ON THE ANGULAR DISTRIBUTION OF THE
BROOKHAVEN 1962 NEUTRINO EXPERIMENT

Jørgen Løvseth

CERN - Geneva

6584/p/TH. 349
28 May 1963
The angular distribution for the BNL experiment \(^1\) has been given by J.H. Gaillard \(^2\). It is shown in Fig. 1 and contains the 34 events with "long" single tracks corresponding to muons with a visible momentum larger than 300 MeV/c. The experimental distribution is thought to have no particular angular bias, except that it contains \(\sim 5\) cosmic ray events, mostly at high angles \(^3\).

The purpose of this note is to present a calculation which shows that this experimental angular distribution, within the rather large experimental uncertainties from bad statistics and uncertainties about the neutrino spectrum, is compatible with the currently accepted ideas of weak interactions \(^4\).

The interaction for the process

\[ \nu + n \rightarrow p + e^- \]  

is assumed to be of the form

\[ H = \frac{G}{\sqrt{2}} \langle K' | F_1 \gamma_\nu \gamma_\mu \frac{\mu_c}{2M} F_2 \sigma_{\alpha\beta} Q_\beta + \lambda F_1 (\gamma_\nu \gamma_\mu - i \frac{\sigma_{\alpha\beta}}{2M} F_2 Q_\beta) | K \rangle \times \]

\[ \times i \langle K' | \gamma_\alpha (1 + \gamma_5) | k \rangle \]  

where \(K,K'\) and \(k,k'\) are the initial, final 4-momenta and \(M\) and \(m\) the masses for nucleons and leptons respectively. \(G\) is the vector coupling constant and the parameters \(\mu, \lambda\) and \(b\) give the relative strength of the other couplings. The form factors \(F_1\) are functions of

\[ Q^2 = (k - k')^2 \]

and normalized so that

\[ F(0) = 1. \]
As nuclear model we will use the ideal Fermi gas. For simplicity we will consider only the case \( N = Z \) and neglect effects of an effective nucleon mass.

In the initial state all levels will be filled up to the Fermi momentum \( P_F \). For \( r_0 = 1.12 \) Fermi one has \(^5\)

\[
P_F = 2.67 \text{ MeV} / c.
\]

For a scattering event to take place, the exclusion principle requires that the \( 3 \)-momentum of the final nucleon, \( P' \), satisfies

\[
P' > P_F.
\]

To be able to compare with the experimental distribution, we require that the final lepton momentum \( p' \) satisfies

\[
p' > 300 \text{ MeV} / c.
\]

By neglecting the lepton mass except in the pseudoscalar term and assuming the form factors to be real, the differential cross-section can be obtained as

\[
\frac{d\sigma}{d\Omega} = \frac{G^2 M}{8 \sqrt{2}} \frac{3}{4 \pi E_F} \int d^3p' e^{i\mathbf{q}\cdot\mathbf{p}'} \frac{e^{i\mathbf{q}\cdot\mathbf{p}'}}{E_F} \left[ A(q^2) + (\cdot\cdot\cdot) B(q^2) + (\cdot\cdot\cdot) C(q^2) \right].
\]
The prime (on the integral) indicates that the conditions in Eqs. (5) and (6) must be fulfilled. $e$ and $e'$ are initial and final lepton energy, $E$ initial nucleon energy ($= \sqrt{p^2 + m^2}$). The expression in the square brackets represents the invariant part where

$$A(q^2) = \frac{q^2}{4} \left( 1 + \frac{q^2}{4m^2} \right) \left( -F_1^2 + \lambda F_A - \frac{q^2}{4m^2} \right) +$$

$$+ \frac{q^2}{2m^2} \left( F_1 + \mu F_2 \right)^2 - \frac{m}{4M} \lambda_b F_A F_p + \frac{q^2}{4m^2} b^2 F_A^2 \right) \right) \right)$$

$$B(q^2) = \frac{q^2}{M^2} \left( F_1 + \mu F_2 \right) \lambda F_A$$

$$C(q^2) = \frac{1}{4M} \left( F_1^2 + \lambda^2 F_A^2 + \frac{q^2}{4M^2} \lambda^2 F_A^2 \right)$$

$$s - \mu \equiv - \left( k + k' \right)^2 + \left( k - k' \right)^2 = -2k \left( k + k' \right).$$
The cross-section for the reaction

\[ \bar{v} + p \rightarrow n + \ell^+ \]  

(12)

is obtained by changing the sign of \( B \) in (7)\(^*\).

Some words about form factors. Assuming CVC and \( e-\mu \) universality, then

\[ F_1 = F_2 = F_3 = \left( \frac{4 + 4.25 q^2/M^2}{q^2} \right)^{-1}. \]  

(13)

this being an empirical representation of the nucleon isovector electromagnetic form factors\(^6\). About \( F_A \) and \( F_P \) little is known. For simplicity we will assume that they are of the same form,

\[ F_A = \left( \frac{4}{q^2/M_A^2} \right)^{-1}. \]  

(14)

\[ F_P = \left( \frac{4}{q^2/M_P^2} \right)^{-2}. \]  

(15)

\(^*\) That the covariant matrix element is a quadratic in \((s-u)\) depends solely on the current-current form of the interaction, as shown by Martin and Gourdin\(^7\). That only the sign of the linear term is changed for the antineutrino reaction depends, according to J.S. Bell\(^8\), only on PCT, unitarity, and the neglect of neutron-proton mass difference.
If the interaction is mediated by a vector boson of mass \( M_W \),
then this can be accounted for by using modified form factors \( F' \),
given by

\[
F'_i = F_i \left( 1 + Q^2 / M_{W}^2 \right)^{-1}, \quad i = A, \lambda, A \tag{16}
\]

\[
F'_{\mu} = F_{\mu} + \frac{\lambda^2}{b M_W^2} F_A \left( 1 + Q^2 / M_{W}^2 \right)^{-1}. \tag{17}
\]

For the coupling parameters we will take the usual values

\[
G_N M^4 = 10^{-5}, \quad \lambda = 1.25, \quad \mu = 3.7, \quad b = 8 \lambda = 10. \tag{18}
\]

The differential cross-section \( 17 \) has been calculated by a Monte-Carlo method and averaged over the appropriate neutrino spectrum for the BNL experiment \( 1 \). The calculated curves are shown together with the experimental points in Fig. 1.

As is seen, the curve with all form factors = \( F_0 \) (Eq. \( 13 \)) and \( M_W = \infty \) is very nearly the same as the one with \( F_A = 1 \) and \( M_W = 0.8 \) GeV. A more detailed study shows that it will be difficult to disentangle the influence of the axial form factor and the vector boson, if direct evidence is not obtained for the latter.

Lapidus \( 9 \) has suggested that the predominance of muons in this experiment might be due to a strong pseudoscalar coupling. To test this hypothesis we show a curve labelled "pure FS" which corresponds to
\[ F_P = F_e, \quad b = \sqrt{10} \times 10, \quad F_1 = F_2 = F_A = 0. \] Obviously it does not agree with the experimental distribution. One could make the shape better by reducing \( N_P \) in (15), but the value of \( b \) then required to make the total cross-section of right order would be even more difficult to accept.

A suppression of the pseudoscalar contribution by putting \( b = 0 \) would on the contrary improve the agreement with the experimental distribution. Effectively the same suppression can also be obtained by making the natural assumption that the one-pion pole dominates in the pseudoscalar interaction. Instead of (15) one then has

\[
F_P = F_{e'} = \left( 1 + q^2 / m^2 \right)^{-1} \left( 1 + m^2 / m^2 \right)
\]

This falls off much more rapidly than \( F_e \) with \( q^2 \).

Calculation shows that the pseudoscalar contribution \( (b = 10) \) is then quite negligible for this experiment.

In Fig. 2 we show the calculated contributions from \( \pi^- \) and \( K \) neutrinos separately \(^*)\). The fact that there are slightly more events in the forward direction than calculated, suggests that the intensity of \( K \) neutrinos may have been underestimated. An upward revision would strengthen the evidence against "neutrino flip" \(^1\).

\(^*)\) For the purpose of Fig. 1 the number of neutrinos has been assumed equal to the number of antineutrinos. For Fig. 2 the ratio between neutrinos and antineutrinos has been taken to be 1:1 for the pion and 3:1 for the kaon part, since \( K^+ \) is known to be more abundantly produced than \( K^- \).
Acknowledgements

The author is indebted to Drs. J.S. Bell and J.M. Gaillard for many discussions and helpful criticism.
REFERENCES

3) J.M. Gaillard, private communication.
4) Neutrino reactions have been treated by:
   Y. Yamaguchi, CERN 61-2 and Prog. Theor. Phys. 23, 1117 (1960),
   N. Cabibbo and R. Gatto, Nuovo Cim. 12, 304 (1960),
6) R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956) and 30, 482 (1958).
8) J.S. Bell, Lectures at CERN, unpublished.
9) L. Lapidus, JEFFT, in press.
FIGURE CAPTIONS

Fig. 1.

Data from the 1962 BNL experiment compared with theoretical curves. Unless otherwise indicated, form factors are taken equal to the electromagnetic form factor with $M_Q = \infty$.

Fig. 2.

Calculated cross-section for $\pi$ and $K$ neutrinos compared with experimental results.
\[
\frac{d\sigma}{d\Omega} \left/ \times 10^{-39} \text{cm}^2 / \text{STERAD}\right.
\]
\[
\frac{d\sigma}{d\Omega} \times 10^{-39} \text{cm}^2/\text{STERAD}
\]