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STATISTICAL THEORY RESULTS ON MULTIPLE PARTICLE PRODUCTION FROM

π - N COLLISION AT 3 GEV

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A B S T R A C T

Spectra and mean particle numbers for all produced particles in 3 GeV lab. energy π - N collisions are calculated according to a refined version of the Fermi model. One takes into account the existence of π - N isobars, conservation of isospin and uses unbiased value of the phase space integrals. Finally γ spectra from π^0 decay are given.

In this paper we propose to apply the statistical theory to $\pi - N$ collisions. We shall follow closely the methods and notations of Cerulus and Hagedorn ^{1),2)}, to which the reader is requested to refer for a discussion of the method.

The pion nucleon collision at 3 GeV gives rise to a set of many particle final states. Taking into account the conservation of strangeness and baryon number, these particles are:

$$\begin{aligned} N + n\pi & \quad ; \quad n = 1 - 6 \\ N^* + n\pi & \quad ; \quad n = 1 - 5 \\ \Lambda + K + n\pi & \quad ; \quad n = 0 - 3 \\ \Sigma + K + n\pi & \quad ; \quad n = 0 - 2 \\ \Xi + 2K + n\pi & \quad ; \quad n = 0 - 2 \\ N + K + \bar{K} + n\pi & \quad ; \quad n = 0 - 2 \\ N^* + K + \bar{K} + n\pi & \quad ; \quad n = 0 - 2 \end{aligned}$$

Here N^* is the $\pi - N$ isobar with mass 1.31 (in nucleon mass units) and mean life (according to its width) of 10^{-23} sec.

The probability for a certain set b , of states, can be written ^{1),2)}

$$P_b = f_{TS} \cdot \Omega_1^{n_1} \cdot \Omega_2^{n_2} \cdot \Omega_3^{n_3} \cdot g_b^*(E, m_1, \dots, m_n; \vec{P}=0) \quad (1)$$

In this formula the factor f_{TS} depends on isopin and spin multiplicity and also on the number of equal particles.

Ω_1 , Ω_2 and Ω_3 are the interaction volumes which in this statistical theory stand for the average of the square of the transition matrix elements. One uses Ω_1 for nucleons, isobars and pions (total number of such particles n_1) Ω_2 and Ω_3 are respectively for hyperons and K mesons (total number n_2 and n_3).

Finally we can write the probability for the final state as

$$\rho^*(E, m_1, \dots, m_n, \vec{P}) = \int d\vec{p}_1 \dots \int d\vec{p}_n \delta(E - \sum_i \sqrt{p_i^2 + m_i^2}) \delta(\sum_i \vec{p}_i - \vec{P}) \quad (2)$$

where the phase space integral is over all possible final states.

is the phase space integral for the final state; ($\vec{p} = 0$ in the c.m.s.).

This last factor, which is the most important, was calculated by a Monte Carlo method proposed by Cerulus and Hagedorn ³⁾. One obtains also by the same calculation the momentum spectra of each particle in the c.m.s., averaged over all angles.

$$\omega_n(\vec{p}) d\vec{p} \propto 4\pi p^2 dp \rho^*(E - \sqrt{m_n^2 + p^2}, m_1, \dots, m_n, \vec{p} = -\vec{p}_n) \quad (3)$$

gives the probability to have a momentum between $|\vec{p}|$ and $|\vec{p} + d\vec{p}|$ for the particle of mass m_n .

Results

We used for the Lorentz contracted interaction volumes

$$\Omega_1 = \Omega_2 = \frac{4\pi}{3} \lambda_\pi^3 \left(\frac{2}{E_{\text{c.m.s.}}} \right) \left(\frac{1}{2\pi} \right)^3 = 3.576$$

$$\Omega_3 = 1.788 = 0.5 \Omega_1$$

in units $\hbar = c = 1$

These volumes are suggested a.o. by a comparison of the statistical theory with experiments in $p\bar{p}$ annihilation at rest ⁴⁾.

For the probabilities P_b in percent of the different possible end states, we obtained :

Final states	$T = \frac{1}{2}$	$T = 3/2$
$N + \pi$	$P_b = 2.95$	$P_b = 2.51$
$N + 2\pi$	20.3	17.36
$N + 3\pi$	15.5	16.13
$N + 4\pi$	2.6	2.87
$N + 5\pi$	0.12	0.15
$N^* + \pi$	5.09	4.34
$N^* + 2\pi$	19.94	25.43
$N^* + 3\pi$	9.67	11.48
$N^* + 4\pi$	0.87	1.20
$\Lambda + K$	1.29	----
$\Lambda + K + \pi$	4.04	3.44
$\Lambda + K + 2\pi$	1.63	1.39
$\Lambda + K + 3\pi$	0.07	0.07
$\Sigma + K$	1.24	1.05
$\Sigma + K + \pi$	6.44	5.48
$\Sigma + K + 2\pi$	1.87	1.98
$\Sigma + K + 3\pi$	0.007	0.07
$\Xi + 2K$	0.30	0.13
$\Xi + 2K + \pi$	0.005	0.000
$N + K + \bar{K}$	3.65	1.57
$N + K + \bar{K} + \pi$	1.41	1.20
$N + K + \bar{K} + 2\pi$	0.05	0.05
$N^* + K + \bar{K}$	1.15	1.94
$N^* + K + \bar{K} + \pi$	0.13	0.15

Kinetic energy spectra of all particles, (in the centre of mass system) normalized to unit area are given at the end of this paper. One observes a peak at the end of the K and the Σ spectra in both cases with total isospin $T = \frac{1}{2}$ and $T = \frac{3}{2}$. These peaks arise from the reaction $\pi + N \rightarrow \Sigma + K$. The same remark is true for the $\sqrt{s}$ spectrum with $T = \frac{1}{2}$. For the average particle numbers $\langle n \rangle$ and average kinetic energies $\langle \epsilon \rangle$ in the centre of mass system (units 0.938 GeV) we have calculated the following numbers :

Particle	$T = 3/2$		$T = \frac{1}{2}$	
	$\langle n \rangle$	$\langle \epsilon \rangle$	$\langle n \rangle$	$\langle \epsilon \rangle$
N	0.864	0.217	0.831	0.229
π	2.54	0.387	2.34	0.401
N^*	0.446	0.194	0.369	0.203
Λ	0.0491	0.165	0.070	0.206
Σ	0.086	0.175	0.096	0.179
Ξ	0.0013	0.069	0.0030	0.0702
K	0.187	0.251	0.236	0.279
$\bar{K}$	0.0491	0.185	0.0639	0.214
γ	1.68	0.260	1.54	0.267

Nucleons and pions are either directly produced or come from the decay of N^* . End states with one nucleon and two or three pions are probably produced in non-central collisions. Statistical theory does not say anything about such peripheral interactions ⁵⁾.

It is conceivable that a statistical theory of the type we have considered is able to account well for the central collisions ; the over-all picture of $\pi^- - N$ collisions would then be obtained by combining our results with those of the peripheral collision model.

We think that the χ spectrum would be useful for experimentalists who want to know more about peripheral collisions. Our χ spectra would give an idea of the "central background" if one wants to select from the over-all χ spectrum, those χ coming from π^0 produced in peripheral collisions ^{6)*}).

Let us remark finally that the $\pi^- - p$ initial state is a mixture of $|T = 3/2, T_z = -\frac{1}{2}\rangle$ and $|T = \frac{1}{2}, T_z = -\frac{1}{2}\rangle$ states, in the $\pi^- - p$ case one has therefore to multiply the results of the statistical theory where $T = 3/2$ by $1/3$ and those where $T = \frac{1}{2}$ by $2/3$.

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Fig 1 and 2; π - spectrum; $\Delta \epsilon = 34.7 \text{ MeV}$; $\Sigma \frac{\Delta n}{\Delta \epsilon} = 1$

Fig 3 and 4; N - spectrum; $\Delta \epsilon = 34.7 \text{ MeV}$; $\Sigma \frac{\Delta n}{\Delta \epsilon} = 1$

Fig 5 and 6; K - spectrum; $\Delta \epsilon = 34.7 \text{ MeV}$; $\Sigma \frac{\Delta n}{\Delta \epsilon} = 1$

Fig 7 and 8: $\bar{K}$ -spectrum; $\Delta \epsilon = 34.7$ MeV; $\Sigma \frac{\Delta n}{\Delta \epsilon} = 1$

Fig 9 and 10; Λ -spectrum; $\Delta \epsilon = 34.7$ MeV; $\Sigma \frac{\Delta n}{\Delta \epsilon} = 1$

Fig 11 and 12; Σ - spectrum; $\Delta \epsilon = 34.7 MeV$; $\frac{\Delta n}{\Delta \epsilon} = 1$

Fig 13 and 14; Ξ - spectrum; $\Delta \epsilon = 34.7 \text{ MeV}$; $\Sigma \frac{\Delta N}{\Delta \epsilon} = 1$

Fig 15 and 16; γ - spectrum; $\Delta\varepsilon = 34.7$ MeV; $\Sigma \frac{\Delta n}{\Delta\varepsilon} = 1$