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ON PROBABILITY DISTRIBUTIONS OF NUCLEAR CROSS-SECTIONS

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The study of nuclear cross-sections in the "continuum" with high energy resolution promises to throw interesting light on the validity of the statistical model of nuclear reactions and on its parameters. It will probably also permit us a glimpse of the general microstructure of thermodynamic systems. At the excitation energies corresponding to the "continuum" the number of available exit channels is so high that the average width Γ is much larger than the level spacing D . As a consequence of this, all the many intermediate states contained in an energy interval Γ around the incident energy contribute coherently, though randomly, to the scattering amplitude, thereby producing statistical fluctuations in the cross-section as a function of energy ¹⁾.

We have recently developed a quantitative theory for those aspects of the fluctuations which can be studied by correlation functions for the nuclear cross-sections ²⁾. The shape of the energy correlation function and the mean square variation of the fluctuations are the main quantities obtained in this way. It is the purpose of this letter to point out that the probability distribution of the cross-section ^{*)} is expected to have a simple form. Let us assume we are dealing with a heavy nucleus with an average width which, while much larger than the level spacing, still is very small compared to the region of appreciable variation in the average cross-section. Under these conditions the width amplitudes can be expected to have an important random component. We will for the moment take their phases to be completely random. Consider for simplicity the scattering amplitude $f(\vartheta)$ for an inelastic process in which all initial and final spins are zero. Then the scattering amplitude is

$$f(\vartheta) = \sum_{\ell} A_{\ell} P_{\ell}(\cos \vartheta) = \xi(\vartheta) + i\eta(\vartheta) \quad . \quad (1)$$

where we have separated $f(\vartheta)$ into its real and imaginary parts.

*) Cross-section probability distributions have recently been discussed for the case of isolated resonances in connection with generalizations of the optical model ³⁾. See also Ref. ⁴⁾.

We can write the coefficients A_ℓ in terms of intermediate states of complex energy $E_i^{(\ell)}$ as

$$A_\ell = \sum_i \frac{a_i^{(\ell)}}{E - E_i^{(\ell)}} \equiv \xi^{(\ell)} + i\eta^{(\ell)} \quad (2)$$

It is now easy to show that if $a_i^{(\ell)}$ is a random number (real or complex) and if $\Gamma \gg D$, then $\xi^{(\ell)}$ and $\eta^{(\ell)}$ will have random, i.e., Gaussian, probability distribution, with the same dispersion. The same property applies hence also to $\xi(\vartheta)$ and $\eta(\vartheta)$ according to a well-known theorem of statistics, since they are just sums of the random numbers $\xi^{(\ell)}$ and $\eta^{(\ell)}$ with the same weight factors $P_\ell(\cos\vartheta)$. Since the differential cross-section $\sigma(\vartheta)$ is

$$\sigma(\vartheta) = \left(\xi(\vartheta)\right)^2 + \left(\eta(\vartheta)\right)^2 \quad (3)$$

we can hence immediately conclude that it must have a χ^2 distribution of two degrees of freedom. In units $x = \sigma(\vartheta)/\langle\sigma(\vartheta)\rangle$ the probability distribution $P_1(x)$ is therefore

$$P_1(x) = \exp(-x) \quad (4)$$

The characteristics of this distribution are a rather important probability of low cross-sections as well as a long tail of high cross-sections. As a consequence the ratio between the highest and lowest cross-section in a sample is generally large (more exactly, of the order $n \ln n$, where n is the number in the sample).

The result (4) for zero spin reactions can be generalized. First, the effect of a coherent, non-random, component $f_{DI}(\vartheta)$ can be included in the scattering amplitude. Then,

$$f(\vartheta) = f_{DI}(\vartheta) + f_{f\ell}(\vartheta) \quad (5)$$

with a corresponding average cross-section

$$\langle \sigma(\vartheta) \rangle = |f_{DI}(\vartheta)|^2 + \langle |f_{f\ell}(\vartheta)|^2 \rangle = \sigma_{DI}(\vartheta) + \langle \sigma_{f\ell}(\vartheta) \rangle \quad (6)$$

The corresponding probability distribution is

$$P_1(x) = \exp\{-(x+x_{DI})\} J_0(2ix^{\frac{1}{2}} x_{DI}^{\frac{1}{2}}) \quad (7)$$

where x and x_{DI} are $\sigma(\vartheta)$ and $\sigma_{DI}(\vartheta)$ measured in units of $\langle \sigma_{f\ell}(\vartheta) \rangle$. The function $J_0(x)$ is the cylindrical Bessel function of zeroth order. For $\sigma_{DI}(\vartheta)$ smaller than $\langle \sigma_{f\ell}(\vartheta) \rangle$ the probability distribution (7) does not differ appreciably from that of Eq. (4) (see Fig. 1). On the other hand it has a maximum for a non-zero value of $\sigma(\vartheta)$ if $\sigma_{DI}(\vartheta)$ is larger than $\langle \sigma_{f\ell}(\vartheta) \rangle$, so that this situation is clearly discernible experimentally, and can be used, as the correlation function, for a determination of $\sigma_{DI}(\vartheta)$ and $\langle \sigma_{f\ell}(\vartheta) \rangle$.

A second generalization of Eq. (4) can be made to the case of non-zero spins and to integrated cross-sections within the frame of the random phase approximation. As is known from the theory of the correlation function¹⁾ the number of effective, incoherent channels increases from unity to N in this case. We hence expect that the probability distribution now is given by the χ^2 distribution of $2N$ degrees of freedom.

$$P_N(x) = \frac{N}{(N-1)!} (Nx)^{N-1} \exp(-Nx) \quad (8)$$

This form is only approximate, even within the random phase assumption, since it requires all the N effective channels to have the same dispersion. It should, however, be useful for a comparison to experimental results, since the general shape and a reasonable value of N should be significant. The number of effective channels N is for small spin (not all zero) approximately half the joint spin

weight of the initial and final systems. For the integrated cross-section this has to be multiplied by the effective number of orbital angular momenta ¹⁾. Use of dynamics permits more exact, though analytically much more complicated, predictions for these probability distributions.

The results of Eqs. (4), (7) and (8) show that the cross-section probability distributions in heavy nuclei yield information on the reaction mechanisms and the structure of the scattering amplitude. The form of the probability distribution will furthermore provide a consistency check on the results obtained from cross-section correlation functions. We therefore suggest that the high resolution experiments suitable for the study of correlation functions are sampled also for the cross-section distribution.

R E F E R E N C E S

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 - 3) C.E. Porter, Phys. Rev. Letters 9, 429 (1962).
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FIGURE CAPTION

Fig. 1 Shape of the probability distributions of the differential cross-section in zero spin reactions ($P \gg D$)

- a) for $\sigma_{DI}(\vartheta) = 0$ (Eq. (4))
- b) for $\sigma_{DI}(\vartheta) = \langle \sigma_{f\ell}(\vartheta) \rangle$ (Eq. (7))
- c) for $\sigma_{DI}(\vartheta) = 2 \langle \sigma_{f\ell}(\vartheta) \rangle$ (Eq. (7))