Factorization, charming penguins, and all that

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Abstract

We discuss few selected topics related to the calculation of hadronic amplitudes relevant for two-body non-leptonic $B$ decays.

Introduction

It is likely that most of the future results in $B$ phenomenology, including the study of CP violation, will come from the measurements of two-body non-leptonic decays. The major theoretical problem in predicting the rates of these decays is the evaluation of hadronic amplitudes. In general the solution to this problem is unknown as it contains all the difficulties of low-energy strong interactions and hadronization. In QCD, the best one can do is using the operator product expansion to separate the short- and long-distance scales. In the resulting effective Hamiltonian, the effect of short-distance physics can be computed perturbatively and it is described by the notorious Wilson coefficients. Long-distance, non-perturbative physics is contained

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in the hadronic matrix elements of a set of local operators. In particular, for $B$ decays into two mesons, one needs to compute matrix elements of dimension-six four-fermion operators (we neglect in the following magnetic dipole transitions),

$$\langle M_1 M_2 | Q_i | B \rangle = \langle M_1 M_2 | \bar{q}_1 \Gamma q_1 \bar{q}_2 \Gamma' q_3 | B \rangle , \quad i = 1, \ldots, 10 ,$$

where $b, q_j$ are the appropriate quark fields and $\Gamma, \Gamma'$ are various combinations of Dirac and colour matrices. Unlike $K$ physics, neither analytic computation techniques nor systematic expansions are known for these matrix elements. Numerical approaches to QCD are severely limited by the present computing power, since a small lattice spacing is required to simulate the heavy $B$ field.

These problems have not prevented theorists from making predictions of interesting quantities, such as BRs, asymmetries, etc. On the one hand, observables free from hadronic uncertainties have been identified, the best (and apparently unique) example being the CP asymmetry in $B_d \to J/\Psi K_s$. On the other hand, when the evaluation of hadronic matrix elements could not be avoided, as for the BRs, various theoretical approaches, such as flavour symmetry, factorization and form-factor models, have been developed and used.

Although these approaches have been successful in some applications, their theoretical soundness is questionable. From a phenomenological point of view, this means that the theoretical error affecting the predicted amplitudes can only be guessed.

**Factorization**

To be concrete, let us consider the popular approach based on the factorization hypothesis applied to an emission-dominated decay, say $B^+ \to D^+ \pi^0$. Using Fierz and colour rearrangement, one is left only with the matrix element

$$\langle D^+ \pi^0 | \bar{b} \gamma_\mu (1 - \gamma_5) u \bar{c} \gamma^\mu (1 - \gamma_5) d | B^+ \rangle ,$$

where the quark fields are contracted according to the disconnected emission diagram $DE$ in fig. 1. The factorization hypothesis states that

$$\langle D^+ \pi^0 | \bar{b} \gamma_\mu (1 - \gamma_5) u \bar{c} \gamma^\mu (1 - \gamma_5) d | B^+ \rangle = \langle \pi^0 | \bar{b} \gamma_\mu (1 - \gamma_5) u | B^+ \rangle \langle D^+ | \bar{c} \gamma^\mu (1 - \gamma_5) d | 0 \rangle ,$$

namely the four-fermion operator matrix element is given by the product of the matrix elements of two currents. Pictorially this means that the two disconnected
Figure 1: Diagrams representing the Wick contraction of a four-fermion operator between a $B$ and two mesons $M_1$ and $M_2$. The operator insertion is represented by the double black dots.Disconnected emission ($DE$), penguin ($DP$) and annihilation ($DA$) diagrams are shown.

branches of the emission diagram do not interact. The heuristic physical argument is appealing [1]: if a large energy is transferred to the emitted meson, as in the case at hand, the emitted quarks have not “enough” time to interact before going far from the interaction point and hadronizing. Starting from this picture, Dugan and Grinstein introduced the large energy effective theory (LEET) [2], in which factorization holds at the lowest order of a systematic expansion in powers of $\Lambda_{QCD}/E_{\text{emit}}$. Besides the objections that can be raised even on the lowest-order result [3], higher order corrections in LEET, or equivalently corrections to factorization, are not known. This is already a crucial point: even accepting that factorization holds in some effective expansion of QCD, there is no indication that it is an accurate approximation, and corrections at the level of 10–20%, or even more, are not surprising at all.

Charming penguins

It is easy to find decays where a moderate deviation of the matrix elements from their factorized values changes the predicted BRs by orders of magnitude. The reason is simply that some matrix elements, which vanish in the factorization limit, are Cabibbo-enhanced with respect to emission diagrams. This is precisely the mechanism which produces the large charming-penguin enhancement [4, 5].

In general, it is useful to identify two classes of operators: i) current-current operators of the form $\bar{b}\Gamma q_u q'_u\Gamma'$s which have $O(1)$ Wilson coefficients and ii) penguin operators like $\bar{b}\Gamma\sum_q q\Gamma'q$, the Wilson coefficients of which are $\lesssim 0.03$ at a scale $\sim M_B$. In $B \to K\pi$ and $B \to K\rho$, for example, the only operators of class
Figure 2: Dependence of $BR(B^+ \to \rho^+ K^0)$ on $\eta_L$, the ratio of charming penguin to emission amplitudes. Exact factorization prediction corresponds to $\eta_L = 0$.

i) which have non-vanishing factorized matrix elements are those containing up-quark fields. They are doubly Cabibbo-suppressed with respect to the operators in ii), so that their Wilson coefficient enhancement is compensated and the contribution of the two classes is comparable. However, if we allow for a violation of the factorization, class i) operators containing charm-quark fields can also have non-vanishing matrix elements. Contractions of these operators, like $DP$ in fig. 1, are called charming penguins and contribute to the decays we are considering. Since they have large Wilson coefficients and are not Cabibbo-suppressed, their contribution easily becomes dominant, already assuming corrections to factorization at the level of $10 - 20\%$ [4]. Striking examples of charming-penguin dominance are given by $B \to \rho K$ channels. For instance, $BR(B^+ \to \rho^+ K^0)$, shown in fig. 2, changes by three orders of magnitude for modest values of $\eta_L$, which is the ratio of charming-penguin to emission amplitudes. Exact factorization prediction, namely $\eta_L = 0$, is definitely unreliable for this decay. Notice that the value of the charming-penguin parameters suggested by $B \to K\pi$ measurements brings the $BR(B \to K\rho)$ near to their present experimental bounds [5].

This discussion leads to the following conclusion: there is a class of rare $B$ decays for which the use of factorized amplitudes gives very unstable predictions, which are drastically changed by moderate corrections to exact factorization. These decays,
including $B \to K\pi/K\rho/K\eta$, etc. have been extensively studied in ref. [5].

The recent CLEO measurements of $BR(B \to K\pi)$ [6] actually call for some enhancement over the predictions obtained with factorized amplitudes. If charming penguins have to explain the data, their value must be about 20-30% of the corresponding emission diagrams [5]. Notice that the recent analysis of ref. [7], which claims to be able to reproduce the data using factorized amplitudes, somehow takes into account charm-loop effects in perturbation theory by enhancing the penguin-operator Wilson coefficients. The underlying physical process providing the enhancement is the same in the two approaches, but we believe that the perturbative treatment is not appropriate.

A somewhat related argument, which has recently become popular, is the effect of the final state interaction (FSI) in $B$ decays. In two-body decays, FSI is neglected in factorized amplitudes by definition. Also in this case, there are amplitudes that are neglected on the basis of factorization. For example, the annihilation diagrams, like $DA$ in fig. 1, are usually neglected on the basis of the following argument: the annihilating quarks must be near enough for the weak current to annihilate them, implying a suppression proportional to the $B$ wave function at the origin, namely a factor $f_B/M_B$. However this and other diagrammatic arguments may not hold in presence of FSI, which mixes up different classes of diagrams [8]. Since recent theoretical estimates [9] suggest, at variance with factorization, that the final-state interaction cannot be neglected in $B$ decays, many phenomenological analyses relying on neglecting this or that amplitude on the basis of factorization should be reconsidered. FSI effects in $B \to \pi\pi$, $B \to K\pi$ have been studied in the analysis of ref. [4]. Moreover, it has been recently shown [10] that rescattering effects invalidate the Fleischer-Mannel bound [11] on $\cos(\gamma)$ and affect bounds on new physics [12].

**And all that**

Charming-penguin enhancement is not always effective: there are decay channels for which penguins are not present or not enhanced. Table 1 contains a list of measured emission-dominated channels. In these cases, factorized amplitudes are expected to give more reliable predictions, yet further assumptions are required. Using Lorentz invariance, matrix elements of currents can be parameterized in terms of form factors, which however are known only in some special cases. If the external states are both heavy, HQET helps to express them in terms of few known quantities, such as the heavy masses, the Isgur-Wise function slope $\tilde{\rho}^2$, etc [13]. On the
Table 1: Predictions for measured emission-dominated decays. Two form-factor models, NRSX and QCDSR, are considered for different values of $\rho^2$. Other required input parameters are chosen according to the central values of ref. [5]. The values of the fitted parameters $\xi$ and $\delta\xi$ are shown in the two cases together with the values of $\chi^2$/dof.

<table>
<thead>
<tr>
<th>Channel</th>
<th>QCDSR ($\rho^2 = 0.65$)</th>
<th>NRSX ($\rho^2 = 1.1$)</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_d \to \pi^+ D^-$</td>
<td>301</td>
<td>331</td>
<td>$310 \pm 44$</td>
</tr>
<tr>
<td>$B_d \to \pi^+ D^0$</td>
<td>323</td>
<td>308</td>
<td>$280 \pm 41$</td>
</tr>
<tr>
<td>$B_d \to \rho^+ D^-$</td>
<td>794</td>
<td>866</td>
<td>$840 \pm 175$</td>
</tr>
<tr>
<td>$B_d \to \rho^+ D^0$</td>
<td>994</td>
<td>949</td>
<td>$730 \pm 153$</td>
</tr>
<tr>
<td>$B^+ \to \pi^+ \tilde{D}^0$</td>
<td>508</td>
<td>534</td>
<td>$500 \pm 54$</td>
</tr>
<tr>
<td>$B^+ \to \pi^+ \tilde{D}^{*0}$</td>
<td>605</td>
<td>567</td>
<td>$520 \pm 82$</td>
</tr>
<tr>
<td>$B^+ \to \rho^+ \tilde{D}^0$</td>
<td>1015</td>
<td>112</td>
<td>$1370 \pm 187$</td>
</tr>
<tr>
<td>$B^+ \to \rho^+ \tilde{D}^{*0}$</td>
<td>1396</td>
<td>1339</td>
<td>$1510 \pm 301$</td>
</tr>
<tr>
<td>$B_d \to K^{0} J/\Psi$</td>
<td>81</td>
<td>75</td>
<td>$85 \pm 14$</td>
</tr>
<tr>
<td>$B_d \to K^{*0} J/\Psi$</td>
<td>164</td>
<td>173</td>
<td>$132 \pm 24$</td>
</tr>
<tr>
<td>$B^+ \to K^{+} J/\Psi$</td>
<td>84</td>
<td>78</td>
<td>$102 \pm 11$</td>
</tr>
<tr>
<td>$B^+ \to K^{*+} J/\Psi$</td>
<td>171</td>
<td>180</td>
<td>$141 \pm 33$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.47</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>$\delta\xi$</td>
<td>0.42</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>$\chi^2$/dof</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

Other hand, heavy-light form factors need phenomenological models to be evaluated. Input-parameter dependence in the heavy-heavy case, and model dependence in the heavy-light one, introduce further theoretical errors, which sum up with the uncertainty on factorization and should be taken into account in the phenomenological analyses.

Once form factors are chosen, no other free parameters are present in exact factorization and emission-dominated decay rates should be predicted without further assumptions. However it has become a standard procedure, starting with the well-known $a_1$ and $a_2$ of BSW [14], to introduce more parameters to account for possible deviation from factorization and fit them to the experimental data, see e.g. refs. [5, 6, 15]. The comparison between these fitted parameters and the factorization expectation provides a test of the reliability of factorization. Our parameterization
is the following: connected ($CE$) and disconnected ($DE$) emissions are related according to $CE = \xi e^{\delta_\xi} DE$, where $\xi$ and $\delta_\xi$ are the parameters to be fitted, then $DE$ is expressed in terms of form factors using factorization. Colour rearrangement and exact factorization would give $\xi = 1/3$ and $\delta_\xi = 0$. Fitting the decay channels in tab. 1, we find, in agreement with other analyses [6, 15], that experimental data are reasonably well described by factorized amplitudes for reasonable values of $\xi$ and $\delta_\xi$. Surprisingly enough, we have found a strong dependence on the Isgur-Wise function slope $\rho^2$ appearing in heavy-heavy form factors. Indeed, by changing $\rho^2$, the value of $\chi^2$ in the fit also changes as shown in fig. 3. In the global fit, the details of the model used for the heavy-light form factors turn out to be hidden by this large dependence. We have considered two popular models: NRSX [16], which is a modern version of the original BSW model, and QCDSR [17], a model based on light-cone QCD sum rules. We have found that NRSX, QCDSR and the other models considered in ref. [5] fit well the data, once a suitable value of $\rho^2$ is assumed. However, for example, the best fit within QCDSR and NRSX is given by quite different values of $\rho^2$ ($\rho^2 \sim 0.65$ and $\sim 1.1$ respectively). Moreover there exist experimentally allowed values of $\rho^2$ for which no model gives a good fit.

It is reassuring that it is always possible to fit the data with acceptable val-
ues of the parameters. Within a given class of two-body final states with definite Lorentz properties, this may simply be a consequence of the dominance of emission diagrams and of SU(3) flavour symmetry, which reduce the number of independent amplitudes, rather than a test of the factorization hypothesis. However the global fit of many decay channels, including pseudoscalar and/or vector mesons in the final state, actually probes the different form-factor models and the factorization itself.

On the other hand, the strong $\rho^2$ dependence in the fit calls for a careful study of the uncertainties affecting the factorized amplitudes. Indeed, the common procedure of comparing different heavy-light form-factor models for fixed value of $\rho^2$, certainly leads to underestimate the theoretical error on $\xi$ and $\delta \xi$ (or whatever parameters are fitted). On the contrary, $\rho^2$ should be varied within its experimentally allowed range in order to estimate the actual theoretical uncertainty.

References


