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DO STRANGE PARTICLES OBEY PARASTATISTICS ?

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Replace page 6 and the first line of page 7 by :

the following local interactions are not allowed :

$$\bar{N}NK, \bar{N}N\bar{K}, \bar{N}\wedge\pi, \bar{N}\equiv\pi \text{ and their Hermitian conjugates.}$$

Our second general theorem concerns Yukawa interactions involving a baryon field and its charge conjugate anti-baryon field. We find that the third field must be an ordinary boson. For example consider an interaction of the form $H \bar{\Lambda} \wedge \phi$ where we do not specify the multiplicity of the field ϕ . Introducing the $\bar{\Lambda} \wedge \phi$ interactions in condition (5) yields :

$$\left[\bar{\Lambda}(x_1) \wedge (x_1) \phi(x_1), \bar{\Lambda}(x_2) \wedge (x_2) \phi(x_2) \right] = 0.$$

Inserting the commutation relations (3) this condition gives

$$\left[\phi(x_1), \phi(x_2) \right] = 0$$

the result quoted above. It follows that the following interactions are not allowed

$$\bar{\Lambda} \wedge K, \bar{\Lambda} \wedge \bar{K}, \bar{N}NK, \bar{N}N\bar{K}, \bar{\Xi} \Xi K, \bar{\Xi} \Xi \bar{K}.$$

Our third general theorem is concerned with the consistency of two interaction Hamiltonians. If two of the three interacting fields are common to both, the third for each of the interactions must have identical multiplicities. For example, the existence of the $H_{\bar{N}\wedge K}$ Eq. (6) interaction precludes the

existence of the $H \equiv \wedge K$ interaction. The proof is simple. Introduce the $\overline{\equiv} \wedge K$ interaction into the integrability condition (5) :

$$\left[\overline{\equiv}(x_1) \wedge (x_1) K(x_1), \overline{\equiv}(x_2) \wedge (x_2) K(x_2) \right] = 0 .$$

Now inserting Eq. (8) into this equation gives

$$\left\{ \overline{\equiv}(x_1), \overline{\equiv}(x_2) \right\} = 0$$

which is contrary to hypothesis.

On the basis of this theorem and the existence of interaction (6) the following local interactions are forbidden :

$$\overline{N} \wedge \Pi, \overline{\equiv} \wedge K, \overline{N} \equiv K, \text{ and their Hermitian conjugates.}$$

On the basis of the same theorem and the existence of interaction (7) the following local interactions are forbidden

$$\overline{N} \wedge \overline{K}, \overline{\equiv} \wedge \Pi, \overline{\equiv} N \overline{K}, \text{ and their Hermitian conjugates.}$$

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existence of the $H \Xi \wedge K$ interaction. The proof is simple. Introduce the $\Xi \wedge K$ interaction into the integrability condition (5) :

$$\left[\Xi(x_1) \wedge (x_1) K(x_1), \Xi(x_2) \wedge (x_2) K(x_2) \right] = 0.$$

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On the basis of the same theorem and the existence of interaction (7) the following local interactions are forbidden

$$\bar{N} \wedge \bar{K}, \Xi \wedge \pi, \Xi N \bar{K}, \text{ and their Hermitian conjugates.}$$

Conservation of strangeness and the associated Gell-Mann/Nishijima relations were introduced as phenomenological rules. Whether strangeness is a new primordial quantity to be considered on the same footing as charge or a derived quantity is an open question. In this note we explore the possibility that strange particles obey parastatistics and that the strangeness selection rules and the phenomena of associated production in strong interactions are a consequence of what might be loosely called "conservation of statistics". As we shall see such a scheme is possible. Parastatistics by itself does not suffice but some additional simple postulates are required.

We shall need to first fix the nomenclature and notations and review some of the elements of the theory of parastatistics. Nucleons will be referred to as ordinary fermions or fermions of unit multiplicity while pions will be, correspondingly, ordinary bosons or bosons of unit multiplicity. The field operators for ordinary fermion or ordinary boson particles satisfy bilinear anti-commutation or commutation rules respectively. Fields obeying the intermediate or parafermion statistics in which the maximum occupation number is two rather than unity will be called fermions of multiplicity two. These fields characteristically obey a trilinear anti-commutation rule. If following Volkov¹⁾ we define the symbol $\{ , \}_F$ by :

$$\{ \psi_\alpha(x), \bar{\psi}_\beta(x') \}_F \equiv -i \int_{\alpha\beta} (x-x') \quad (1)$$

$$\{ \psi_\alpha(x), \psi_\beta(x') \}_F = 0 \quad (2)$$

where $\int_{\alpha\beta}$ is the well-known singular function. Then if $\phi_\alpha, \phi_\beta, \phi_\gamma$ are Dirac spinors or conjugate spinors, the commutation rule for parafermions of multiplicity two is :

2.

$$\begin{aligned} & \phi_{\alpha}(x)\phi_{\beta}(x')\phi_{\gamma}(x'')+\phi_{\gamma}(x'')\phi_{\beta}(x')\phi_{\alpha}(x) = \\ & = \left\{ \phi_{\alpha}(x), \phi_{\beta}(x') \right\}_F \phi_{\gamma}(x'') + \left\{ \phi_{\gamma}(x''), \phi_{\beta}(x') \right\} \phi_{\alpha}(x). \end{aligned} \quad (3)$$

For example if $\phi_{\alpha} = \psi_{\alpha}$, $\phi_{\beta} = \bar{\psi}_{\beta}$ and $\phi_{\gamma} = \psi_{\gamma}$, Eq. (3) becomes

$$\begin{aligned} & \psi_{\alpha}(x) \bar{\psi}_{\beta}(x') \psi_{\gamma}(x'') + \psi_{\gamma}(x'') \bar{\psi}_{\beta}(x') \psi_{\alpha}(x) = \\ & = -i \int_{\alpha\beta} (x-x') \psi_{\gamma}(x'') - i \int_{\gamma\beta} (x''-x') \psi_{\alpha}(x). \end{aligned} \quad (4)$$

Paraboson fields of multiplicity two satisfy a trilinear commutation rather than an anti-commutation rule, the plus sign on the left hand side of Eq. (3) being replaced by a minus sign and with the fermion singular functions on the right hand side being replaced by the corresponding boson ones. Parafermion and boson fields of multiplicity three obey quadrilinear commutation rules, etc. A field theoretic treatment of parastatistics was first given by Green²⁾. Volkov¹⁾ treated the parafermion field of multiplicity two showing that the theory was T.C.P. invariant. In a later paper³⁾ he determined the changes in the rules for constructing amplitudes from Feynman diagrams. These results have recently been refined by Dell'Antonio, Greenberg and Sudarshan⁴⁾. These authors derive a generalization of the Pauli spin and statistics theorem to the effect that parafermion fields must have odd half integral spin, paraboson fields integral spin. Kamefuchi and Takahashi⁵⁾ discuss these general questions also and give the quadrilinear commutation relations for parafields of multiplicity. Finally, as Galindo, Morales and Nuñez-Lagos⁶⁾ and Jauch⁷⁾ have demonstrated, there are some very interesting questions with regard to the physical interpretation of the wave functions associated with systems of parafermions or parabosons.

We will now consider, within a field theoretic framework, reactions in which parabosons and parafermions can interact. We shall assume :

- 1) the integrability condition, which requires that the interaction Hamiltonian evaluated at two different spacetime points commutes if the points are separated by a space-like distance ^{*)} :

$$\left[H(x_1), H(x_2) \right] = 0 \quad \text{if} \quad |x_1 - x_2| \quad \text{is space-like} \quad (5)$$

- 2) that all local interactions are of the Yukawa type involving only three fields ;
- 3) the $\bigwedge$ particle ^{**)} and Ξ are parafermions of multiplicity two. No assumptions as to the symmetry nature of the K particle will be made ;
- 4) that there are at least the following two local interactions

$$H_{\bar{N}\bigwedge K} = (\bar{N}\bigwedge - \bigwedge \bar{N})K + \text{Hermitian conj.} \quad (6)$$

and

$$H_{\Xi \bigwedge \bar{K}} = (\Xi \bigwedge - \bigwedge \Xi) \bar{K} + \text{Hermitian conj.} \quad (7)$$

^{*)} We shall use this in a strong form in that we shall assume that there must be commutation for $|x_1 - x_2|$ space-like between the different terms in the interaction Hamiltonian. This is not a necessary consequence of (5) since it is possible, though unlikely, that the total interaction will satisfy (5) but the individual terms do not.

^{**)} We shall designate both the $\bigwedge$ and $\sum$ particles by $\bigwedge$.

Here we have made only the dependence on the field operator explicit. The dependence on spinor or isotopic spin components has been suppressed ^{*)} ;

- 5) the conservation of parity, angular momentum, baryon number ;
- 6) that the commutation rules between different fields are anti-commutation between parafermion fields, commutation between paraboson fields and either parafermion or paraboson fields.

We shall show that these assumptions and here assumption (3) plays an essential role, lead to the experimentally determined selection rules from which the strangeness selection rules are inferred. The method consists in examining all possible local interactions of the Yukawa type (assumption 2)) involving the baryon fields and the π and K meson fields. We shall then find that only those local interactions are allowed which are "strangeness" conserving. This result, the "strangeness" conservation, remains valid if not all these interactions are in fact local.

Some simple theorems giving the statistics of systems obtained by combining particles of the same or different statistics by means of local Yukawa interactions are needed. Some of these hold more generally as for example the

*) These interactions do not satisfy the criteria proposed by Kamefuchi and Strathee ⁵⁾ which are based on the requirement that the equations of motion obtained from the commutator relation $[\mathcal{H}, \psi] = i \dot{\psi}$ agree with those obtained from the Lagrange/Euler equations. For interactions (6) and (7), the Lagrange equations of motion are consistent with the commutator relation in the sense that if the Lagrange equations are inserted into the commutator in a suitable fashion an identity results. Secondly, the equation $[\mathcal{H}, H_{\text{int.}}] = i \dot{H}_{\text{int.}}$ holds in virtue of Eq. (8) so that the interaction representation and the S matrix can be defined. It seems thus entirely possible that the requirements of Kamefuchi and Strathee may be too stringent.

the following theorem which is rather obvious intuitively. If one of the three fields involved in the Yukawa interaction is an ordinary field and a second obeys parastatistics ^{*}), the third must also obey parastatistics. This gives the phenomena of associated prediction when one of the fields obeys ordinary statistics. We illustrate the proof of this theorem using as an example the interaction Hamiltonian (6) $H_{\bar{N}\wedge K}$. Inserting Eq. (6) into Eq. (5) we obtain

$$\begin{aligned} \left\{ \wedge(x_1)K(x_1), \wedge(x_2)K(x_2) \right\} &= 0 \\ \left\{ \bar{\wedge}(x_1)\bar{K}(x_1), \wedge(x_2)K(x_2) \right\} &= 0 \quad \text{when } |x_1-x_2| \text{ is space-like} \\ \left\{ \bar{\wedge}(x_1)\bar{K}(x_1), \bar{\wedge}(x_2)\bar{K}(x_2) \right\} &= 0. \end{aligned} \quad (8)$$

We now note that since $\wedge$ is a parafermion field of multiplicity two it follows that the K field must be a paraboson field. For if we assumed the opposite, namely that the K field were an ordinary boson field, we would find from Eq. (8) that the $\wedge$ field would satisfy the anti-commutation rules of the ordinary fermion contrary to hypothesis. Indeed the K field must satisfy paraboson statistics of multiplicity two. To prove this we employ a necessary condition for the consistency of Eq. (8) namely that the commutator

$$\left[\left\{ \wedge(x_1)K(x_1), \wedge(x_2)K(x_2) \right\}, \left\{ \wedge(x_3)K(x_3), \wedge(x_4)K(x_4) \right\} \right]$$

and other like it which can be found from Eq. (8) are zero if all the points are separated by space-like distances. If we now insert the anti-commutation rules for the $\wedge$ field, the result follows. Using this first theorem we find that

^{*}) We shall use this term in the future to designate fields whose multiplicity is greater than one and employ the term ordinary for unit multiplicity.

the following local interactions are not allowed :

$$\bar{N}NK, \bar{N}N\bar{K}, \bar{N}\wedge\pi, \bar{N}\equiv\pi \quad \text{and their Hermitian conjugates.}$$

Our second general theorem is concerned with the consistency of two interaction Hamiltonians. If two of the three interacting fields are common to both, the third for each of the interactions must have identical multiplicities. For example, the existence of the $H_{\bar{N}\wedge K}$ Eq. (6) interaction precludes the existence of the $H_{\bar{A}\wedge K}$ interaction. The proof is simple. Introduce the $\bar{A}\wedge K$ interaction into the integrability condition (5) :

$$\left[\bar{A}(x_1)\wedge(x_1)K(x_1), \bar{A}(x_2)\wedge(x_2)K(x_2) \right] = 0.$$

Now inserting Eq. (8) into this equation gives

$$\left\{ \bar{A}(x_1), \bar{A}(x_2) \right\} = 0$$

which is contrary to hypothesis.

On the basis of this theorem and the existence of interaction (6) the following local interactions are forbidden :

$$\bar{N}\wedge\pi, \bar{A}\wedge K, \bar{A}\bar{K}\wedge, \bar{\Xi}\wedge K, \bar{N}\Xi K, \bar{N}NK.$$

On the basis of the same theorem and the existence of interaction (7) the following local interactions are forbidden

$\bar{N}\Lambda\bar{K}$, $\bar{\Lambda}\Lambda\bar{K}$, $\Xi\Lambda\pi$, $\Xi\bar{N}\bar{K}$, $\Xi\Xi\bar{K}$, $\Xi\Xi K$ and their Hermitian conjugates.

Finally, the one remaining combination we have not yet considered is $\Xi\bar{N}K$ and its Hermitian conjugate. This term evaluated at x_1 should commute with $H_{\bar{N}\Lambda K}$, Eq. (6), evaluated at x_2 , $|x_2-x_1|$ space-like. This leads immediately to the condition that K is an ordinary boson which is, of course, in contradiction with the earlier result that K is a paraboson.

With this we have considered all Yukawa vertices involving two baryons and one meson (π or K) permitted by conservation of baryons. Interactions involving mesons only need to be considered next. Since the π is pseudoscalar, conservation of parity prohibits interactions of the form $KK\pi$, or $\bar{K}K\pi$. The $\pi\pi K$ and the KKK interactions are forbidden because of their lack of commutativity when evaluated at x_1 with interaction (6) evaluated at x_2 , $|x_2-x_1|$ space-like.

We may now draw the conclusion that the only local interactions which are allowed are identical with those for which strangeness conservation holds. Thus the selection rules provided by the strangeness scheme and those following from our assumptions are identical.

These results suggest that it would be fruitful to experimentally determine the statistics of strange particles. Experiments in which pairs of Λ 's are produced or doubly strange hypernuclei are formed by the absorption of Ξ^- or Ξ^0 particles by nucleons or nuclei provide possibilities for the measurement of the relative probability of the various internal states of the two Λ system. One could then determine whether the Λ 's act like ordinary or parafermions. Similarly experiments which would test the statistics of the K particles are possible. Such experiments have been considered by de-Shalit⁸⁾ and by Messiah and Greenberg⁹⁾.

It follows that in strangeness violating transitions, the integrability condition (5) is violated. As far as this writer knows, this involves no contradiction with experimental evidence. The integrability condition is one of the sufficient conditions needed in some derivations of T.C.P. invariance¹⁰⁾. It is not clear whether it is a necessary condition. In any event, T.C.P. invariance has not been verified experimentally for strangeness violating transitions. The theoretical treatment of these weak decays in the present context would involve the introduction of new particles which would provide the required non-locality.

Finally, our theory suggests the possibility that paraparticles of higher multiplicity than two may exist. These would manifest themselves by weak decays into strange particles.

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R E F E R E N C E S

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