Gluon Radiation in Top Production and Decay at Lepton Colliders\textsuperscript{1}

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Abstract. In this talk we discuss gluon radiation in top production and decay. After reviewing results for hadron colliders we consider soft gluon radiation at lepton colliders and present gluon distributions that are potentially sensitive to production-decay interference effects.

INTRODUCTION

Top quark events are often accompanied by gluons. This gluon radiation can occur in association with both the top production and top decay processes, and must be understood if we are to make sense of top physics. This is especially true for top momentum reconstruction, where a jet originating as a gluon may or may not be a top quark decay product. Top momentum reconstruction can play a crucial role not only in precision top mass measurements but also in identification of top events by using a mass cut. In addition, the pattern of the gluon radiation itself contains information about the top production and decay processes and their color structure, and can be sensitive to new physics.

In this talk we discuss results for gluon radiation in top events at hadron colliders, and then focus on lepton colliders, noting some similarities and differences along the way. Further details can be found in the references.

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Review of Hadron Collider Results

In top production and decay at hadron colliders, strongly interacting particles in the initial, intermediate, and final states all give rise to gluon radiation. In reconstructing the top mass, it matters whether jets from these gluons are part of the top production or decay process. If the gluons arise in top production, they should be ignored in mass reconstruction, but if they are part of the top decay process then they must be included.\textsuperscript{2}

In practice that seemingly straightforward rule is hard to follow for several reasons. First, gluon jets are not necessarily distinguishable from other jets from top decays. Second, even if the gluon jets can be identified one cannot easily distinguish between production- and decay-stage radiation. In a study of gluon radiation in top production and decay at the Tevatron \cite{1} we found that for kinematic cuts meant to mimic typical detector capabilities, the amounts of production- and decay-stage radiation were comparable (ignoring radiation from $W$ decay products). Furthermore, the two were not easily separated. Production-stage radiation is well spread out in rapidity, and although decay-stage radiation is more central, there is significant overlap in the relevant detector regions. Even proximity to one of the $b$-quark jets is not sufficient to uniquely identify a gluon as being associated with decay, for example. This underscores the importance of understanding gluon radiation in top processes.

At the LHC $pp$ collider, the situation is slightly different. \cite{2} Its higher energy and luminosity give a vast increase in the top production cross section, but at the same time the amount of gluon radiation is also increased, notably at the production stage. This is illustrated in Figure 1, which shows the ratio of cross sections for $t\bar{t}j$ and $t\bar{t}$ production at the Tevatron (solid line) and LHC (dashed line) as a function of the minimum transverse energy $E_T$ of the gluon. The LHC is seen to have a vast increase in production-stage radiation over the Tevatron for any given $E_T$ cut.

In contrast, the decay-stage radiation at the LHC looks similar to that at the Tevatron, because the kinematics of the produced top quarks are similar. The main difference is that the $t$'s are more spread out in rapidity at the LHC, which gives rise to a decay-stage gluon distribution that is also more spread out.

At the LHC, then, the individual distributions of production- and decay-stage radiation are similar to those at the Tevatron, but production-stage radiation dominates by far. This can be a problem, for example, when those gluons overlap with jets from $t$ decays, causing mismeasurement of parton energies that can feed into momentum reconstruction. Such effects cannot be

\textsuperscript{2) Interference between gluons from production and those from decay is negligible provided their energies are large compared to the top width, a reasonable assumption for gluons that are to be detected as jets at hadron colliders. See the lepton collider section for further discussion.}
avoided but they can be taken into account as long as they are incorporated into the relevant analyses.

**GLUON RADIATION AT LEPTON COLLIDERS**

At lepton colliders there is no gluon emission from the initial state. That means that, for purposes of studying gluon radiation in top production and decay, there is no difference between $e^+e^-$ and $\mu^+\mu^-$ colliders. (There are important differences in, for example, precision studies at the $t\bar{t}$ threshold; see the talk by M. Berger. [3]) The absence of initial-state gluon radiation and the relatively simpler kinematics compared to hadron colliders allows for the possibility of more detailed study of radiation patterns at lepton colliders. The pattern of gluon radiation can give information about the dynamics of

![Figure 1](image-url) **FIGURE 1.** Ratio of $\sigma(t\bar{t}j)$ to $\sigma(t\bar{t})$ for $E_T^j > E_T$ at the Tevatron (solid line) and LHC (dashed line). [2]
the underlying process, including interplay between top production and decay, and possible signals of new physics.

**Soft Gluon Radiation**

The study of soft gluon radiation patterns in high energy process, sometimes called “partonometry,” is useful for a number of reasons. The infrared singularity in the cross section means that there is a high probability for emitting soft gluons, whose distribution determines the distributions of soft hadrons or minijets in such events. Studying these distributions then gives information about the underlying process. In doing calculations, the soft limit is useful because the cross section factorizes into the lowest order (differential) cross section and a piece due solely to the gluon radiation:

\[
dN \equiv \frac{1}{\sigma_0 d\sigma_g} = \frac{dE_g d\Omega}{E_g \frac{C_F}{4\pi} \pi} R, \tag{1}
\]

where \( E_g \) and \( \Omega \) denote the gluon energy and solid angle.

In \( tt \) production and decay at lepton colliders, although there is no initial state gluon radiation, there are still gluons emitted in both the top production and decay stages (respectively before and after the top quark goes on shell). It is straightforward to find \( R \) and decompose it into the contributions corresponding to radiation from the various stages; see [4].

One interesting result of such an analysis is that there can be interference between production- and decay-stage gluons when the gluon energy is roughly comparable to the top width \( \Gamma \). This can be understood heuristically in terms of Breit-Wigner distributions for the square of the top momentum with and without the gluon, corresponding to radiation in the decay and production stages, respectively. The peak separation is roughly the gluon energy \( E_g \), and since each distribution has width \( \Gamma \), when \( E_g \approx \Gamma \), the two distributions overlap, giving rise to non-negligible interference.\(^3\) Because the presence of this interference for a given gluon energy depends on the value of \( \Gamma \), it follows that gluon distributions can be sensitive to the top width.

In Ref. [4] the gluon distribution \( R \) was studied for specific kinematic configurations for a top mass of 140 GeV. Here we update those results for a 175 GeV top and for energies appropriate to muon collider studies. In Figure 2 we show the gluon distribution for center of mass energy 1 TeV for a configuration where the \( t \) quark decays to a backwards \( b \) and the \( \bar{t} \) also decays to a backwards \( \bar{b} \). The distribution is shown as a function of \( \theta_g \), the angle between the gluon and the top quark. For this configuration, the \( t \) and \( \bar{b} \) quarks are

\(^3\) We can now see why the production-decay interference is negligible at hadron colliders. We consider extra jets with minimum transverse energies of 20 and 40 GeV at the Tevatron and LHC, respectively; such energies are much larger than the Standard Model top width of about 1.5 GeV.
located at $\theta_g = 0^\circ$ and the $t$ and $b$ quarks are at $\theta_g = 180^\circ$. The gluon energy is taken to be 5 GeV. The overall shape shows the “dead cone” behavior characteristic of radiation from heavy quarks — radiation along the quark direction is suppressed by the mass. The curves are for different values of the top width $\Gamma$, as labeled. We see that radiation in the SM case ($\Gamma = 1.5$ GeV) is suppressed compared to that for $\Gamma = 0$, due to destructive interference between the production and decay stages. As the width increases so does this suppression, and in the limit $\Gamma \to \infty$, the gluon distribution is what it would be if the $b$ quarks were produced directly: the contribution of radiation from the top quark disappears altogether.

Top quarks at high energies are not terribly likely to decay to backward $b$’s, but interference effects remain if we look at a more likely configuration in Figure 3, where the $b$ quarks come out at $90^\circ$ to their parent top quarks.

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**FIGURE 2.** Soft gluon distribution in top production and decay at lepton colliders as described in the text, for $t$’s decaying to backward $b$’s and collision energy 1 TeV.
Here the $t$ and $\bar{t}$ quarks are at $0^\circ$ and $180^\circ$, respectively, and the $b$ and $\bar{b}$ are at $90^\circ$ and $270^\circ$. We see more interesting structure in this case, because the $b$ directions do not coincide with those of the $t$'s. Again we see destructive production-decay interference that increases with the top width, and in the $\Gamma \to \infty$ limit (dotted curve) there is no evidence of the top direction in the distribution.

The 1 TeV collision energy discussed above is certainly desirable from the point of view of a variety of physics topics. However, as stated in the workshop guidelines, a first muon collider is more likely to operate at the lower energy of 500 GeV. How do top production-decay interference effects look in the gluon distribution at the lower collision energy? Not too promising, unfortunately, as can be seen in Figure 4, which is the same as Figure 2 except that the collision energy is 500 GeV. There is little sensitivity to the top width $\Gamma$. The reason is

![FIGURE 3. Soft gluon distribution in top production and decay at lepton colliders as described in the text, for $t$’s decaying to $b$’s at $90^\circ$ and collision energy 1 TeV.](image_url)
that the top quarks come out with lower energy and the radiation is dominated
by that from the $b$ quarks. Results for the configuration corresponding to
Figure 3 show similar behavior.

At higher collision energies it appears that the distribution of soft gluon
radiation may be sensitive to the top quark width, which suggests this as
a method for measuring $\Gamma$. This will be challenging, to say the least. The
results presented here are at the parton level only, and moreover they are for
fixed kinematic configurations. A more realistic assessment requires a more
detailed study of integrated cross sections. The result is most likely to be
that the sensitivity leaves something to be desired. However, it will still be
worth pursuing such measurements, because simply seeing the interference
effects will be interesting. Moreover, the width that appears here is the total
top width, independent of decay mode, and would serve as a consistency
check to compare to other measurements. And it should be remembered that

$\text{FIGURE 4. Same as Figure 2, except for collision energy 500 GeV.}$
additional top decay modes, for example to supersymmetric particles, are likely to increase the total width, which would make the effects discussed here more easily observed.

CONCLUSION

In summary, we have discussed issues associated with gluon radiation in top production and decay at hadron and lepton colliders. We noted that for purposes of gluon radiation, there are no significant differences between muon and electron colliders, since there is no initial state gluon radiation. We presented some gluon distributions at lepton colliders that show interference effects between production- and decay-stage gluon radiation, and that are potentially sensitive to the value of the top quark width. Finally, we note that the clean environment of lepton colliders allows for detailed experimental studies of QCD effects in top quark physics, both within the Standard Model and beyond.

REFERENCES

3. Berger, M., these proceedings.
\[ \frac{dN}{d\cos \theta_g} \]

\( \Gamma(\text{GeV}) \)

\( M = 175 \text{ GeV} \)
\( W = 1000 \text{ GeV} \)
\( \theta_b = 90^\circ \)
$M = 175$ GeV
$W = 500$ GeV
$\theta_b = 180^\circ$