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TRIANGLE INEQUALITIES FOR SPIN EFFECTS

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When the amplitudes for three reactions obey a triangle relation, because of charge independence or some other law, the three cross-sections obey familiar inequalities ¹⁾. Michel ²⁾ has recently given further inequalities for the polarizations. The present letter shows how to derive inequalities for more complicated spin effects, like depolarization and spin correlation, and discusses their usefulness.

Suppose three reactions A,B,C are each described by N independent scalar amplitudes f_i ($i = 1, 2, \dots, N$), which are defined so that the differential cross-section is

$$\sigma = \sum_{i=1}^N |f_i|^2, \quad (1)$$

and which obey the triangle relations

$$f_i^A + f_i^B + f_i^C = 0 \quad (i = 1, 2, \dots, N). \quad (2)$$

It follows at once, using Schwartz's inequality, that

$$\sqrt{\sigma_A} + \sqrt{\sigma_B} \geq \sqrt{\sigma_C}, \quad (3)$$

(and permutations of A,B,C), or equivalently,

$$2(\sigma_A \sigma_B + \sigma_B \sigma_C + \sigma_C \sigma_A) \geq \sigma_A^2 + \sigma_B^2 + \sigma_C^2. \quad (4)$$

We now make three remarks.

- i) These inequalities depend only on σ being a positive definite quadratic function of the f_i . Polarization, depolarization, spin correlation etc., parameters as usually defined are not positive definite. However, we can combine them with one another and with the cross-section to form positive

definite quantities, which then obey inequalities similar to Eqs. (3) and (4).

ii) Equality holds in Eqs. (3) and (4) only if the amplitudes are proportional:

$$f_i^A = \lambda f_i^B = -\frac{\lambda}{1+\lambda} f_i^C, \quad (i = 1, 2, \dots, N) \quad (5)$$

for some real λ . If the f_i are replaced by N independent linear combinations, the condition is the same. But if the f_i are replaced by fewer than N terms, the condition becomes less restrictive.

iii) Suppose each amplitude f_i contains a part which satisfies the triangle law (2) plus a small part which violates it. Our inequalities will reveal the latter only if the former yield equality or near-equality. In this sense, an inequality based on a positive definite quantity like Eq. (1) becomes intrinsically more powerful as the number N of independent terms decreases.

As an illustration, consider nucleon-nucleon scattering. The amplitude for scattering given nucleons between initial and final c.m. momenta $\underline{p}_i$ and $\underline{p}_f$ is a matrix $\underline{M}$ in spin space ³⁾

$$\begin{aligned} \underline{M} = & a + c(\underline{\sigma}^{(1)} + \underline{\sigma}^{(2)}) \cdot \underline{N} + m \underline{\sigma}^{(1)} \cdot \underline{N} \underline{\sigma}^{(2)} \cdot \underline{N} + (g+h) \underline{\sigma}^{(1)} \cdot \underline{P} \underline{\sigma}^{(2)} \cdot \underline{P} + \\ & + (g-h) \underline{\sigma}^{(1)} \cdot \underline{K} \underline{\sigma}^{(2)} \cdot \underline{K}, \end{aligned} \quad (6)$$

where $\underline{\sigma}^{(1)}$ and $\underline{\sigma}^{(2)}$ are the Pauli spin operators of the two nucleons. $\underline{N}$, $\underline{P}$ and $\underline{K}$ are unit vectors in the directions $\underline{p}_i \times \underline{p}_f$, $\underline{p}_f + \underline{p}_i$ and $\underline{p}_f - \underline{p}_i$. The scalar amplitudes a, c, m, g, h are functions only of energy and scattering angle θ ($\underline{p}_i \cdot \underline{p}_f = p^2 \cos \theta$). Because of charge independence, the amplitudes for the three reactions

$$\begin{aligned}
A : n+p &\rightarrow n+p \\
B : n+p &\rightarrow p+n \\
C : p+p &\rightarrow p+p
\end{aligned} \tag{7}$$

obey a triangle law. Thus for every positive definite quadratic quantity constructed from a, c, m, g and h there are inequalities like (3) and (4). Many authors ^{3),4),5)} give the various observables in terms of these amplitudes. From the cross-section σ and polarization P we can form

$$\sigma = |a|^2 + |m|^2 + 2|c|^2 + 2|g|^2 + 2|h|^2 \tag{8}$$

$$\sigma(1 \pm P) = \frac{1}{2} |a-m|^2 + \frac{1}{2} |a+m \pm 2c|^2 + 2|g|^2 + 2|h|^2. \tag{9}$$

These lead to known inequalities ^{1),2)}. But it is simple to construct further independent positive definite quantities, using for example the depolarization parameters D, R, A and the spin-correlation parameter C_{NN} :

$$\sigma(1-D) = 4|g|^2 + 4|h|^2, \tag{10}$$

$$\sigma(1+D) = 2|a|^2 + 2|m|^2 + 4|c|^2, \tag{11}$$

$$\sigma(1 \pm 2P+D) = |a-m|^2 + |a+m \pm 2c|^2, \tag{12}$$

$$\sigma(1 + R \cos \frac{\theta}{2} - A \sin \frac{\theta}{2}) = 2|a|^2 + 2|c|^2 + 2|g-h|^2, \tag{13}$$

$$\sigma(1 - R \cos \frac{\theta}{2} + A \sin \frac{\theta}{2}) = 2|m|^2 + 2|c|^2 + 2|g+h|^2, \tag{14}$$

$$\sigma(1-C_{NN}) = |a-m|^2 + 4|g|^2, \quad (15)$$

$$\sigma(1+C_{NN}) = |a+m|^2 + 4|c|^2 + 4|h|^2, \quad (16)$$

$$\sigma(1 \pm 2P + C_{NN}) = |a+m \pm 2c|^2 + 4|h|^2, \quad (17)$$

and we could construct a great many more. The quadratic forms (10) - (17) contain only two or three terms each: according to remark iii) above, we expect the corresponding inequalities to be more powerful than those based on (8) or (9).

Now channels A and B are simply related, for σ , P and C_{NN} :

$$\sigma^A(\theta) = \sigma^B(\pi - \theta)$$

$$P^A(\theta) = -P^B(\pi - \theta) \quad (18)$$

$$C_{NN}^A(\theta) = C_{NN}^B(\pi - \theta).$$

Inequalities involving the above are especially simple at $\theta = \pi/2$: at this one angle we find ^{*)} from (9), (15) and (16)

$$P_{np}^2 \leq (4\sigma_{np}\sigma_{pp} - \sigma_{pp}^2)/4\sigma_{np}^2, \quad (19)$$

$$4\sigma_{np}(1 \pm C_{NN})_{np} \geq \sigma_{pp}(1 \pm C_{NN})_{pp}. \quad (20)$$

In general, however, the relation between A and B involves more than a sign and a change of angle: depolarization in B is the same as polarization transfer (or "exchange depolarization") in A, and the corresponding inequalities

*) In fact, the meaning of (20) is rather simple: it states that for isospin zero scattering, the singlet and triplet cross-sections are both ≥ 0 .

appear less simple. Present cross-section and polarization data seem to satisfy the appropriate inequalities, but there are not enough n-p data yet to test the new inequalities.

The best test of charge independence is a complete analysis of the scattering matrix. When inelastic scattering is negligible, not many of the more complicated spin experiments are needed ⁴⁾ to do this for the nucleon-nucleon system. However, the new inequalities we have discussed may have some temporary usefulness at these energies, and a more lasting value at higher energies. They should also be useful for other complicated spin systems.

R E F E R E N C E S

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