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ANTIBOUND STATES OF THE PION-PION SYSTEM

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The purpose of this paper is to show that dispersion relations strongly indicate that there is an antibound state in the $I = 0$ S state of the reaction $\pi + \pi \rightarrow \pi + \pi$, close to threshold (corresponding perhaps to the ABC "particle")¹⁾. A possible antibound state occurs in the $I = 2$ S wave, but much further from the threshold; none is indicated for the $I = 1$ amplitude.

The simplest demonstration uses zero momentum transfer invariant amplitudes :

$$A(s, t) \equiv \begin{bmatrix} A^0 \\ A^1 \\ A^2 \end{bmatrix} \quad (1)$$

s and t are the Mandelstam variables and the superscripts are the isospin labels. Writing $A_1^I(s)$ for the partial wave amplitudes, one has

$$\begin{aligned} A^I(4, 0) &= A_0^I(4) = a_I \quad (\text{say}) \quad I=0, 2 \\ A^1(4, 0) &= 0 \end{aligned} \quad (2)$$

where a_0 and a_2 are the S wave scattering lengths. The values of these amplitudes at the crossed threshold, $s = 0$, are given by the crossing relation

$$A(s, t) = \beta A(4-s-t, t) \quad (3)$$

$$\beta = \begin{bmatrix} 1/3 & -1 & 5/3 \\ -1/3 & 1/2 & 5/6 \\ 1/3 & 1/2 & 1/6 \end{bmatrix} \quad (4)$$

Hence

$$\begin{aligned} A^0(0, 0) &= \frac{1}{3}(a_0 + 5a_2) \\ A^1(0, 0) &= -\frac{1}{3}(a_0 - \frac{5}{2}a_2) \\ A^2(0, 0) &= \frac{1}{3}(a_0 + \frac{1}{2}a_2) \end{aligned} \quad (5)$$

2.

Most dynamical analyses give positive values for a_0 and a_2 , so that A^0 and A^2 are positive at $s = 0$ and $s = 4$.

The condition for an antibound state in a partial wave I, l is that a pole should appear between $s = 0$ and $s = 4$ on the second Riemann sheet. The function on the second sheet is defined by

$$B_l^I(s) = A_l^I(s) \left[1 - 2 \left(\frac{4-s}{s} \right)^{1/2} A_l^I(s) \right]^{-1} \quad (6)$$

Here the surd is defined to be cut for $(-\infty < s \leq 0)$ and $(4 \leq s < \infty)$ and to be positive for $(0 < s < 4)$ on sheet I. Then the condition for a pole of $B_l^I(s)$ at $s = m^2$ is

$$A_l^I(m^2) = \frac{1}{2} \left[\frac{m^2}{4 - m^2} \right]^{1/2} \quad (7)$$

If one makes the simple approximation

$$\begin{aligned} A^I(s, 0) &= A_0^I(s) & I = 0, 2 \\ A^I(s, 0) &= 3 A_1^I(s) & \end{aligned} \quad (8)$$

where partial waves for $l = 2, 3, \dots$ have been neglected, it follows that the condition for an antibound state for $I = 0, l = 0$ is

$$A^0(m^2, 0) = \frac{1}{2} \left(\frac{m^2}{4 - m^2} \right)^{1/2} \quad (9)$$

Since the right-hand side of this equation is 0 at $m^2 = 0$ and $+\infty$ at $m^2 = 4$, it follows from the fact that $A^0(s, 0)$ is positive at $s = 0$ and $s = 4$ that (9) must be satisfied for some m , which is therefore

the mass of an antibound state. A similar consideration applies to the $I = 2$, $l = 0$ amplitude. On the other hand $A^1(s, 0)$ is 0 at $s = 4$ and from (5) could be positive or negative at $s = 0$. A more sensitive test is necessary for this amplitude.

A further approximation will be made in order to derive a simple relation between the mass of the antibound state and the $I = 0$, $l = 0$ scattering length. The following relation is exact at the symmetry point $s = \frac{4}{2} = t$:

$$A^0(s, t) = \frac{5}{2} A^2(s, t) \quad (10)$$

It will be assumed to be approximately true at $s = 4$, $t = 0$. Then

$$a_0 \doteq \frac{5}{2} a_2 \quad (11)$$

Thus (5) gives

$$\begin{aligned} A^0(0, 0) &= a_0 = A^0(4, 0) \\ A^1(0, 0) &= 0 \\ A^2(0, 0) &= a_2 = A^2(4, 0) \end{aligned} \quad (12)$$

Hence A^0 and A^2 are separately equal at the direct and crossed thresholds $s = 4$ and $s = 0$. They will be assumed to be approximately constant throughout $0 < s < 4$ (this cannot be exactly true, of course).

Then (7) is easily solved, to give

$$m_I^2 = \frac{16 a_I^2}{1 + 4 a_I^2} \quad I = 0, 2. \quad (13)$$

together with (11).

4.

Here m_0 and m_2 are the masses of the $I = 0$ and 2 antibound states. Fig. 1 shows m_0^2 and m_2^2 plotted against a_0 . Thus, for $a_0 = 1$, $m_0^2 = 3.2$ and $m_2^2 = 1.6$; and for $a_0 = 1.5$ (which is entirely possible), $m_0^2 = 3.6$ and $m_2^2 = 2.4$. (The unit is the pion mass.)

Hence one expects an $I = 0$ antibound state quite close to the normal threshold. This will be identified with the ABC "particle", and can be invoked to "explain" the large $I = 0$ scattering length. There may also be an $I = 2$ antibound state, but this is further from the threshold, and would have a smaller effect on low energy scattering. Moreover, it is more likely to be strongly affected by the approximations in this treatment; and an accurate analysis may even remove it.

The analysis can be repeated with the quantity $F(s) \equiv \frac{A^1(s,0)}{4-s}$ in order to determine whether there is an $I = 1$ antibound state. Defining the P wave scattering length by

$$a_1 = \lim_{q \rightarrow 0+} A_1^1(4(q^2+1))/q^2 \quad (14)$$

one has $F(4) = -\frac{3}{4} a_1$

and $F(0) = \frac{3}{2} a_1$ (15)

with the same approximations as before. Now a_1 is known to be small and positive, of the order of 0.05^2). On the other hand, the antibound state condition (7) reads, for $F(s)$,

$$F(m^2) = \frac{3}{2} \frac{1}{\sqrt{m^2(4-m^2)}} \quad (16)$$

and the minimum of the modulus of this function is $\frac{3}{4}$. Accordingly (16) is probably never satisfied and there is no P wave antibound state.

In conclusion a few words can be said about improving these approximations. Firstly, instead of assuming constancy in $(0 < s < 4)$, the amplitudes can be evaluated in this interval by using a dispersion relation. Secondly, combinations of the amplitudes and their first derivatives can be considered which eliminate the D wave identically. Calculations embodying these improvements will be published later.

A still better approximation would probably be to use the partial wave dispersion relations directly to find the solution of (7).

REFERENCES

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FIGURE CAPTION

Relation between the $I = 0$ $\pi\pi$ S wave scattering length a_0 and the antibound state masses. m_0 is the $I = 0$ state mass, m_2 is the $I = 2$ state mass.

Fig. 1