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ON THE CORRESPONDENCE BETWEEN RESONANCES AND S MATRIX
POLES FOR BROKEN SYMMETRIES

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Much interest has been devoted recently on how a higher symmetry in elementary particle interaction can be broken due to a mass difference. In particular, Oakes and Yang¹⁾ have questioned the meaning of symmetric considerations (mass formulae, dynamical calculations) for resonances when the mass differences between elementary particles are so big that some thresholds can overcome the position of resonances. On the basis of the assumption that the poles of the S matrix must be only those corresponding to the observed resonances and must, therefore, be on the sheet that can be obtained from the physical one by crossing the real energy axis at the position of the resonance, they arrived at the conclusion that when the elementary particle mass differences are eliminated, either the resonances become bound states or they behave in such a different way under the symmetry breaking interaction that there is not much hope that any consideration based on the symmetry can retain some meaning in the actual physical situation.

We would like to show in a particularly simple example (a dynamical calculation in a two-channel case) that this situation is not necessarily the case, the initial assumption made by Oakes and Yang being an unrealistic one. In this example we shall be able to trace the movement of the poles in the different sheets in the complex energy plane, relate them to the presence of a resonance, and discuss the validity of a dynamical result concerning the mass difference of the resonances in terms of the mass difference of the elementary particles.

The two-channel situation we shall discuss will be that of $\pi\Lambda$ and $\pi\Sigma$ interaction in the "limited symmetry" (i.e., equal pion interaction with Λ and Σ which is broken by their mass difference). A fixed source calculation in which the $\Sigma - \Lambda$ mass difference (called Δ) was explicitly taken into consideration (in a non-perturbative way), was performed some years ago²⁾. Two resonances were found to be present, respectively, in the $T=1$ and $T=2$ π -hyperon $p_{\frac{3}{2}}$ states which would have had the same position if $\Delta = 0$.

2.

Let us write the partial wave S matrix in terms of the K matrix

$$S = \frac{1 + iK}{1 - iK} \quad (1)$$

The K matrix elements for the cases we are interested in ($p_{\frac{3}{2}}$, $T=1$ and $T=2$) were written in Eq. (12) of (I)³⁾ and can be written in the following form

$$K^{(2)} = [(\omega - \Delta)^2 - 1]^{3/2} \frac{N}{D_2} \quad (2)$$

$$\left. \begin{aligned} K_{\Lambda\Lambda}^{(1)} &= (\omega^2 - 1)^{3/2} \frac{N_{\Lambda\Lambda}}{D_1} \\ K_{\Lambda\Sigma}^{(1)} &= (\omega^2 - 1)^{3/4} [(\omega - \Delta)^2 - 1]^{3/4} \frac{N_{\Lambda\Sigma}}{D_1} \\ K_{\Sigma\Sigma}^{(1)} &= [(\omega - \Delta)^2 - 1]^{3/2} \frac{N_{\Sigma\Sigma}}{D_1} \end{aligned} \right\} \quad (3)$$

where ω is related to the total energy by

$$E = \omega + m_\Lambda \quad (4)$$

and N and D are functions of ω and Δ which are single valued over the entire energy plane. For the case $T=2$ we have only one channel ($\pi\Sigma$) and therefore only one elastic threshold (at the value $1+\Delta$) while for the case $T=1$ we have two coupled channels ($\pi\Lambda$ and $\pi\Sigma$) and two thresholds (1 and $1+\Delta$).

For $T=2$, cf., Fig. 1, we have one unphysical sheet (sheet α) which we find in passing over the elastic cut, while for $T=1$ we shall find two of them: one (β) which we shall find in passing over both elastic cuts, the other (β') by passing through the gap between the two thresholds. Near the real axis, and beyond both thresholds, in β both $(\omega^2 - 1)^{3/2}$ and $((\omega - \Delta)^2 - 1)^{3/2}$ in Eq. (3) are positive while in β' the first is positive while the second one is negative.

The pole of the S matrix corresponding to the $p_{\frac{3}{2}}$ resonance shall be given for $T=2$ by the solution of

$$D_2 = i [(\omega - \Delta)^2 - 1]^{3/2} N \quad (5)$$

while for $T=1$ it shall satisfy the following equation

$$\begin{aligned} D_1^2 \pm (\omega^2 - 1)^{3/2} [(\omega - \Delta)^2 - 1]^{3/2} (N_{\Lambda\Sigma}^2 - N_{\Lambda\Lambda} N_{\Sigma\Sigma}) = \\ = i D_1 [(\omega^2 - 1)^{3/2} N_{\Lambda\Lambda} \pm [(\omega - \Delta)^2 - 1]^{3/2} N_{\Sigma\Sigma}] \end{aligned} \quad (6)$$

where the upper signs hold for the pole in β and the lowest ones for that in β' . In Eqs. (5) and (6) the square roots are all defined so that their arguments are half of the argument of their radical *).

From the explicit forms of N and D 's it is possible to see that for $\Delta \rightarrow 0$ the position of the pole in β tends to the position of the pole in α while that in β' differs from that position. This difference is however small if the width of the resonance is small. This situation is physically reasonable, when $\Delta \rightarrow 0$ the sheet β' loses physical meaning (it cannot be reached from the physical sheet) and the position of the poles in the two channels is completely degenerate due to the fact that the symmetry is satisfied.

For Δ increasing from zero all the poles move in their respective sheet of the complex energy plane, the position of those in β and β' remaining however rather similar in the case of narrow resonances. Let us discuss the physical meaning of this situation, restricting the discussion to the case of narrow resonances. For small values of Δ (as compared to the resonance Q values) the pole in β corresponds to the physical $T=1$ resonance, while that in β' has no physical meaning.

*) The arguments being defined so as to be smaller than π in absolute value.

The distance between the $T=2$ and $T=1$ resonance shall increase with Δ . We shall postpone the discussion of the rate of this separation. Simultaneously, the second threshold for $T=1$ separates from the first one. When this threshold, for Δ sufficiently big, overcomes the position of the poles in β and β' (which do not lie far apart) then the physical role of the two poles in β and β' is exchanged: the pole in β' shall represent the physical resonance while that in β shall have not much physical meaning because it shall not be near the physical region.

From the K matrix formalism one can easily see that the arguments regarding the movement of the poles for symmetry violation are not restricted to the particular model we are discussing. In general, due to a mass difference, in a particular channel there shall be several different thresholds and, therefore, several unphysical sheets. For a narrow resonance we shall find in every sheet a pole, whose position will not be very different from each other. Clearly only one of them shall be near the physical region and, therefore, shall have a physical meaning. Varying the symmetry breaking interaction, this bunch of poles shall travel smoothly, each one on its unphysical sheet ⁴⁾. When this bunch shall overcome a threshold then the one which had a physical meaning just loses it and another one acquires this distinction. We see therefore that if we disregard effects of the order of the widths, a dynamical calculation or a mass formula is valid independently on the relative positions of the resonances and the thresholds. On the other hand, formulae regarding the widths - or effects of that order of magnitude - can be affected by different relative positions of resonance and thresholds in the different states. In this sense it is possible that in higher symmetries as in the octet model - because due to the large mass difference some thresholds overcome the position of resonances - some "sheet corrections" to a width formula are expected to be significant while their influence on a mass formula can be less important (at least if the widths are smaller than the mass differences).

For completeness, we give in Fig. 2 the actual results of Eqs. (5) and (6) for different values of Δ , the other parameters in the equations being arbitrarily chosen to be the same as the ones appearing in the πN problem. In Fig. 2 one finds the curves that the poles describe as function

of Δ . To each value of Δ corresponds a point on every curve which corresponds to the position of the pole on the corresponding sheet. The position of the pole is given in terms of $\omega - \Delta$ (complex ω) so that the second ($\pi\Sigma$) threshold is fixed in the figure at the value $\omega - \Delta = 1$. Above such a value the point lying in β corresponds to the physical resonance, below it is the one in β' which corresponds to it.

Let us discuss briefly the problem of the separation of the two resonances $T=1$ and $T=2$ due to the $\Sigma - \Lambda$ mass difference which breaks the interaction. As was already noted in (I), in the case of small width

$$\omega_{r,2} - \omega_{r,1} = 2\Delta \quad (7)$$

Eq. (7) is not a mass formula in the sense used frequently in the higher symmetry language: in our case we have only two levels and therefore only one splitting for every multiplet. Anyhow, Eq. (7) shows the interesting feature that the splitting is linear in the mass term and does not depend on the other parameters of the model, even if the result is not perturbative in Δ or in the coupling constants. However, the result of Eq. (7) [as well as all the expressions for D in (I)] can easily be obtained from the considerations of just the lowest order Feynman diagrams. This coincidence can throw some light on the validity of a perturbative mass formula if this simple method is extended to a higher symmetry, and, therefore, it is perhaps worthwhile to comment on the reason of its occurrence. This reason is due to two facts that are inherent to the dynamical model. The first reason is that the method - even if it is not perturbative - considers only a family of graphs, the sum of which can be represented by an integral equation with separable kernels. In these cases, it is well-known that the first order approximation to the Fredholm determinant is actually its exact evaluation. The second reason is the effective range approximation due to which the energy dependence of some cut-off integrals is neglected. This approximation could probably not be done for s wave resonances in which case an extension to higher

symmetries of this simple method could not justify a simple linear mass formula. For the moment, this is only a conjecture based on the results of the simple calculation of (I) : we shall further investigate, however, the idea that even if a mass breaking interaction is big, an eventual explanation of a perturbative mass formula can perhaps be provided by some characteristics of the interaction as the short range of the interaction combined with the centrifugal barrier and the dominance of a family of diagrams.

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REFERENCES

- 1) R.J. Oakes and C.N. Yang, Phys.Rev. Letters 11, 174 (1963).
- 2) D. Amati, A. Stanghellini and B. Vitale, Nuovo Cimento 13, 1143 (1959). We shall refer to this paper hereafter as (I).
- 3) In (I) was studied how the symmetry was broken both by a mass difference and an unsymmetric interaction ($f_{\Lambda}^2 \neq f_{\Sigma}^2$). In this note, we shall restrict our discussion to the only effect of the mass difference. We use units in which the pion mass is equal to one.
- 4) If a resonance becomes a bound state, then the pole which was near the physical region above the threshold of the last open channel, is the one that passes to the real axis below all thresholds in the physical sheet, while the others shall continue to be in unphysical sheets.

FIGURE CAPTIONS

Figure 1 Definition of the unphysical sheets in the complex w plane.

Figure 2 Movement of the poles in the different sheets of the complex w plane as function of Δ (different values of Δ are indicated along the paths). The $\pi\Sigma$ threshold is at the value 1 in the abscissa axis.

FIG. 1

FIG. 2