/2} = 1.15 \sqrt{\frac{10^{-19}}{\tau_{\Sigma^0}}} \times 10^{-3} \text{ MeV}^{-1} \quad (6)$$

where μ_x is the $\Sigma - \Lambda$ transition magnetic moment and τ_{Σ^0} is the lifetime for the electromagnetic decay of the Σ^0 hyperon. As a result the vector current gives only first forbidden contributions. It follows that, if the decay rate for processes (1) is of the order to be expected for an allowed transition ($\Gamma \approx 10^6 \text{ sec}^{-1}$), this points to a large axial contribution. Such a contribution could be expected from an extension of the Goldberger and Treiman ⁷⁾ relation which would give ⁸⁾

$$c = \frac{G_A}{G} \frac{2M_N}{M_\Sigma + M_\Lambda} \frac{g_{\pi\Sigma\Lambda}}{g_{\pi NN}} \quad (7)$$

An experiment to test C.V.C. in processes (1) should therefore be able to prove the absence of allowed vector contributions and possibly detect the small forbidden contribution which comes from the weak magnetism term (Eq. (6)).

In Ref. ¹⁾ it has already been proposed to look at the Λ^0 longitudinal polarization at a fixed momentum and averaged over the leptons configurations corresponding to that momentum, in the case of unpolarized Σ^0 's or at the Λ^0 asymmetry in the same conditions as above but from polarized Σ^0 's. These effects ⁹⁾ come from A-V interference terms. They should, therefore, be small if one of the currents is predominant. This is still true, of course, after averaging over the Λ^0 spectrum, which is more convenient for an experimental test due to the smallness of the branching ratio.

It is important to note that this further averaging does not decrease the sensitivity of the test. For example, if both A and V give allowed contributions ($a = c \neq 0$) one obtains $\langle P_{\Lambda^0} \rangle = 0.63$. Still assuming C.V.C. $\langle P_{\Lambda^0} \rangle$ can be expressed as a function of the Σ^0 decay rate and of $c(0)$, if we neglect forbidden axial contributions. The parameter $c(0)$ can be eliminated by making use of formulae (A.1) to (A.3) of the Appendix and it is possible to express directly $\langle P_{\Lambda^0} \rangle$ as a function of the branching ratio

$$R^{\pm} = \frac{\Sigma^{\pm} \rightarrow \Lambda^0 + e^{\pm} + \nu}{\text{all } \Sigma^{\pm}}$$

and of the Σ^0 lifetime.

In Fig. 1 we show the value of $\langle P_{\Lambda} \rangle$ against R for different values of the Σ^0 lifetime. Reasonable estimates of the Σ^0 lifetime (i.e., of the $\Sigma^0 - \Lambda^0$ transition magnetic moment) result in values around 10^{-19} sec ¹⁰⁾. The corresponding value for $\langle P_{\Lambda} \rangle$ is 0.06 for a branching ratio $R^- = 10^{-4}$ and 0.20 for $R^- = 10^{-5}$. If the measured polarization results to be small, this means that one of the currents is small, it is, however, desirable to prove that the dominant one is the axial current. This can be accomplished by a measurement of the Λ^0 spectrum.

As shown in Fig. 2 the shape of the spectrum is very different for allowed vector or axial contributions. It is in general true that for the vector case the spectrum rises at zero momentum with vanishing derivative while in the axial case - the derivative - is infinity. In fact, all vector contributions are proportional to the square of the Λ^0 momentum as can be seen in the Appendix.

In the case of odd $\Sigma - \Lambda$ parity one still expects small average longitudinal polarization of the Λ^0 hyperon. The conclusions regarding the shape of the spectra are, however, opposite to the ones of the even parity case. It is now the axial part which vanishes at the low end of the spectrum. For allowed contributions Fig. 2 can still be used where curve 1 corresponds to the vector case and curve 2 to the axial case (expected on the basis of C.V.C. hypothesis).

Informations about the relative magnitude of the b and c terms can be obtained, in principle, also from the electron spectrum but the main features of the spectrum are the same for an allowed vector or an allowed axial vector contribution. Hence, the electron spectrum together with the measurement of the average Λ^0 polarization is not enough to fully verify the C.V.C. hypothesis. Although the proposed use of the Λ^0 spectrum in the test of C.V.C. depends on the relative $\Sigma - \Lambda$ parity, such knowledge will probably be available by the time enough statistics have been collected on the processes considered here.

The present test of C.V.C. is invalidated if the baryon vector current contains a term odd under G , which is coupled to the leptons. In fact, there is no reason why such a term (a second class current according to Weinberg's definition ¹¹⁾) should be conserved. One could in that case expect an allowed vector contribution to processes (1) as well as a large longitudinal Λ^0 polarization. The proposed test, therefore, applies in the case in which only first class currents are present. Under such an assumption the six form factors

a, b, b', c, d and d' are the same for the two decays and this could be tested by comparing the two processes. For example, the two rates should be in the ratio of the phase spaces (1.57) considering only allowed contributions and the longitudinal Λ^0 polarizations equal.

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After completion of the present work we have been informed that a similar argument has been discussed by W. Alles¹²⁾.

A P P E N D I X

The decay probability for processes (1) as a function of the Λ^0 momentum $\vec{k}$ in the rest frame of the Σ hyperon is obtained from Eqs. (2) to (4) averaging over the spins and momenta of the leptons as

$$W(\vec{k}, \vec{P}_\Sigma, \vec{P}_\Lambda) d^3k = \frac{G^2}{96\pi^4} \left\{ A + B(\vec{P}_\Lambda \cdot \vec{P}_\Sigma) + \right. \quad (A.1)$$

$$\left. + B'(\vec{P}_\Lambda \cdot \vec{k})(\vec{P}_\Sigma \cdot \vec{k})/|\vec{k}|^2 + C(\vec{P}_\Lambda \cdot \vec{k}) + D(\vec{P}_\Sigma \cdot \vec{k}) + E(\vec{k} \cdot \vec{P}_\Lambda \times \vec{P}_\Sigma) \right\}$$

where

$$A = |\vec{k}|^2 |\alpha|^2 + |\beta|^2 - 2k^2(|\gamma|^2 |\vec{k}|^2 + |\delta|^2)$$

$$B = |\vec{k}|^2 |\alpha|^2 - |\beta|^2 \quad B' = 2|\beta|^2 - 2k^2(|\gamma|^2 |\vec{k}|^2 - |\delta|^2)$$

$$C = 2 \operatorname{Re} \alpha \beta^* - 4k^2 \operatorname{Re} \gamma \delta^* \quad (A.2)$$

$$D = 2 \operatorname{Re} \alpha \beta^* + 4k^2 \operatorname{Re} \gamma \delta^*$$

$$E = 2 \operatorname{Im} \alpha^* \beta$$

(if T is conserved and we neglect the final state e.m. interaction, E should vanish), where in turn

$$\alpha = a \left(1 + \frac{k_0}{2M_\Lambda} \right) + b \frac{k_0^2 - |\vec{k}|^2}{2M_\Lambda} \quad (A.3)$$

$$\beta = c \left(k_0 + \frac{|\vec{k}|^2}{2M_\Lambda} \right) + d (|\vec{k}|^2 - k_0^2)$$

$$\begin{aligned} \gamma &= a \frac{1}{2M_\Lambda} + b \left(\frac{k_0}{2M_\Lambda} - 1 \right) \\ \delta &= c + d \left(\frac{|\vec{k}|^2}{2M_\Lambda} - k_0 \right) \end{aligned} \quad (\text{A.3})$$

k_0 is the energy given to the lepton pair ($\sim M_\Sigma - M_\Lambda$) and $k^2 = \vec{k}^2 - k_0^2$ (a negative quantity).

The longitudinal polarization of the Λ^0 hyperons with momentum $\vec{k}$ from unpolarized Σ 's ($P_\Sigma = 0$) is given by

$$P_{\Lambda^0}(\vec{k}) = \frac{C|\vec{k}|}{A} \quad (\text{A.4})$$

and the average polarization is obtained by performing the integration

$$\langle P_{\Lambda^0} \rangle = \frac{\int C |\vec{k}|^3 d|\vec{k}|}{\int A |\vec{k}|^2 d|\vec{k}|} \quad (\text{A.5})$$

In the case of opposite $\Sigma - \Lambda$ parity Eq. (3) and (4) of the text go over to

$$(2\pi)^3 \langle \Lambda^0 | J_\mu^\nu | \Sigma^- \rangle = \bar{u}^{\Lambda^0} [c \gamma_\mu \gamma_5 + d \sigma_{\mu\nu} q_\nu \gamma_5] u^\Sigma \quad (\text{A.6})$$

$$(2\pi)^3 \langle \Lambda^0 | J_\mu^A | \Sigma^- \rangle = \bar{u}^{\Lambda^0} [a \gamma_\mu + b \sigma_{\mu\nu} q_\nu] u^\Sigma \quad (\text{A.7})$$

where the b' and d' terms have been dropped. The formulae for the rate (A.1) to (A.5) remain obviously unchanged, only the meaning of the various form factors is now interchanged and the predictions of C.V.C. are

$$c(0) = 0 \quad (\text{A.8})$$

$$d(0) = \frac{\mu_x}{M_\Lambda + M_\Sigma} = 1.15 \sqrt{\frac{10^{-19}}{\tau_{\Sigma^0}}} \times 10^{-3} \text{ MeV}^{-1} \quad (\text{A.9})$$

The extension to this case of the Goldberger and Treiman relation has also been proposed ⁸⁾ giving an even larger contribution for the axial current [term a in (A.7)].

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- 6) One could still expect $a(0)$ to be of the order of e^2 due to e.m. effects; this will not affect the conclusions of the present discussion. We note also that since the range of variation of the momentum transfer in the considered processes is rather small the momentum dependence of the form factors is not expected to be important. Taking into account such dependence is equivalent to the introduction of higher forbidden terms.
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FIGURE CAPTIONS

Fig. 1 Average longitudinal Λ^0 polarization as a function of the Σ^0 lifetime and of the branching ratio R , as predicted by C.V.C.

Fig. 2 Λ^0 spectrum as a function of the square of the Λ^0 momentum ($x = |\vec{k}|^2$, in units of $|\vec{k}_{\max}|^2$) for the case of even $\Sigma - \Lambda$ parity. For the case of parity see **text**.

Fig. 1

Fig. 2