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POSSIBLE RESONANT ENHANCEMENT OF THE TWO-PHOTON EXCHANGE IN
ELECTRON-PROTON SCATTERING

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A B S T R A C T

We investigate the corrections to electron-proton scattering and proton-antiproton annihilation into an electron-positron pair due to the two-photon intermediate state in a dispersion-theoretic treatment in terms of the proton Compton scattering amplitude. The possible resonances in the annihilation channel which might enhance this contribution are discussed. It turns out in the case of negligible electron mass that up to spin 2 only a 2^+ particle can contribute via real Compton scattering, which could give a 10 % effect in the cross-section with reasonable choice of the coupling constants. Even if we relax the restriction that the two-photon intermediate state should be real, scalar and pseudoscalar particles do not contribute; however, a 1^+ particle might appear.

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Introduction

Higher order electromagnetic corrections in electron-proton scattering and annihilation of proton-antiproton pairs into an electron-positron pair are again of interest, since in the last year so many new resonances have been discovered. There is now the possibility that the smallness of the electromagnetic coupling constant may be, at least partially, compensated by the enhancement due to a resonant state. In this connection, the $\bar{33}$ resonance has been investigated by several authors ^{1), 2)}. They have shown that this contribution is small up to 1 GeV. The radiative corrections of quantum electrodynamics have been calculated up to 5 GeV ³⁾. In the scattering experiments at very high momentum transfers, planned with the new machines in Cambridge, Hamburg and Stanford, the latter can easily be taken into account.

In this paper, we want to consider resonances appearing in the annihilation channel and the deviations caused by them from the Rosenbluth formula ⁴⁾ of electron-proton scattering and from the one-photon exchange differential cross-section for nucleon-antinucleon annihilation into an electron-positron pair. We use dispersion relations in t as already proposed in this connection by Drell and Fubini ²⁾ and retain only the one and two-photon intermediate state in the evaluation of the spectral function, neglecting the contribution of the s cut which has been proved by these authors to be small up to 1 GeV.

After introducing kinematics and useful shorthand notations we describe in the first Section a suitable set of invariants for the scattering amplitude. In Section II the absorptive parts of the amplitudes are calculated in terms of the amplitudes of Compton scattering. We give the formulae for the differential cross-sections for scattering and pair annihilation in Section III and survey the possible resonances in the t channel in Sections IV and V. In the last Section the results are discussed.

I. Kinematics and invariants

We shall consider simultaneously the two processes: electron-proton scattering and proton-antiproton annihilation into an electron-positron pair (Fig. 1).

Fig. 1

The scattering amplitude defined by

$$S = 1 - \frac{(2\pi)^4 i T}{(16 q_1^0 q_2^0 p_1^0 p_2^0)^{1/2}} \quad (1)$$

depends on the three variables *)

$$\begin{aligned} s &= (p_1 + q_1)^2 \\ t &= (q_1 - q_2)^2 \\ \bar{s} &= (p_1 - q_2)^2. \end{aligned} \quad (2)$$

*) We use the metric $ab = a_0 b_0 - \vec{a} \vec{b}$

Further, we use the four vectors

$$\begin{aligned}
 P &= \frac{1}{2}(p_1 + p_2) \\
 Q &= \frac{1}{2}(p_1 - p_2) \\
 R &= \frac{1}{2}(q_1 + q_2) \\
 \mathcal{P} &= P - \frac{PR}{R^2}R \\
 \mathcal{R} &= R - \frac{PR}{P^2}P \\
 S_\mu &= \varepsilon_{\mu\sigma\tau} \mathcal{P}^\sigma \mathcal{R}^\tau Q^\tau.
 \end{aligned} \tag{3}$$

A normalized four-vector V is called

$$\tilde{V} = \frac{V}{|V|}.$$

In the annihilation channel of the Compton amplitude which appears also in our work (Fig. 2) the following additional variables and vectors are used

$$\begin{aligned}
 \sigma &= (p_1 - k_1)^2 \\
 \tau &= t \\
 \bar{\sigma} &= (p_1 - k_2)^2
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 K &= \frac{1}{2}(k_2 - k_1) \\
 \Pi &= P - \frac{PK}{K^2}K \\
 N_\mu &= \varepsilon_{\mu\sigma\tau} \Pi^\sigma K^\tau Q^\tau.
 \end{aligned} \tag{5}$$

In general, six invariant amplitudes are sufficient to describe the scattering of two fermions of unequal masses. One can choose these to be the five Fermi invariants and

$$U = \bar{u}_e \tilde{\gamma} u_e \bar{u}_N \tilde{\gamma} u_N \quad (6) ^*)$$

which have the advantage of being free from kinematical singularities. In our approximation with vanishing mass of the electron, which is justified at high electron energies, S, P, and T cannot appear if we suppose the electron has only ordinary electromagnetic interactions and weak interactions of V-A type ^{**)}. Therefore, the invariant scattering amplitude T can be written

$$T = B_1 V + B_2 A + B_3 U \quad (7)$$

$$\begin{aligned} V &= \bar{u}_e \gamma^\mu u_e \bar{u}_N \gamma_\mu u_N \\ A &= \bar{u}_e \gamma_5 \gamma^\mu u_e \bar{u}_N \gamma_5 \gamma_\mu u_N \end{aligned} \quad (8)$$

*) The spinors are normalized to $u^\dagger u = 2E$.

**) In the framework of Lagrangian theory this can, for instance, be seen by observing that the electromagnetic Lagrangian is invariant under the transformation $\Psi_e \rightarrow \gamma_5 \Psi_e$, $m_e \rightarrow -m_e$. Requiring the same invariance for the matrix elements we find that S, P, and T must be multiplied by an odd power of the electron mass ⁵⁾.

The corresponding decomposition of the Compton amplitude is, according to Hearn and Leader ⁶⁾, $\bar{u}_N F^{\mu\nu} u_N$, where ^{*})

$$F^{\mu\nu} = A_1(\sigma_1 \tau_1 \bar{\sigma}) \tilde{\pi}^\mu \tilde{\pi}^\nu + A_2 \tilde{N}^\mu \tilde{N}^\nu + \\ A_3(\tilde{\pi}^\mu \tilde{N}^\nu - \tilde{N}^\mu \tilde{\pi}^\nu) \gamma_5 + A_4 \tilde{\pi}^\mu \tilde{\pi}^\nu \gamma_K + \quad (9) \\ A_5 \tilde{N}^\mu \tilde{N}^\nu \gamma_K + A_6(\tilde{\pi}^\mu \tilde{N}^\nu + \tilde{N}^\mu \tilde{\pi}^\nu) \gamma_5 \gamma_K.$$

^{*}) Our A_i differ from the definition in Ref. ⁶⁾ by a factor $(2\pi)^6$.

II. Dispersion relations

If we neglect the s cut of the amplitude, Fig. 1, and retain only the one and two-photon exchange intermediate state in the t channel the absorptive part $\beta_{2\gamma}$ of $T_{2\gamma}$ in $T = T_{1\gamma} + T_{2\gamma}$ reads

$$\beta_{2\gamma} = -\pi (2\pi)^3 \sum_{\lambda, \mu} \int \frac{d^3 k_1 d^3 k_2}{4k_1^0 k_2^0} \bar{u}_e \langle \bar{q}_1 | j_e(0) | k_1, k_2, \lambda, \mu \rangle \quad (10)$$

$$\cdot \langle k_1, k_2, \lambda, \mu | j_N(0) | \bar{p}_2 \rangle u_N \delta(k_1 + k_2 - q_2 - \bar{q}_1)$$

Fig. 2

Using perturbation theory in the electron part one arrives at

$$\beta_{2\gamma} = \pi \frac{e^2}{(2\pi)^3} \int \frac{d^4 K}{D} \delta(Q^2 - K^2) \delta(4QK) \theta(Q-K) \theta(Q+K) \cdot$$

$$\bar{u}_e \left\{ Q^2 [R^\mu \gamma^\nu + \gamma^\mu R^\nu + \frac{1}{2} (\gamma^\mu \not{K} \gamma^\nu + \gamma^\nu \not{K} \gamma^\mu)] + \right. \quad (11)$$

$$\left. \frac{KR}{2} [\gamma^\mu \not{K} \gamma^\nu - \gamma^\nu \not{K} \gamma^\mu] \right\} v_e \cdot \bar{u}_N F_{\mu\nu} u_N$$

$$D = Q^4 - (RK)^2 \quad (12)$$

which can be written

$$\beta_{2\gamma} = \frac{\pi e^2}{(2\pi)^3} \bar{u}_e \gamma^\mu v_e \bar{u}_N [J_\mu + (J_{\mu\beta} + \gamma_5 \mathcal{L}_{\mu\beta}) \gamma^\beta] u_N \quad (13)$$

with

$$J_\mu = \int \frac{d^4 K}{D} \delta(Q^2 + K^2) \delta(4QK) (A_1 - A_2) Z_\mu$$

$$J_{\mu\beta} = \int \frac{d^4 K}{D} \delta(Q^2 + K^2) \delta(4QK) (A_4 - A_5) Z_\mu K_\beta$$

(14)

$$\mathcal{L}_{\mu\beta} = \int \frac{d^4 K}{D} \delta(Q^2 + K^2) \delta(4QK) A_6 Y_\mu K_\beta$$

$$Z_\mu = 2Q^2 R^\sigma (\tilde{\pi}_\mu \tilde{\pi}_\sigma + \frac{1}{2} \tilde{K}_\mu \tilde{K}_\sigma)$$

$$Y_\mu = 2Q^2 R^\sigma (\tilde{\pi}_\mu \tilde{N}_\sigma + \tilde{N}_\mu \tilde{\pi}_\sigma).$$

(15)

Some simplification was achieved putting the electron mass m equal zero.

This makes A_3 disappear since the corresponding contribution is proportional to m . E.g., the π^0 intermediate state is contained in the Compton amplitude in A_3 ⁶⁾ and one knows that the decay amplitude of π^0 into an electron positron pair is proportional to m . Due to the orthogonality of Q and K the integrals (14) must be linear combinations of products of the three independent vectors P , R and S . Because of symmetry arguments I_μ can only depend on P and R , whereas the other two integrals contain also S , but $I_{\mu\beta}$ only bilinearly and $\mathcal{L}_{\mu\beta}$ linearly. By straightforward calculations one obtains for the absorptive parts $B_{i,A}$ of the amplitudes B_i from §2.8

$$\begin{aligned}
 B_{1,A} = & \frac{\pi e^2}{(2\pi)^3} \frac{1}{\beta^2 R^2} \int \frac{d^4 K}{D} \delta(Q^2 + K^2) \delta(4QK) \left\{ \frac{(RP)^2 (ZP)}{M} (A_1 - A_2) + \right. \\
 & + \left[(ZP)(KR)(PR) + \frac{1}{Q^2} (ZS)(KS) \right] (A_4 - A_5) + \\
 & + \frac{1}{R^2} \left[(YP)(KS)(RP) - (YS)(KQ)P^2 \right] A_6 \left. \right\}
 \end{aligned}
 \tag{16}$$

$$\begin{aligned}
 B_{2,A} = & \frac{\pi e^2}{(2\pi)^3} \frac{1}{\beta^2} \int \frac{d^4 K}{D} \delta(Q^2 + K^2) \delta(4QK) \left\{ - \frac{(ZP)(RP)}{M} \cdot \right. \\
 & \cdot (A_1 - A_2) - \frac{1}{\beta^2} \left[(ZP)(KS)(RP) + (ZP)(KQ)P^2 \right] (A_4 - A_5) - \\
 & - \frac{1}{R^2} (YP)(KS) A_6 \left. \right\}
 \end{aligned}
 \tag{17}$$

$$\begin{aligned}
B_{3,A} = & \frac{\pi e^2}{(2\pi)^3} \int \frac{d^4 K}{D} \delta(Q^2 + K^2) \delta(4QK) \left\{ \frac{ZP}{M} (A_1 - A_2) + \right. \\
& + [(Z\hat{P})(K\hat{P}) - (Z\hat{S})(K\hat{S})] (A_4 - A_5) + \\
& \left. + \frac{1}{S^2} [(Y\hat{P})(K\hat{S})(PR) - (YS)(KQ)P^2] A_6 \right\}
\end{aligned} \quad (18)$$

The dispersion relations for the amplitudes are given by

$$B_i = \frac{1}{\pi} \int_0^{\infty} \frac{B_{i,A}(t', \bar{s}) dt'}{t' - t} \quad (19)$$

The contribution to the amplitudes B_i from the one-photon-exchange diagram is

$$\begin{aligned}
B_1^{1\gamma} &= \frac{e^2}{(2\pi)^6} \frac{1}{t} \left[G_2(t) - \frac{(PR)^2}{MR^2} 2 F_2(t) \right] \\
B_2^{1\gamma} &= \frac{e^2}{(2\pi)^6} \frac{1}{t} 2 \frac{PR}{M} F_2(t) \\
B_3^{1\gamma} &= - \frac{e^2}{(2\pi)^6} \frac{1}{t} 2 \frac{P^2}{M} F_2(t)
\end{aligned} \quad (19')$$

where $F_1(t)$ and $F_2(t)$ are the normalized proton form factors and G_2 is defined in Eq. (20').

III. Cross-sections

For unpolarized electrons and protons the scattering cross-section in the laboratory system, splitting the interference term between the one and two-photon exchange amplitude from the Rosenbluth cross-section is given by (ϑ is the scattering angle in the laboratory system)

$$d\sigma = (d\sigma)_{\text{Rosenbluth}} + (d\sigma)_{\text{Mott}} (2\pi)^6 t \cdot$$

$$\cdot 2 \operatorname{Re} \left\{ - \left[-F_1 + \frac{t}{2M^2} \tan^2 \frac{\vartheta}{2} G_2 \right] B_1 + G_2 \frac{t}{2M^2} B_2 \tan^2 \frac{\vartheta}{2} \right. \quad (20)$$

$$\left. \cdot \left[1 - \frac{4M^2}{t} \sin^2 \frac{\vartheta}{2} \right]^{\frac{1}{2}} - \left[-F_1 + \frac{t}{4M^2} \tan^2 \frac{\vartheta}{2} G_2 \right] B_3 \right\}$$

$$G_1 = F_1 + \frac{t}{2M} F_2, \quad G_2 = F_1 + 2M F_2. \quad (20')$$

Similarly, for proton-antiproton annihilation into an electron-positron pair the differential cross-section in the centre of mass reads

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{c.m.}} = \frac{e^4}{4(4\pi)^2} (t(t-4M^2))^{-\frac{1}{2}} \left\{ |G_2|^2 (1 + \cos^2 \psi) + |G_1|^2 \frac{4M^2}{t} \sin^2 \psi + \right.$$

$$+ (2\pi)^6 t + 2 \operatorname{Re} \left\{ B_1 \left[G_2^* (1 + \cos^2 \psi) + G_1^* \frac{4M^2}{t} \sin^2 \psi \right] + \right. \quad (21)$$

$$+ 2 B_2 G_2^* \cos \psi \left(1 - \frac{4M^2}{t} \right)^{\frac{1}{2}} + B_3 \left[G_2^* \cos^2 \psi + \frac{4M^2}{t} \right.$$

$$\left. \left. , G_1^* \sin^2 \psi \right] \right\}$$

where t is now the total energy in the centre of mass and ψ the angle between the incoming proton and outgoing electron. In the GeV region both formulae can be taken over for muon scattering and annihilation into muons respectively. Clearly, the angular dependence of the corrections depends on the specific form of the $B_i(\bar{s}, t)$.

IV. Resonances in the Compton amplitude

Of course, the correction terms in Eqs. (20) and (21) are smaller by a factor α than the pure one-photon exchange contribution. Therefore they should have an appreciable influence only if they are enhanced by resonances in the Compton amplitude. We shall not consider resonances in the G cut, since it has been shown in an equivalent treatment in Ref. ²⁾ that, even though they produce a big imaginary part, the real part of T is small and hence also the interference term with the purely real Rosenbluth amplitude is small. The A_i which follow from resonances in the γ channel are listed in Table I in terms of the possible photon and nucleon vertices, which are given in the appendix.

The sign of proportionality in Table I indicates that a factor $\text{const.} \cdot (t-t_r)^{-1}$ is omitted. One sees that the only resonance which can contribute to electron-proton scattering and pair annihilation of protons into an electron pair via real Compton scattering is the tensor particle 2^+ . If the G parity is $+1$ the tensor resonance is to be included into the isoscalar part of $F^{\mu\nu}$ with $G = -1$ on the other hand in the isovector part. The way in which the A_i enters Eqs. (10), (17), (18) shows together with the coupling possibilities in Table I that there are only two independent coupling parameters left.

For $II^{2^+} \times 1^{2^+}$ and $III^{2^+} \times 1^{2^+}$ we arrive at

$$\begin{aligned} B_{1,A}^{(1)} &\sim - \frac{(\tilde{P}\tilde{R})^3}{M} \left(\frac{t-4M^2}{t} \right)^{1/2} \\ B_{2,A}^{(1)} &\sim - \frac{(\tilde{P}\tilde{R})^2}{M} \\ B_{3,A}^{(1)} &\sim \frac{\tilde{P}^2(\tilde{P}\tilde{R})}{M} \left(\frac{t-4M^2}{t} \right)^{1/2} \end{aligned} \quad (22)$$

TABLE I

A_i from resonances in the γ channel;
the symbols are explained in the appendix.

resonance	photon vertex	nucleon vertex	combination	contribution to A_i	contr. to B_i
0^+	I^{0^+}	1^{0^+}	$I^{0^+} \times 1^{0^+}$	$A_1 = A_2$	no
0^-	I^{0^-}	1^{0^-}	$I^{0^-} \times 1^{0^-}$	A_3	no
1^+					no
1^-					no
2^+	$I^{2^+}, II^{2^+}, III^{2^+}$	$1^{2^+}, 2^{2^+}$	$I^{2^+} \times 1^{2^+}$	$A_1 = A_2$	no
			$I^{2^+} \times 2^{2^+}$	$A_1 = A_2, A_4 = A_5$	no
			$II^{2^+} \times 1^{2^+}$	$A_1 \sim \pi^2 + A'_1, A_2 \sim A'_1$	yes
			$II^{2^+} \times 2^{2^+}$	$A_1 \sim M + A'_1, A_2 \sim A'_1$ $A_4 \sim \frac{(PK)}{Q^2}, A_6 \sim \frac{Q^2 \pi }{2 N }$	yes
			$III^{2^+} \times 1^{2^+}$	$A_1 \sim A'_2, A_2 \sim \pi^2$	yes
			$III^{2^+} \times 2^{2^+}$	$A_1 \sim A'_2, A_2 \sim M + A'_2,$ $A_5 \sim \frac{(PK)}{Q^2}, A_4 \sim -\frac{Q^2 \pi }{2 N }$	yes
2^-	I^{2^-}, II^{2^-}	1^{2^-}	$I^{2^-} \times 1^{2^-}$	A_3	no
			$II^{2^-} \times 1^{2^-}$	A_3	no

and for $\text{II}^{2+} x_2^{2+}$ and $\text{III}^{2+} x_2^{2+}$ we get

$$\begin{aligned} B_{1,A} = & \frac{x}{1-x^2} \frac{1}{\mu} \left\{ 4(2-x^2) \ln 2 - \frac{1}{2}(1-x^2) - (2-x^2)(y+z) \right\} + \\ & + \frac{x}{1-x^2} \frac{\lambda_r}{\mu_r} \left\{ (4-x^2) 2 \ln 2 - 1 + x^2 + \frac{2-x^2}{x} (y-z) - \right. \\ & \left. - 2(y+z) \right\} \end{aligned} \quad (23)$$

$$\begin{aligned} B_{2,A} = & \frac{1}{1-x^2} \left\{ 6x^2 \ln 2 - 1 + x^2 + x[(2-x)y - (2+x)z] \right\} + \\ & + \frac{\mu}{2} \frac{\lambda_r}{\mu_r} \frac{1}{1-x^2} \left\{ 2x^2 \ln 2 + \frac{1-x^2}{6} + x(y-z) \right\} \end{aligned}$$

$$\begin{aligned} B_{3,A} = & \frac{x}{1-x^2} \frac{1}{\mu} \left\{ \frac{1}{2}(1-x^2) - 2(5-x^2) \ln 2 + (1-x^2)(y+z) \right\} - \\ & - \frac{x}{1-x^2} \frac{\lambda_r}{\mu_r} \left\{ 4(3+x^2) \ln 2 - \frac{5}{6}(1-x^2) + \frac{2}{x} [(1-x)^2 y - \right. \\ & \left. - (1+x)^2 z] \right\} \end{aligned}$$

with

$$\mu = \frac{\sqrt{t}}{2}, \quad \mu_r = \frac{\sqrt{t_r}}{2}$$

$$\lambda = \frac{(t-4M^2)^{1/2}}{2}, \quad \lambda_r = \frac{(t_r-4M^2)^{1/2}}{2}$$

$$x = \cos \theta = -\frac{PR}{2\mu}, \quad y = (1-x) \ln(1-x)$$

$$z = (1+x) \ln(1+x).$$

Since in this model for the imaginary part (Fig. 3)

Fig. 3

Imaginary part for e - p scattering with a resonance in the t channel. The broken line indicates that the photons are real.

we only take the contribution of a tensor particle this means that in a partial wave expansion in the t channel we take only d waves. The imaginary part then takes the form

$$M = a(t) P_2(\cos \theta).$$

Separating out a single term of high spin in the expansion in partial waves clearly causes a lacking of the unitarity properties of the amplitude with respect to this variable. A dispersion relation in t with $\cos \theta = \frac{s-s_0}{t-t_0}$ fixed avoids this difficulty, whereby the cut in t remains unchanged.

V. Simple tensor exchange model

The essential features of the tensor resonance contribution to electron-proton scattering can also be derived from the direct exchange of a tensor particle between electron and nucleon (Fig. 4)

Fig. 4

Exchange of a resonance particle

using the effective interaction density $\mathcal{H}_N$ with the nucleon and $\mathcal{H}_e$ with the electron respectively

$$\begin{aligned}
 \mathcal{H}_N = & g_{N1} \frac{1}{2} \{ (\partial_\mu \bar{\Psi}_N) (\partial_\nu \Psi_N) + (\partial_\nu \bar{\Psi}_N) (\partial_\mu \Psi_N) \} B^{\mu\nu} + \\
 & + g_{N2} \frac{i}{8} \{ [\bar{\Psi}_N \gamma_\mu, (\partial_\nu \Psi_N)] - [(\partial_\nu \bar{\Psi}_N), \gamma_\mu \Psi_N] + \\
 & + (\mu \leftrightarrow \nu) \} B^{\mu\nu}
 \end{aligned} \tag{24}$$

$$\mathcal{H}_e = e^4 g_e \frac{i}{s} \left\{ [\bar{\Psi}_e \gamma_\mu (\partial_\nu \Psi_e)] - [(\partial_\nu \bar{\Psi}_e), \gamma_\mu \Psi_e] + \right. \\ \left. + (\mu \leftrightarrow \nu) \right\} B^{\mu\nu}. \quad (25)$$

Calculating the matrix element in this way one finds

$$T_{2\gamma} = \bar{u}_e \gamma^\mu u_e \bar{u}_N [C_1 + \gamma^\mu C_2] u_N + \\ + \bar{u}_e \gamma^\mu u_e \bar{u}_N \gamma^\mu u_N C_3 \quad (26)$$

where

$$C_1 = \frac{e^4 g_e g_{N1}}{(2\pi)^6} \frac{2(P \cdot R)}{t - t_m} \\ C_2 = \frac{e^4 g_e g_{N2}}{(2\pi)^6} \frac{1}{t - t_m} \\ C_3 = \frac{e^4 g_e g_{N2}}{(2\pi)^6} \frac{R \cdot P}{t - t_m}. \quad (27)$$

The invariants in (26) can be transformed to the set (6), (8) and we can express the B_i in terms of C_i

$$\begin{aligned}
 B_1 &= \frac{(PR)^2}{MR^2} C_1 + (RP) C_2 + C_3 \\
 B_2 &= -\frac{PR}{M} C_1 + Q^2 C_2 \\
 B_3 &= \frac{P^2}{M} C_1
 \end{aligned} \tag{28}$$

Calculating the cross-section (20) the C_1 terms in B_1 and B_2 just cancel.

The result obtained from the more complicated model in Section IV (Fig. 3) differs from (27) and (28) only by logarithmic and weakly varying functions which play the role of form factors.

VI. Conclusions and experimental consequences

According to the Rosenbluth formula plotting $d\sigma/d\sigma_{\text{Mott}}$ at constant momentum transfer t against $\tan^2 \theta/2$ should give a straight line whose slope is limited ⁷⁾. The latest experimental data seem to be in good agreement with such a behaviour ⁸⁾.

The model of Section V gives a simple and transparent formula for the correction terms to the differential cross-section due to the exchange of a tensor particle

$$d\sigma_{\text{lab}} = d\sigma_{\text{Rosenbluth}} + d\sigma_{\text{Mott}} \frac{e^2 g_e t}{1 - t_r/t} \cdot \frac{1}{4} \left[\frac{t - 4M^2}{t} - \frac{4M^2}{t} \tan^2 \frac{\theta}{2} \right]^{1/2} \cdot \left\{ 4M G_1 g_{N_1} - g_{N_2} \left[G_2 + F_1 + \frac{t}{M^2} G_2 \tan^2 \frac{\theta}{2} \right] \right\}. \quad (29)$$

To achieve a 10% deviation from the Rosenbluth formula at a momentum transfer $t = 30\text{f}^{-2}$ the products of the coupling constants $g_e g_{N_1}$ have to be of the order one if we take t_r in the region of the known pion resonances. This result is not altered by taking into account the "form factors" of the model of Section IV. However, it should be stressed that even if the total deviation from the Rosenbluth terms is about 10% as above the deviation from the straight line behaviour in $\tan^2 \theta/2$ will at most be 3.5% at $\tan^2 \theta/2 = 1$ and smaller at larger angles. The biggest deviations from the straight line behaviour occur at small angles and are mainly due to the g_{N_1} term in (29). Since, at present, the experimental errors are of the order of the above corrections and, within this accuracy, experimental data seem to agree with a straight line; a detailed numerical analysis of the correction terms seems not worthwhile at the moment. But in future experiments the two-photon contribution might be measured directly.

The contribution from the two-photon exchange diagram enters with opposite sign into the electron-proton and positron-proton scattering cross-sections. Hence, it can explicitly be separated by performing both experiments. This also provides a means of distinguishing if observed deviations from the Rosenbluth formula stem from two-photon contributions or from a Regge pole nature of the photon ⁹⁾ as conjectured by Blankenbecler, Cook and Goldberger ¹⁰⁾; since in the latter case the corrections should have the same sign.

For annihilation of protons into an electron-positron pair there are yet no data available but experiments are proposed for the near future ¹¹⁾. Here the one and the two-photon exchange contribution can be separated even more easily due to their different properties under charge conjugation by simply changing the symmetry of detecting electrons and positrons.

Summarizing, we can say that a scalar and a pseudoscalar particle cannot contribute in the t channel since they cannot decay into an electron-positron pair if $m_e = 0$. In the case of a scalar particle, because of parity and angular momentum conservation, the only state available for the pair is 3P_0 but this state does not exist for mass zero particles since their spin points in the direction of motion which is perpendicular to the orbital angular momentum and hence cannot add up to $J = 0$. For a pseudoscalar particle the pair must be in a 1S_0 state but with $m_e = 0$ the electromagnetic interactions cannot mediate an effective pseudoscalar coupling to the electron ^{*)} and for the axial vector interaction this state is not available because of the helicity conservation. A vector particle contributes to the form factor in the Rosenbluth formula and an axial vector particle is excluded in the case of a two-photon intermediate state. However, taking other than the two-photon state in the unitarity relation it may contribute as shown at the end of the appendix. It has been discussed in the framework of a partial wave analysis by Gourdin and Martin ¹²⁾. In the direct exchange model it would lead to a cross-section

*) See footnote ^{**) on page 4.}

$$d\sigma_{\text{cal}} = d\sigma_{\text{Rosenbluth}} - d\sigma_{\text{Mott}} \frac{e^2 g_{eA} g_{NA}}{1 - t r/t} \cdot \frac{t}{M^2} G_2 \frac{t_0^2}{2} \left[1 - \frac{4M^2}{t} - \frac{4M^2}{t} \frac{t_0^2}{2} \right]^{1/2}. \quad (30)$$

Equation (30) has the same shape as the last term in (29) but a weaker t dependence. Further, the deviations from the straight line behaviour are much smaller than the ones stemming from the first term in (29). It should also be mentioned that only one of the two possible couplings of the axial vector particle to the nucleon gives interference with the Rosenbluth amplitude, hence there is only one coupling parameter in (30) in contrast to (29).

The tensor resonance is of particular interest in connection with the prediction of a 2^{++} particle from the concept of Regge poles in the theory of elementary particles ¹³⁾. Indications of a particle with the predicted mass of one GeV have been found in the $\pi - \pi$ system ¹⁴⁾. Therefore, electron-proton scattering at high momentum transfer above 1 GeV and annihilation experiments possibly provide a means to decide whether it exists.

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APPENDIX

We discuss first the vertices between the resonance particle and the two photons. Starting from the most general expression formed with the two independent four vectors Q, K and the tensors $g^{\mu\nu}$ and

$$S^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} Q_\rho K_\sigma = Q^2 (\tilde{\pi}^\mu \tilde{N}^\nu - \tilde{N}^\mu \tilde{\pi}^\nu)$$

and taking into account crossing symmetry, gauge invariance and the subsidiary condition to fix the angular momentum of the resonance particle we arrive at the expressions of Table I, where the following symbols have been used (μ, ν are always polarization indices of the photons, ρ, σ of the tensor particle)

$$I^{0+} = w^{\mu\nu}$$

$$I^{0-} = S^{\mu\nu}$$

$$I^{2+} = w^{\mu\nu} (K^\rho K^\sigma + \frac{Q^2}{3} d^{\rho\sigma})$$

$$II^{2+} = w^{\mu\rho} w^{\nu\sigma} + w^{\nu\rho} w^{\mu\sigma} - \frac{2}{3} d^{\rho\sigma} w^{\mu\nu}$$

$$III^{2+} = S^{\mu\rho} S^{\nu\sigma} + S^{\nu\rho} S^{\mu\sigma} + \frac{2}{3} Q^4 d^{\rho\sigma} w^{\mu\nu}$$

$$I^{2-} = S^{\mu\nu} (K^\rho K^\sigma + \frac{Q^2}{3} d^{\rho\sigma})$$

$$II^{2-} = (S^{\mu\rho} w^{\nu\sigma} - S^{\nu\rho} w^{\mu\sigma} + S^{\mu\sigma} w^{\nu\rho} - S^{\nu\sigma} w^{\mu\rho})$$

with

$$w^{\mu\nu} = g^{\mu\nu} - \tilde{Q}^\mu \tilde{Q}^\nu - \tilde{K}^\mu \tilde{K}^\nu = \tilde{\pi}^\mu \tilde{\pi}^\nu + \tilde{N}^\mu \tilde{N}^\nu$$

$$d^{\rho\sigma} = g^{\rho\sigma} - \tilde{Q}^\rho \tilde{Q}^\sigma.$$

Similarly, the possible nucleon-resonance vertices that can be constructed with $C = +1$ are the following

$$1^{0+} = I$$

$$1^{0-} = \gamma_5$$

$$1^{2+} = p^\rho p^\sigma - \frac{p^2}{3} d^{\rho\sigma}$$

$$2^{2+} = p^\rho \gamma^\sigma + p^\sigma \gamma^\rho - \frac{2M}{3} d^{\rho\sigma}$$

$$1^{2-} = (p^\rho p^\sigma - \frac{p^2}{3} d^{\rho\sigma}) \gamma_5.$$

For the vertex of an axial vector particle with two photons one can, in general, write down six invariants, but applying gauge invariance and crossing symmetry only two remain and we can write for it (K_1, K_2 and $K = K_1 + K_2$ being the momenta of the photons $a_{1,2}$ and the axial vector particle b^ν respectively):

$$\begin{aligned} & \varepsilon_{\mu\nu\rho\sigma} a_1^\mu a_2^\nu K_1^\rho K_2^\sigma [(K_1 - K_2) b] (K_1^2 - K_2^2) F_1(K_1^2, K_2^2, K^2) + \\ & + \varepsilon_{\mu\nu\rho\sigma} a_1^\mu a_2^\nu K_1^\rho K_2^\sigma (K b) F_2(K_1^2, K_2^2, K^2). \end{aligned}$$

The first term vanishes if the photons are on the mass shell but can mediate an effective axial vector coupling of b^ν to the electron via two virtual photons. The contribution of the second term to the decay into an electron-positron pair with $m_e = 0$ vanishes as does the one of a pseudoscalar particle; furthermore, the whole term vanishes if the subsidiary condition $Kb = 0$ is satisfied.

R E F E R E N C E S

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