Vector-tensor multiplets\textsuperscript{†}

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Abstract

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Vector-Tensor Multiplets

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Abstract: We review the coupling of \(N = 2\) supergravity to vector-tensor multiplets, based on the method of superconformal multiplet calculus.

1 Introduction

We review the couplings of vector-tensor multiplets to supergravity and to generic configurations of vector multiplets [1, 2, 3], using the method of superconformal multiplet calculus. The full supergravity theory involving vector-tensor multiplets can be found in [3] and an extensive list of references is given there. Here we explain the essential features of the construction which could possibly have been obscured by the many details given in [1, 2, 3]. Couplings of \(N = 2\) supergravity in four-dimensional spacetime to a number of matter multiplets have already been studied extensively in the past. The most well-known couplings involve vector multiplets and hypermultiplets. Theories with \(N = 2\) supersymmetry have provided a laboratory for exploring and testing many phenomena in perturbative and non-perturbative field theory and in string theory (for some reviews, see e.g. [4]).

Besides the vector multiplet, the hypermultiplet and the vector-tensor multiplet there exist two other multiplets that describe 4 + 4 bosonic and fermionic physical degrees of freedom. They are the tensor multiplet and the double-tensor multiplet. All these multiplets can be described in terms of 8+8 off-shell degrees of freedom. The fields of the vector-tensor multiplet are a scalar \(\phi\), a doublet of Majorana spinors \(\lambda_i\), a vector gauge field \(V_\mu\), an antisymmetric tensor gauge field \(B_{\mu\nu}\) and an auxiliary scalar \(\phi^{(2)}\). These fields are subject to an off-shell central charge, which will be discussed later. The interactions of the tensor gauge field necessarily involve Chern-Simons couplings to vector gauge fields. Based on these Chern-Simons couplings vector-tensor multiplets can be classified into two inequivalent classes. When the tensor gauge field couples to its own vector \(V_\mu\), the transformation rules are non-linear in the components of the multiplet and the action contains a self-coupling. The other class is characterized by the fact that the tensor does not couple to its own vector and the transformation rules become linear in the vector-tensor fields.

Recently there has been quite some interest in setting up a suitable superspace formulation for the vector-tensor multiplet. In view of the complexity of our results [3], a superspace description would be
very welcome, for instance to analyze possible non-renormalization properties of this multiplet. As a first step in this direction, the description of the linearized vector-tensor multiplet in $N = 2$ central-charge superspace was given in terms of a constrained super 1-form [5] and in terms of a constrained super 2-form in [6, 7]. In the last two papers the linearized version of the multiplet with Chern-Simons couplings was constructed. However, the central charge gives rise to constraints on the superfields, which have a rather complicated structure.

As is well known for the massless hypermultiplet, one can circumvent an off-shell central charge by adopting a description in terms of an infinite number of fields. A natural setting for this construction is provided by harmonic superspace [8], where hypermultiplets are described by unconstrained analytic superfields. The hope was that a similar construction of the vector-tensor multiplet in harmonic superspace would be possible. However, it turned out that a central charge could not be avoided, although the superfield constraints can be formulated more concisely in harmonic superspace. The harmonic superspace version of the linearized multiplet was given in [9] and the nonlinear version exhibiting a self-coupling was derived in [10, 11]. Gauging of the central charge in superspace has been considered in [12]. To date no full supergravity coupling in superspace has been constructed.

2 Superconformal multiplet calculus and the construction of $N=2$ supergravity-matter couplings

In [3] the method of superconformal multiplet calculus has been reviewed, based on [13]. It allows a systematic construction of off-shell supersymmetric Lagrangians in terms of relatively small supermultiplets. These multiplets are controlled by a large set of gauge transformations which contains the superconformal group. Here we demonstrate the method by showing how the field content of minimal $N = 2$ Poincaré supergravity is described in a gauge equivalent way in terms of 2 superconformal multiplets. One is the Weyl multiplet, which contains the gauge fields of the superconformal algebra, the other an (abelian) vector multiplet.

The superconformal group in four dimensions is $SU(2, 2|N)$ [14]. For $N = 2$ supersymmetry the relevant algebra contains general-coordinate ($GCT$), local Lorentz ($M$), scale ($D$), special conformal ($K$), chiral $U(1)$ and $SU(2)$ and two types of supersymmetry transformations, called $Q$-supersymmetry and $S$-supersymmetry transformations. The Weyl multiplet contains the corresponding gauge fields and a number of auxiliary fields. These fields are

\[
\begin{align*}
\text{GCT} & \quad M_{ab} \quad D \quad K_a \quad U(1) \quad SU(2)^i_j \quad Q_i \quad S_i \quad T_{ij}^a \quad \chi_i \quad D \\
5 & \quad 0 \quad 3 & \quad 9 & \quad 16 & \quad 6 & \quad 8 & \quad 1
\end{align*}
\]

(1)

The fields that are underlined depend on the other fields through conventional constraints [13]. These constraints involve the matter fields $T_{ij}^a$, $\chi_i$, and $D$, such that the multiplet has (24+24) off-shell degrees of freedom, as specified in the bottom line of (1). The fields given in (1) transform irreducibly under the superconformal transformations. The algebra associated with these transformations has field-dependent structure constants. The vierbein and the gravitini of the $N = 2$ Poincaré supergravity multiplet are contained in the Weyl multiplet, but there is no candidate for the graviphoton. The graviphoton in the gauge-equivalent description resides in a superconformal ‘compensating’ vector multiplet

\[
(X^0, \Omega^0, 0^0, W_0^a),
\]

(2)

which contains (8+8) off-shell degrees of freedom. Because these two multiplets contain the Poincaré supergravity fields, the resulting field configuration is called the ‘minimal field representation’. For a consistent gauge-equivalent formulation of the action, a second compensating multiplet is required, e.g. a hypermultiplet

\[
(A_i^\alpha, \zeta^\alpha, A_i^{\alpha(z)}),
\]

(3)
which contains (8+8) off-shell degrees of freedom; here $A_i^{\alpha(z)}$ is an auxiliary field related to $A_i^\alpha$ by the action of the central charge. The other central-charge transformed fields $(X_i^{(z)}, A_i^{\alpha(z)}, ...)$ all depend algebraically on the fields given in (3). We will comment on the central-charge transformation in due course.

To construct a theory invariant under the superconformal group, we have to impose scale, chiral $U(1)$ and $SU(2)$ invariance. Therefore fields of a superconformal multiplet acquire definite scaling and chiral $U(1)$ weights $(w, c)$,

$$
\phi \rightarrow \exp[w \Lambda_D + ic \Lambda_{U(1)}] \phi.
$$

(4)

In the gauge-equivalent form, some of the multiplet components may act as compensating fields. This has a profound effect on the super-Poincaré theory. First of all, some of the superconformal gauge symmetries are no longer present, in particular the dilatations, the chiral $SU(2)$ and $U(1)$ transformations and $S$-supersymmetry. Secondly, the Poincaré supersymmetry transformations take a more complicated form and consist of a field-dependent linear combination of the original $Q$- and $S$-supersymmetry variations.

In the construction of theories with rigid supersymmetry it is not imperative to impose invariance under scale and chiral transformations. Nevertheless, this is what we did in our initial construction of the coupling of vector-tensor to vector multiplets. This approach is tailor-made for constructing the coupling to supergravity. However in the context of rigid supersymmetry, imposing scale and chiral invariances does not amount to an extra restriction, as it is always possible to freeze the vector multiplet to a constant, i.e. $(X_i, \Omega_i, Y_{ij}, W_\mu^0) \rightarrow (1, 0, 0, 0)$.

When based on a finite number of fields, the off-shell representation of some multiplets requires the presence of off-shell central charges. This is for instance the case for the hypermultiplet used in (3) and also for the vector-tensor multiplet. The central charge acts as a translation on the fields generating an infinite sequence of central-charge transformed fields,

$$
\phi \rightarrow \phi^{(z)} \rightarrow \phi^{(zz)} \rightarrow \cdots
$$

(5)

This central-charge transformation appears in the commutator of two (rigid) supersymmetry transformations,

$$
[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \cdots + \delta_Z(z + \bar{z}),
$$

(6)

with $z = 4\epsilon^{ij}\bar{\epsilon}_{2i}\epsilon_{1j}X^0$. Imposing this algebra on the fields of the multiplet requires constraints on the images of the fields under the central-charge transformation. When we consider local supersymmetry this central charge has to be realized in a local way as it appears in the commutator of two local supersymmetry transformations. So we have to provide a gauge field for this symmetry, which is found in the compensating vector-multiplet (2) of the minimal field representation. However, on this multiplet the supersymmetry algebra closes into a field-dependent gauge transformation (which is identified with the central-charge transformation). For $W_\mu^0$ we have

$$
[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]W_\mu^0 = \cdots + \partial_\mu(z + \bar{z}),
$$

(7)

where now $z = 4\epsilon^{ij}\bar{\epsilon}_{2i}\epsilon_{1j}X^0$. The key observation here is that realizing the central charge in a local fashion, as is required for local supersymmetry, gives rise to a field-dependent central-charge transformation in the commutator of two supersymmetries. With respect to the super-Poincaré transformations the Weyl multiplet and the compensating vector-multiplet define an off-shell irreducible multiplet, known as the minimal field representation and consisting of 32+32 degrees of freedom.

The Poincaré supergravity action can be conveniently derived by adding the compensating hypermultiplet and imposing gauge-conditions on the components of the compensating fields,

$$
b_\mu = 0; \quad X^0 = \kappa^{-1}; \quad \Omega_i^0 = 0; \quad A_i^\alpha = \delta_i^\alpha; \quad \zeta^\alpha = 0,
$$

(8)

where $\kappa$ is the gravitational coupling constant. The fields $T_{ij}^{ab}, \chi_i, D, A_\mu, V_{ij}, X_i^0$ and $A_i^{\alpha(z)}$ are auxiliary and can be eliminated as independent fields. In this way, we end up with the on-shell $N = 2$ Poincaré supergravity multiplet with the fields $(e_\mu^a, \psi_{\mu i}, W_\mu^0)$, associated with the graviton, gravitini and graviphoton.
3 Vector-tensor multiplets coupled to N=2 supergravity

In this section we apply the method outlined above to the supergravity coupling of vector-tensor multiplets. The linearized transformation rules for this multiplet under rigid supersymmetry were given in [15, 16] and it was shown that the (rigid) algebra closes into an off-shell central-charge transformation. A finite number of degrees of freedom describing the multiplet consists of a scalar $\phi$, a doublet of Majorana spinors $\lambda_i$, a vector gauge field $V_\mu$, with field strength $F_{\mu\nu}$, an antisymmetric tensor gauge field $B_{\mu\nu}$, with field strength $B_{\mu\nu\rho}$ and an auxiliary scalar $\phi^{(i)}$. With the linearized transformation rules given in [16], the commutator of two (rigid) supersymmetries on $B_{\mu\nu}$ yields

$$[\delta Q(\epsilon_1), \delta Q(\epsilon_2)]B_{\mu\nu} = (2\epsilon_2^i\gamma^\rho\epsilon_{i1} + \text{h.c.})(\partial_\rho B_{\mu\nu} + 2\partial_{[\mu} B_{\nu]\rho}) - 2\partial_\rho (\epsilon_2^i\gamma_\rho\epsilon_{i1} + \text{h.c.}) - \frac{1}{4}(z + \bar{z})\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} + \frac{1}{4}(z - \bar{z}) F_{\mu\nu}.$$  

(9)

The first, second and third term on the right-hand side of this equation represent an infinitesimal translation, a tensor gauge transformation and a central-charge transformation, respectively. The complex parameter $z$ is equal to $4\epsilon_2^i\epsilon_{2i}\epsilon_{1j}$, which is constant at this stage. At the linearized level the last term can be interpreted in two ways. For the first interpretation, one moves $z - \bar{z}$ under the derivative. The transformation on $B_{\mu\nu}$ then takes the form of a tensor-gauge transformation

$$\delta\lambda B_{\mu\nu} = 2\partial_\rho (\frac{1}{2}(z - \bar{z}) V_{\rho i}).$$  

(10)

However, when requiring scale and chiral invariance of the transformation rules of the vector-tensor multiplet, the parameter $z$ acquires a field-dependent modification $z = 4\epsilon_2^i\epsilon_{2i} X^0$. Also for a locally supersymmetric version of the vector-tensor multiplet, $z$ becomes spacetime dependent through the supersymmetry parameters $\epsilon_i$. In those cases of a non-constant $z$, the last term in (9) can no longer be interpreted as a tensor-gauge transformation and can only be identified as a new gauge transformation associated with a Chern-Simons coupling between $B_{\mu\nu}$ and $V_\mu$,

$$\delta\theta^i B_{\mu\nu} = i(z - \bar{z}) \partial_{[\mu} V_{\nu]}.$$  

(11)

The result is that gauging the central charge in the supersymmetric theory leads necessarily to a Chern-Simons coupling for the tensor field. Demanding that the multiplet has balanced scaling and chiral $U(1)$ weights forces the fields of the vector multiplet to act as compensators. The minimal configuration that only involves the vector-tensor multiplet and the central-charge vector multiplet was constructed in [1]. The direct Chern-Simons coupling (11) of the vector and the tensor of the vector-tensor multiplet leads to unavoidable non-linearities of the action and the transformation rules. Freezing the vector multiplet to a constant and after suitable rescalings of the fields, its rigidly supersymmetric action takes the form

$$\mathcal{L} \propto \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\phi (\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + \frac{1}{2}\phi^{-1} \left(\partial_{[\mu} B_{\nu]\rho] - V_{[\mu} \partial_\nu V_\rho}\right)^2 \\
+ \frac{1}{2}\phi \lambda^i \tilde{\phi} \lambda_i - 2\phi (\phi^{(i)})^2 - \frac{1}{2}i \left(\epsilon^{ij} \lambda_i \sigma^{\mu\nu} \lambda_j + \text{h.c.}\right) (\partial_\nu V_{\rho} - \partial_\rho V_\nu).$$

4
are encoded in specific Chern-Simons terms. We have reviewed some essential features of the coupling of vector-tensor multiplets to 

\[ N = 2 \] supergravity with Chern-Simons couplings of the tensor to its own vector and to a generic configuration of (background) vector multiplets [2].

In four spacetime dimensions one can dualize an antisymmetric tensor to a scalar \( a \). Together with \( \phi \) it combines to the scalar of a vector multiplet \( X^1 = X^0(a + i\phi) \). Vector-multiplet couplings in \( N = 2 \) supergravity can be characterized by a prepotential \( F \). This prepotential is intimately related to the Chern-Simons couplings. Summarizing we have the following two versions:

**the non-linear vector-tensor multiplet**

\[
\delta B_{\mu
u} = \eta_1 X^0 X^1 X^A X^B \]
and the linear vector-tensor multiplet

$$\delta B_{\mu\nu} = \eta_{1A} \theta^1 \partial_{[\mu} W_3^A + \eta_{AB} \theta^A \partial_{[\mu} W_5^B \rightarrow F = \eta_{1A} \frac{(X^1)^2}{X_0} X^A + \eta_{AB} \frac{X^1}{X_0} X^A X^B. \quad (16)$$

Possible applications of vector-tensor multiplets in string theory have been commented on in [3] and we refer the interested reader to references given there.

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