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AN EXTENSION OF POMERANCHUK'S THEOREM TO DIFFRACTION SCATTERING

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Consider the invariant scattering amplitude $T(st)$ for the elastic scattering $A + B \rightarrow A + B$, s and t being $(\text{c.m. energy})^2$ and $-(\text{c.m. momentum transfer})^2$. Spins are neglected. Consider its real and imaginary parts

$$D(st) = \text{Re } T(st), \quad A(st) = \text{Im } T(st).$$

The corresponding functions $\bar{D}(ut)$, $\bar{A}(ut)$ for the crossed reaction $\bar{A} + B \rightarrow \bar{A} + B$, with $u = 2m_A^2 + 2m_B^2 - s - t$ its $(\text{c.m. energy})^2$, are obtained by analytic continuation of T through the upper half s -plane

$$\bar{D}(ut) = \text{Re } T(st), \quad \bar{A}(ut) = -\text{Im } T(st).$$

We make the following assumptions

- i) the differential cross-section ($k = \text{c.m. momentum}$)

$$\left(\frac{d\sigma_{el}}{dt} \right)_{A+B} = \frac{4\pi^3}{k^2 s} \left[|A(st)|^2 + |D(st)|^2 \right]$$

tends to a finite, non vanishing limit for $s \rightarrow +\infty$;

- ii) this limiting cross-section is at least partially due to the imaginary part of T , or more precisely $s^{-1}A(st)$ has a non vanishing limit $a(t)$ for $s \rightarrow +\infty$;

- iii) $u^{-1}\bar{A}(ut)$ has a non infinite limit $\bar{a}(t)$ for $u \rightarrow +\infty$.

2.

With these assumptions we apply to the dispersion relation at constant $t \neq 0$ the well-known argument leading for $t = 0$ to the Pomeranchuk theorem ¹⁾ on equality of total cross-sections $(\sigma_t)_{A+B} = (\sigma_t)_{\bar{A}+B}$. We use the twice subtracted dispersion relation

$$\frac{\text{Re } T(st)}{s^2} = \frac{1}{\pi} \int_{s_0}^{\infty} \left[\frac{A(s't) ds'}{(s'-s) s'^2} + \frac{\bar{A}(u't) du'}{(u'-u) s'^2} \right] + \sum \frac{r_n}{s_n - s} + \frac{c_1}{s} + \frac{c_2}{s^2}, \quad (1)$$

$u' = 2m_A^2 + 2m_B^2 - s' - t$; s_0 = threshold energy; terms in r_n = pole terms; terms in c_1, c_2 = subtraction terms.

Assumption i) implies that for $s \rightarrow +\infty$ the quantity $s^{-1} |\text{Re } T| = s^{-1} |D|$ remains bounded. Just as in the proof of Pomeranchuk's theorem we then conclude from ii) and iii) the equality $a(t) = \bar{a}(t)$ (which reduces to Pomeranchuk's theorem for $t = 0$). Using it in the dispersion relation we calculate

$$\lim_{s \rightarrow \pm \infty} \frac{\text{Re } T(st)}{s} = \frac{1}{\pi} \lim_{s \rightarrow +\infty} \int_{s_0}^{s_1} \frac{\bar{A}(u't) du' - A(s't) ds'}{s'^2} + c_1 - \sum r_n \equiv d(t) \quad (2)$$

Hence

$$\lim_{s \rightarrow +\infty} s^{-1} D(st) = - \lim_{u \rightarrow +\infty} u^{-1} \bar{D}(ut) = d(t) \quad (3)$$

These various results are contained in the single formula

$$\lim_{s \rightarrow \pm \infty} s^{-1} T(st) = d(t) + ia(t) \quad (4)$$

They immediately imply

- 1) the crossed reaction has at high energy the same elastic cross-section as the reaction $A + B$

$$\lim_{u \rightarrow +\infty} \left(\frac{d\sigma_d}{dt} \right)_{\bar{A}+B} = \lim_{s \rightarrow +\infty} \left(\frac{d\sigma_d}{dt} \right)_{A+B}$$

- 2) for a "self-crossed" reaction ($\bar{A} \equiv A$) the ratio $D(st)/A(st)$ vanishes at $s \rightarrow +\infty$.

Conclusion 1) follows from (3) and $a(t) = \bar{a}(t)$. As to conclusion 2), for a self-crossed reaction we have $D(st) = \bar{D}(ut)$ at $s = u$, and (3) then implies $d(t) = 0$.

Conclusion 1) states the identity of diffraction scattering for $A + B$ and $\bar{A} + B$ when the diffraction peak becomes energy independent at high energy. As is well known, the Regge pole description of diffraction by means of the Pomeranchuk trajectory gives the same property for a logarithmically shrinking diffraction peak.

Conclusion 2) can sometimes be generalized by means of isospin considerations. Take for example $A = \pi$, $B = K$. Suppose further, following Pomeranchuk and Okun²⁾, that charge exchange scattering is negligibly rare compared to no-charge exchange at high energies. Then the isospin $\frac{1}{2}$ and $\frac{3}{2}$ amplitudes $T_{1/2}$ and $T_{3/2}$ have equal leading terms at large s . In

4.

view of (4) this can be written

$$\lim_{s \rightarrow \infty} s^{-1} T_{\frac{1}{2}}(st) = \lim_{s \rightarrow \infty} s^{-1} T_{\frac{3}{2}}(st) = d(t) + ia(t).$$

But $\pi^0 + K$ is a self-crossed reaction, hence $d(t) = 0$. Thus both $D_{1/2}/A_{1/2}$ and $D_{3/2}/A_{3/2}$ approach zero for $s \rightarrow \infty$.

The foregoing argument also applies to $\pi + p$ scattering, except that our analysis neglects the proton spin. This is strictly valid for $t = 0$ because of helicity conservation; $d(t) = 0$ then reduces to the sum rule of Goldberger et al.³⁾ For t not too large our neglect of spin will still be justified if helicity flip is rare at high energies.

R E F E R E N C E S

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