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ASYMPTOTIC BEHAVIOUR OF THE NUCLEON-NUCLEON SCATTERING AMPLITUDES

ACCORDING TO THE MULTIPERIPHERAL MODEL

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S U M M A R Y

The asymptotic behaviour of the nucleon-nucleon elastic scattering amplitudes at high energy and small momentum transfer is derived in the framework of the multiperipheral model. The results agree with those obtained according to the "reggeizing" rules, except for the axial vector amplitude which differs in some peculiar features.

The asymptotic behaviour of the nucleon-nucleon elastic scattering amplitudes at high energy and small momentum transfer has been derived by Gribov and Pomeranchuk ¹⁾ and by Gell-Mann ²⁾ by use of the so-called reggeizing rules ^{2), 3)}.

We report here the derivation of the asymptotic behaviour of the nucleon-nucleon amplitudes from the multiperipheral model proposed by Amati, Fubini and Stanghellini ⁴⁾. For the N-N scattering matrix we follow the notation used by Amati, Leader and Vitale ⁵⁾. The process is represented in Fig. 1, where n_1, p_1 and n_2, p_2 are the initial and final 4-momenta, respectively. The scattering matrix can be written in the spinor space in terms of the GNO invariant amplitudes ⁶⁾

$$T_{NN}^{\pm}(s, t) = H_1^{\pm} + (i\gamma^P N + i\gamma^N P) H_2^{\pm} + (i\gamma^P N i\gamma^N P) H_3^{\pm} \\ + (i\gamma^P N \gamma_5^P i\gamma^N P \gamma_5^N) H_4^{\pm} + (\gamma_5^P \gamma_5^N) H_5^{\pm} \quad (1)$$

where

$$N = \frac{1}{2}(n_1 + n_2) \quad , \quad P = \frac{1}{2}(p_1 + p_2)$$

The superscripts $\pm$ refer to the two parts in which the matrix can be splitted in the iso-spin space

$$T = 3 T^{+} + 2 \vec{\tau}^N \cdot \vec{\tau}^P T^{-} \quad (2)$$

2.

According to the peripheral model, the absorptive amplitude A_{NN} is given by the sum of all the diagrams of the kind represented in Fig. 2. Making direct use of the high energy $\pi-N$ absorptive amplitudes, the problem can be reduced to the study of the graph of Fig. 3, in which a bubble representing the high energy $\pi-N$ amplitude is linked to a low energy $\pi-N$ interaction. Following AFS (disregarding, for the moment iso-spin indices) the amplitude A_{NN} is given by the following expression

$$A_{NN}(s, t) = A_{NN}^R(s, t) + \int \frac{ds_0}{8\pi^4} [-A_{N\pi}(s_0, t) - i\gamma^p Q B_{N\pi}^R(s_0, t)] \cdot \\ \cdot \iiint ds' du_1 du_2 \frac{K(s, -u_1^2, -u_2^2; s', u_1, u_2; s_0)}{(u_1 + \mu^2)(u_2 + \mu^2)} [-A_{N\pi}(s', u_1, u_2, t) + \\ + i\gamma^h Q B_{N\pi}(s', u_1, u_2, t)] \quad (3)$$

where A_{NN}^R , $A_{N\pi}^R$ and $B_{N\pi}^R$ stand for low energy absorptive amplitudes. The kernel is given by

$$K(s, -u_1^2, -u_2^2; s', u_1, u_2; s_0) = \int d^4Q \delta[(N+Q)^2 - s'] \delta[(Q - \frac{A}{2})^2 + u_1] \cdot \\ \cdot \delta[(Q + \frac{A}{2})^2 + u_2] \delta[(P-Q)^2 - s_0] \quad (4)$$

and

$$Q = \frac{1}{2}(q_1 + q_2) \\ \Delta = n_1 - n_2 = p_2 - p_1 = q_2 - q_1 \\ s_0 = (p_1 - q_1)^2, \quad u_0 = (p_1 + q_2)^2 \\ s' = (n_1 + q_1)^2, \quad u' = (n_1 - q_2)^2 \\ u_1 = -q_1^2, \quad u_2 = -q_2^2$$

We want now to project out from Eq. (3) the absorptive parts h_i of the invariant amplitudes H_i given in (1). This projection is carried out by decomposing, as done by ALV, the 4-vector Q along the four orthogonal 4-vectors

$$M = N + P \quad ; \quad K = N - P \quad ; \quad \Delta \quad ; \quad L_\lambda = \epsilon_{\lambda\mu\nu\rho} M_\mu K_\nu \Delta_\rho \quad (5)$$

Such decomposition of A_{NN} is not symmetrical with respect to the invariants $(i\gamma^P \cdot N)$, $(i\gamma^N \cdot P)$, since the vectors n_1, n_2 and p_1, p_2 are not treated in a symmetric way in the diagram of Fig. 3. In fact, one has to take into account also the graph represented in Fig. 4, where the rôles of s_0 and s' have been interchanged. In this way, one obtains the following relations

$$\begin{aligned} h_1(s, t) &= h_1^R(s, t) + \int ds_0 ds' du_1 du_2 \frac{K(s, s', s_0, u_1, u_2)}{8\pi^4(u_1 + \mu^2)(u_2 + \mu^2)} [A^R(s_0)A(s') + \\ &\quad + m a A^R(s_0) B(s') - m b B^R(s_0) A(s') - m^2 (ab + \frac{s-u}{su} c^2) B^R(s_0) B(s')] \\ h_2(s, t) &= h_2^R(s, t) + \frac{1}{2} \int ds_0 ds' du_1 du_2 \frac{K(s, s', s_0, u_1, u_2)}{8\pi^4(u_1 + \mu^2)(u_2 + \mu^2)} [a B^R(s_0) A(s') - b A^R(s_0) B(s') + \\ &\quad + m (a^2 + b^2 - \frac{s+u}{su} c^2) B^R(s_0) B(s')] \\ h_3(s, t) &= h_3^R(s, t) - \int ds_0 ds' du_1 du_2 \frac{K(s, s', s_0, u_1, u_2)}{8\pi^4(u_1 + \mu^2)(u_2 + \mu^2)} [ab - \frac{s-u}{su} c^2] \cdot \\ &\quad \cdot B^R(s_0) B(s') \quad (6) \\ h_4(s, t) &= h_4^R(s, t) + \frac{t}{su} \int ds_0 ds' du_1 du_2 \frac{K(s, s', s_0, u_1, u_2)}{8\pi^4(u_1 + \mu^2)(u_2 + \mu^2)} c^2 B^R(s_0) B(s') \\ h_5(s, t) &= h_5^R(s, t) . \end{aligned}$$

4.

where

$$\begin{aligned}
 a &= \frac{1}{4s} (s' - u' - s_0 + u_0) + \frac{1}{4u} (s' - u' + s_0 - u_0) \\
 b &= \frac{1}{4s} (s' - u' - s_0 + u_0) - \frac{1}{4u} (s' - u' + s_0 - u_0) \\
 c^2 &= \frac{1}{2} (u_1 + u_2 + \frac{t}{2}) + \frac{1}{32s} (s' - u' - s_0 + u_0)^2 + \\
 &\quad + \frac{1}{32u} (s' - u' + s_0 - u_0)^2
 \end{aligned} \tag{7}$$

We consider now the high energy limit (s, s' very large with respect to s_0, u_1, u_2, m^2) for each term in (5). The kernel K reduces to

$$\frac{1}{s} Q(-m^2, -m^2, u_1, u_2, x) = \frac{1}{8s} T \left[-t(1-x), u_1 + m^2 x - \frac{s_0 x}{1-x}, u_2 + m^2 x - \frac{s_0 x}{1-x} \right] \tag{8}$$

where T is the triangle function defined by AFS, and $x = s'/s$. The coefficients (7) become

$$\begin{aligned}
 a &\rightarrow -\frac{1}{2s} \left[(4m^2 - t)x + 2s_0 + t + u_1 + u_2 - 2m^2 \right] \\
 b &\rightarrow x \\
 c^2 &\rightarrow -\frac{1}{4s^2} \left[(4m^2 - t)x^2 + 2(2s_0 + t + u_1 + u_2 - 2m^2)x \right. \\
 &\quad \left. - 2(u_1 + u_2 + \frac{t}{2}) \right]
 \end{aligned} \tag{9}$$

The high energy π -N absorptive amplitudes in (6) are to be replaced by their asymptotic expression given in Ref. 4)

$$\begin{aligned} A(s, t) &\rightarrow C(t) \varphi_A(t) s^{\alpha(t)} \\ B(s, t) &\rightarrow C(t) \varphi_B(t) s^{\alpha(t)-1} \end{aligned} \quad (10)$$

where

$$\begin{aligned} \varphi_A(t) = & \int x^\alpha dx \int du_1 du_2 \frac{Q(-m^2, -m^2, u_1, u_2, x)}{8\pi^4(u_1 + \mu^2)(u_2 + \mu^2)} \varphi_{\pi\pi}(u_1, u_2, t) \cdot \\ & \cdot \int ds_0 [A^R(s_0, t) - m x B^R(s_0, t)] \end{aligned}$$

$$\varphi_B(t) = \frac{1}{2} \int x^\alpha dx \int du_1 du_2 \frac{Q(-m^2, -m^2, u_1, u_2, x)}{8\pi^4(u_1 + \mu^2)(u_2 + \mu^2)} \cdot \quad (11)$$

$$\cdot \varphi_{\pi\pi}(u_1, u_2, t) \int ds_0 [(4m^2 - t)x + 2s_0 + t + u_1 + u_2 - 2m^2] B^R(s_0, t).$$

The asymptotic limits of the absorptive amplitudes (6) can then be written in the following way

6.

$$\begin{aligned}
 h_1(s, t) &\rightarrow C(t) \varphi_A^2(t) s^{\alpha(t)} \\
 h_2(s, t) &\rightarrow -C(t) \varphi_A(t) \varphi_B(t) s^{\alpha(t)-1} \\
 h_3(s, t) &\rightarrow C(t) \varphi_B^2(t) s^{\alpha(t)-2} \\
 h_4(s, t) &\rightarrow -\frac{t}{2} C(t) \varphi_B(t) \varphi_C(t) s^{\alpha(t)-3} \\
 h_5(s, t) &\rightarrow 0
 \end{aligned} \tag{12}$$

with the introduction of a new term

$$\begin{aligned}
 \varphi_C(t) &= \frac{1}{2} \int x^\alpha dx \int du_1 du_2 \frac{Q(-u^2, -u^2, u_1, u_2, x)}{8\pi^4(u_1 + \mu^2)(u_2 + \mu^2)} \varphi_{\pi\pi}(u_1, u_2, t) \\
 &\cdot \int ds_0 \left[(4u^2 - t)x + 2(2s_0 + t + u_1 + u_2 - 2u^2) - \frac{2}{x} (u_1 + u_2 + \frac{t}{2}) \right] \cdot B^R(s_0, t)
 \end{aligned} \tag{13}$$

From (12) one can easily derive the total amplitudes. These are obtained by using fixed momentum transfer dispersion relations for the h 's, taking into account their crossing property

$$h_i^\pm(u, s) = \pm (-1)^i h_i^\pm(s, u) \quad (i = 1, 2, 3, 4) \tag{14}$$

The result is for all amplitudes

$$H_i^\pm(s, t) = \frac{e^{-i\pi\alpha(t)} \pm 1}{\sin \pi\alpha(t)} h_i^\pm(s, t) \tag{15}$$

Similar results are obtained in the case of $N-\bar{N}$ elastic scattering.

Before comparing our results with those obtained in Refs. 1), 2), we want to examine the structure of the term $\varphi_c(t)$ contained in H_4 . One sees that for $\alpha \rightarrow 0$, φ_c behaves like $1/\alpha$; in order to avoid that the expression (13) becomes meaningless one should impose the condition $\alpha > 0$.

On the other hand, we know that the factor $S^{\alpha(t)}$, which we have introduced here through the asymptotic $\pi-N$ amplitudes, derives from the structure of the asymptotic integral equation for the $\pi-\pi$ absorptive amplitude ⁴⁾. The parameter $\alpha(t)$ represents the leading Regge trajectory; for the validity of the integral equation one needs the lower bound $\alpha > -1$, which is reached in the weak coupling limit ⁷⁾. This is in contradiction with the condition $\alpha > 0$ implied by the amplitude H_4 , and indicates that the results of the peripheral model for this particular amplitude are no longer reliable. In fact, the asymptotic behaviour of $H_4 (\sim S^{\alpha-3})$ derives from a cancellation of higher order terms ($\sim S^{\alpha-2}$): it might well be that the model does not work for terms lower than these.

We can indicate a qualitative argument in this direction. Let us suppose to add to the peripheral parts H_i^P of the $N-N$ amplitudes (given in (12), (15)) a Born-like contribution H_i^B , corresponding to the graph of Fig. 5, where the nature of the intermediate particle X does not need to be specified (a physical situation would be represented, e.g., taking for X the deuteron in the $N-N$ case, and the pion in the $N-\bar{N}$ case). In general, for the contributions to the amplitudes H_3, H_4 we obtain asymptotically $H_4^B/H_3^B \rightarrow S^0$, while we get from (12): $H_4^P/H_3^P \rightarrow S^{-1}$. This comparison shows that, if the peripheral part H_3^P dominates H_3^B only for $\alpha > -1$, the corresponding part H_4^P will not be taken over by H_4^B only if $\alpha > 0$. Below such limit the peripheral contribution H_4^P would become meaningless.

With the above warning in mind, we compare now our results with those obtained by Gell-Mann ^{*)}. One sees that they agree for the amplitudes H_1, H_2, H_3 . The axial vector amplitude H_4 agrees in the energy dependence but, while for Gell-Mann it is proportional to φ_B^2 , we have in it the new term φ_C , which is not present in the π -N amplitudes. We want to remark that in the model adopted here the amplitude H_4 is originated from a term $(\gamma^{\vec{n}} \cdot \vec{L} \gamma^{\vec{p}} \cdot \vec{L})$ (where $\vec{L}$ is defined by (5)) which mixes in an essential way the spins and momenta of the nucleons of the two external links of the ladder of Fig. 2. Similar terms cannot be present in the π -N scattering, since a single factor $(\gamma \cdot \vec{L})$ is by itself a pseudoscalar and cannot remain if alone. It is not unreasonable a priori that this new term in the N-N amplitudes may not be reduced to the only coefficients which are present in the π -N amplitudes.

This point does not affect at all the factorization principle for the cross sections ⁸⁾. In fact, the amplitude H_4 does not appear in the asymptotic expression for the cross section.

For the sake of completeness, we list here the simplest experimental observables in the N-N analysis ⁹⁾. We write the N-N scattering amplitude as a matrix in the two-dimensional spin space ⁶⁾:

$$T_{NN} = \alpha + \beta (\vec{\sigma}_1 \cdot \vec{n})(\vec{\sigma}_2 \cdot \vec{n}) + i\gamma (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{n} + \delta (\vec{\sigma}_1 \cdot \vec{m})(\vec{\sigma}_2 \cdot \vec{m}) + \epsilon (\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_2 \cdot \vec{l}) \quad (16)$$

*) Disregarding common factors, the functions φ_A, φ_B are related to the η 's of Ref. ²⁾ by the relations

$$\varphi_A \simeq \frac{1}{2m} (\eta_1 - \eta_2)$$

$$\varphi_B \simeq \eta_2$$

where $\vec{l}, \vec{m}, \vec{n}$ are unit vectors in the direction of the 3-vectors

$$\vec{p}_1 + \vec{p}_2, \quad \vec{p}_1 - \vec{p}_2, \quad \vec{p}_1 \times \vec{p}_2$$

The connection between the physical amplitudes in (16) and the invariant amplitudes H_i is given in Ref. ⁶⁾. By using the asymptotic expressions (12), (15) we get (disregarding common factors and keeping only the positive signature, i.e. the leading $I = 0$ trajectory)

$$\begin{aligned} \alpha &\rightarrow \left(\varphi_A(t) - \frac{1}{2m} \varphi_B(t) \right)^2 \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} S^{\alpha(t)} \\ \beta &\rightarrow \frac{t}{4m^2} \varphi_A^2(t) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} S^{\alpha(t)} \\ \gamma &\rightarrow \frac{\sqrt{-t}}{2m} \varphi_A(t) \left(\varphi_A(t) + \frac{1}{2m} \varphi_B(t) \right) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} S^{\alpha(t)} \\ \delta &\rightarrow 0 \\ \epsilon &\rightarrow \frac{t}{8m^2} \varphi_B(t) \varphi_C(t) \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} S^{\alpha(t)-1} \end{aligned} \quad (17)$$

Then the differential elastic cross section and polarization for an initially unpolarized beam are given asymptotically by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &\rightarrow \frac{1}{S} (|\alpha|^2 + |\beta|^2 + 2|\gamma|^2 + |\delta|^2 + |\epsilon|^2) \rightarrow \\ &\rightarrow \left[\left(\varphi_A(t) - \frac{1}{2m} \varphi_B(t) \right)^2 - \frac{t}{4m^2} \varphi_A^2(t) \right] \frac{|1 + e^{-i\pi\alpha(t)}|^2}{8\pi^2 \alpha(t)} S^{2\alpha(t)-1} \end{aligned}$$

$$\frac{d\sigma}{d\Omega} P \rightarrow \frac{2}{S} \operatorname{Im} (\alpha + \beta) \gamma^* \rightarrow 0 \quad (18)$$

The total cross section, according to the optical theorem, becomes ⁶⁾

$$\sigma_{TOT} \rightarrow \frac{16\pi^2}{mS} \operatorname{Im} \alpha(t=0) \rightarrow \frac{16\pi^2}{m} \left(\varphi_A(0) - \frac{1}{2m} \varphi_B(0) \right)^2 S^{\alpha(0)-1} \quad (19)$$

Finally, the depolarization in the triple scattering experiment satisfies the relation ⁹⁾

$$\begin{aligned} \frac{d\sigma}{d\Omega} (1-D) &\rightarrow \frac{2}{S} (|\delta|^2 + |\epsilon|^2) \rightarrow \\ &\rightarrow \frac{1}{2} \left(\frac{t}{4m^2} \right)^2 \varphi_B^2(t) \varphi_C^2(t) \frac{|1 + e^{-i\pi\alpha(t)}|^2}{8\pi^2 \alpha(t)} S^{2\alpha(t)-3} \end{aligned} \quad (20)$$

We see that the elastic and total cross sections contain only the two amplitudes φ_A, φ_B which appear in the π -N scattering; the same holds for the polarization, if one considers the interference between two different trajectories. Instead, only the axial vector amplitude containing φ_C enters the expression (20). This result is not modified if one considers the effect

of other trajectories: in fact, by the general reggeizing rules ²⁾ one can see that the ρ and ω trajectories give the same kind of energy dependence to H_4 as in (12), while the π and η trajectories do not contribute to H_4 . In this way, there seems to be the possibility of detecting a high energy contribution from the axial vector amplitude.

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FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5