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EXCHANGE CONTRIBUTIONS TO HIGH ENERGY SCATTERING
AND IMAGINARY CHARACTER OF THE ELASTIC AMPLITUDE

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As we have shown previously ¹⁾, in an elastic scattering process $A + B \rightarrow A + B$ which is self-crossed ($\bar{A} \equiv A$) the existence at large $s = (\text{c.m. energy})^2$ of an s -independent, non-vanishing differential cross-section $d\sigma_{\text{el}}/dt$ ($t = -(\text{c.m. momentum transfer})^2$) implies that the scattering amplitude $\langle AB | T | AB \rangle \equiv T(st)$ is purely imaginary for $s \rightarrow +\infty$. This was established for every t for which

$$\lim_{s \rightarrow +\infty} s^{-1} \text{Im } T(st) \neq 0$$

using the dispersion relation at constant t . We have also remarked that this conclusion can be extended by means of isospin invariance to the more general case where A belongs to an isospin multiplet containing an element invariant for charge conjugation C . The assumption needed for this generalization is that charge exchange scattering is negligibly rare compared to no-charge exchange at high energy ²⁾.

The assumption just mentioned can be replaced by the weaker one that the contributions of exchange of various isospins to elastic scattering have different asymptotic dependences on s , such that all except one can be neglected for $s \rightarrow +\infty$ and t in a given interval $t_0 \leq t \leq 0$. As follows from a theorem recently established by Foldy and Peierls ³⁾, the non-negligible contribution is then necessarily isospin zero exchange, so that charge exchange scattering becomes indeed negligible at high energy, and the purely imaginary character of T follows as before.

In order to extend these considerations to the general case of elastic scattering we remark that a result of the Foldy-Peierls type obtains almost trivially for exchange of the charge conjugation quantum numbers $C = \pm 1$. Indeed

$$\langle AB | T | AB \rangle \equiv T(st) = T_+(st) + T_-(st) \quad (1)$$

$$\langle \bar{A}B | T | \bar{A}B \rangle = T_+(st) - T_-(st) \quad (2)$$

where $T_{\pm}$ denotes the contribution of $C = \pm 1$ exchange. Since the imaginary part of the forward scattering amplitude can never be negative we conclude

$$| \operatorname{Im} T_-(s0) | \leq \operatorname{Im} T_+(s0) \quad (3)$$

Having this new case of Foldy-Peierls inequality, we make the following assumption :

Assumption a) : the contributions of exchange of the charge conjugation quantum numbers $C = \pm 1$ to the elastic scattering $A + B \rightarrow A + B$ have different asymptotic dependences on s , such that one is negligible compared to the other for $s \rightarrow +\infty$ and t in a given interval $t_0 \leq t \leq 0$.

From inequality (3), at high energy the dominating exchange is $C = 1$, which implies by means of (1) and (2)

$$\lim_{s \rightarrow +\infty} \frac{\langle AB | T | AB \rangle - \langle \bar{A}B | T | \bar{A}B \rangle}{\langle AB | T | AB \rangle} = 0 \quad (4)$$

Making for $t_0 \leq t \leq 0$ assumptions i) and ii) of our previous communication ¹⁾ and noting that (4) then implies assumption iii) of the same reference, the methods there used immediately lead to the conclusion

$$\operatorname{Re} \langle AB | T | AB \rangle / \operatorname{Im} \langle AB | T | AB \rangle \rightarrow 0 \text{ for } s \rightarrow +\infty, \quad t_0 \leq t \leq 0$$

which states that the elastic scattering amplitude is purely imaginary at high energy in the t -interval considered.

To assess the significance of our assumption a) concerning $C = \pm 1$ exchange, or of the analogous assumption mentioned before on isospin exchange, one can compare them with the corresponding assumption in the Regge pole description of elastic scattering, which is that Regge trajectories of different quantum numbers do not intersect each other in an interval $t_0 \leq t \leq 0$. Such assumptions essentially mean that in the process $A + \bar{A} \rightarrow B + \bar{B}$ the channels characterized by various sets of quantum numbers for the system $A + \bar{A}$ have different asymptotic behaviour for $s \rightarrow +\infty$, t fixed. They exclude degeneracies among dynamical systems with different quantum numbers and seem therefore to be quite natural. It is interesting that in the case of energy independent $d\sigma_{el}/dt$ at large s they account for the purely imaginary character of the scattering amplitude. The "shadow" nature of high energy elastic scattering is thereby related to causality through the dispersion relation in s .

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R E F E R E N C E S

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