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REMARKS ON CLASSIFICATIONS OF HADRONS

ACCORDING TO SPIN AND INTERNAL SYMMETRIES

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I. In many recent papers ^{1),2),3)} attempts have been made and theorems have been proved in order to connect the Poincaré group and the internal symmetry groups proposed for hadrons.

Up to now, two main approaches have been followed :

- i) the Poincaré group is enlarged either by extending the Minkowski space (complexification, quaternionization or increase of the dimension) or by considering in addition conformal transformations. Given such a group, one has to interpret its new quantum numbers ¹⁾;
- ii) other authors try to construct a Lie algebra $\mathcal{L}$ containing both the Poincaré algebra $\mathcal{P}$ and the internal symmetry algebra $\mathcal{S}$. In this way several restrictive theorems have been proved ^{2),3)}.

The second approach is certainly more systematic than the first one. Unfortunately, the theorems so far obtained concern only a very particular class of solutions, namely those verifying the relation

$$\mathcal{L} = \mathcal{P} \oplus \mathcal{S}$$

In the present paper, a method is suggested and applied, as an example, to the simple case where the internal symmetry group is restricted to the baryonic gauge group. The masses of the hadrons are supposed different from zero.

A system of commuting observables of the Poincaré group is given by its energy-momentum operators P_μ on one hand and by the commuting observables of its little group SO_3 on the other hand. All these commuting operators have to commute with the system of commuting observables of $\mathcal{S}$. This suggests the search for the "little group" (containing $\mathcal{S} \otimes SO_3$) of an "enlarged Poincaré group".

2.

Instead of giving an abstract description of our method, it is more convenient to apply it to the special and simple case where $\mathcal{S}$ reduces to the baryonic gauge.

Some indications of a more general case are given in Section IV.

II. Hypercharge Y and isospin I satisfy the relation ³⁾

$$(-)^{2I} = (-)^Y. \quad (1)$$

A similar relation holds between the baryonic number B and the usual spin S , namely

$$(-)^{2S} = (-)^B. \quad (2)$$

A closer analogy between Y and B on one hand and I and S on the other hand has been put forward by Lipkin ⁴⁾. Its physical contents is shown in Table 1.

	$Y = 1$ $I = \frac{1}{2}$	$Y = 0$ $I = 1$	$Y = 0$ $I = 0$	$Y = -1$ $I = \frac{1}{2}$
$B = 1$ $S = \frac{1}{2}$	N	Σ	Λ	Ξ
$B = 0$ $S = 1$	K^*	ρ	ω	$\bar{K}^*$
$B = 0$ $S = 0$	K	π	η	$\bar{K}$
$B = -1$ $S = \frac{1}{2}$	$\bar{\Xi}$	$\bar{\Sigma}$	$\bar{\Lambda}$	$\bar{N}$

TABLE 1

It suggests clearly a $SU_3 \otimes SU_3$ classification for hadrons. The first SU_3 contains (S, S_3, B) as the second SU_3 contains (I, I_3, Y) ⁵⁾.

Our aim is to study the possible groups containing both the usual spin group and baryonic gauge preserving relation (2) and providing mathematical support to Table 1.

Let us denote by $\mathcal{P}_E$ a group which includes the connected Poincaré group $\mathcal{P}$ and the baryonic gauge. We define the little group of $\mathcal{P}_E$ as the subgroup G which leaves invariant a given time-like direction of the Minkowski space ⁶⁾. The universal covering group (U.C.G.) $\overline{\mathcal{P}}_E$ of $\mathcal{P}_E$ contains a covering group $\overline{G}$ of G ⁷⁾.

The pair $(G, \overline{G})$ has to verify the following properties :

- a) G contains $SO_3 \otimes U_1$

The fact that G has to contain SO_3 (spin) and U_1 (baryonic gauge) is obvious. Furthermore, spin and baryonic operators commute. Then property a) follows.

- b) $\overline{G}$ is different from G and contains U_2 as a covering group of $SO_3 \otimes U_1$

According to property a) G possesses only integral spin representations (single-valued). The existence of half-integral spins implies that $\overline{G}$ is different from G and contains SU_2 as covering group of SO_3 . From the relation ⁸⁾

$$SO_3 = \frac{SU_2}{Z_2} \quad (3)$$

we can write

$$G = \frac{\overline{G}}{\overline{H}} \quad (4)$$

where H contains Z_2 and is a subgroup of the centre of $\bar{G}$. From these considerations and from relation (2), as proved by Lurçat and Michel ³⁾, property b) follows.

- c) $\bar{G}$ provides a mathematical support to Table 1

Table 1 allows for many interpretations. Conveniently we classify them in the following way :

- c1) $\bar{G}$ is of rank two and possesses a representation 8 with the contents of Table 1;
- c2) $\bar{G}$ is another group of rank two;
- c3) $\bar{G}$ has rank higher than two.

- d) $\bar{G}$ is a compact Lie group

This is a convenient restriction but by no means a necessary property.

III. The essential part of this section is devoted to the groups G of rank two. Investigation of groups of rank higher than two appears as an easy generalization.

We give in Table 2 all compact Lie groups of rank two with their U.C.G. and corresponding Lie algebra.

Algebra	U.C.G.	Compact Groups		
A_2	SU_3	SU_3	SU_3/Z_3	
$B_2 = C_2$	Sp_4	Sp_4	$Sp_4/Z_2 = SO_5$	
G_2	G_2	G_2		
$D_2 = A_1 \otimes A_1$	$SU_2 \otimes SU_2$	$SU_2 \otimes SU_2$	$SO_3 \otimes SU_2$	$SO_3 \otimes SO_3$
			$SO_4 = \frac{(SU_2 \otimes SU_2)}{Z_2}$	$SO_3 \otimes SO_3$
$A_1 \otimes A_0$	$SU_2 \otimes R$	$SU_2 \otimes U_1$	$U_2 = \frac{(SU_2 \otimes U_1)}{Z_2}$	$SO_3 \otimes U_1$
$A_0 \otimes A_0$	$R \otimes R$	$U_1 \otimes U_1$		

TABLE 2 ⁹⁾

The following groups are excluded by conditions a) and b) as G candidates

- i) SU_3 and SU_3/Z_3 satisfy neither condition a) nor condition b) ⁵⁾.

There is thus no place in our framework for a model $SU_3 \otimes SU_3$ as proposed by Lipkin;

- ii) Sp_4 , G_2 , $SU_2 \otimes SU_2$, $SU_2 \otimes U_1$ and $U_1 \otimes U_1$ because they do not have unitary multivalued representations ($\bar{G} = G$);

- iii) SO_4 , U_2 , do not contain $SO_3 \otimes U_1$;

- iv) $SO_3 \otimes SU_2$ is excluded because its only covering group $SU_2 \otimes SU_2$ does not contain U_2 .

We are now left with three different pairs $(G, \bar{G})$ namely $(SO_3 \otimes U_1, U_2)$, $(SO_3 \otimes SO_3, SO_4)$, (SO_5, SO_4) .

Two cases may occur: either integral and half-integral spins are mixed in an irreducible representation of $\bar{G}$ or not. In the first case $\bar{G}$ will be called fantastic.

None of the three solutions possesses an irreducible representation of dimension 8, and moreover none is fantastic. In order to explain Table 1, we are led to examine reducible representations of these three solutions

- a) $(SO_3 \times U_1, U_2)$

Nothing new can be said about this case which is implicitly admitted by physicists. However, the usual choice for $\mathcal{P}_E$ is

$$\mathcal{P}_E = \mathcal{P} \otimes U_1 \quad (5)$$

and this enlarged Poincaré group does not satisfy our axioms. We do not know any proof that there exists a $\mathcal{P}_E$ fulfilling our conditions.

b) $\underline{SO_3 \otimes SO_3}, SO_4$

The irreducible representations of $D(j, j')$ of SO_4 are characterized by two integral or half-integral numbers (j, j') with the condition

$$j + j' = \text{integer}. \quad (6)$$

We identify j with the spin and B with $2m'$ where m' is the eigenvalue of the third component of the "spin" j' .

The representation $D(0, 0)$ (pseudoscalar octet), $D(\frac{1}{2}, \frac{1}{2})$ (baryons and antibaryon octets) and $D(1, 0)$ (vector meson octet) exactly account for Table 1. Moreover we can put the spin $\frac{3}{2}$ decuplet in the representation $D(\frac{3}{2}, \frac{1}{2})$.

c) $\underline{SO_5}, Sp_4^{10}$

This would have been the only solution if we had required the groups $(G$ and $\bar{G})$ to be simple compact Lie groups.

No reducible 8 dimensional representation of the group Sp_4 exists with the contents of Table 1. The simplest way to account for the proposed classification is to put the pseudoscalar octet in the representation (1), the baryons and the antibaryons in the representation (4). The representation (5) contains apart from the three spin states of the vector mesons, a $B = 2$, spin 0 octet with its antiparticles $B = -2$, spin 0 ¹¹⁾.

IV. In this paper, we have presented a method to study the connection between the Poincaré group and the internal symmetry groups (here baryonic gauge).

The problem has been limited to a "compact little group", i.e., essentially to a non-relativistic version. It has been completely solved for groups of rank two. The solution for higher rank groups can be readily found by the same method.

The method can evidently be used when the group $\mathcal{R}$ is the usual eight-fold way group. If we restrict the problem ¹²⁾ to the search of simple compact Lie groups, the pair

$$(G, \bar{G}) = \left(\frac{SU_6}{Z_6}, \frac{SU_6}{Z_3} \right)$$

provides the most economical (groups of lowest ranks) solution ^{13),14)}.

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- 5) For this model, it is tempting to use complex generalization of the Poincaré group as defined by Barut ¹⁾. This group contains apart from the complex translations the transformations Λ verifying

$$\Lambda^{+G} \Lambda = G \quad (G = +, -, -, -).$$

The little group defined as the subgroup which leaves invariant a complex time-like direction is nothing else than SU_3 . This SU_3 generalizes the usual spin group SO_3 but leaves no place for half-integral spin representations. A same comment applies to the quaternionic generalization of the Poincaré group, its little group being Sp_6 .

- 6) More restrictive definition of the little group can be chosen; see, for example, notes 5) and 10).
- 7) In the following, given a group A , $\bar{A}$ will denote its U.C.G. and $\bar{A}$ a covering group of A or A itself.
- 8) Z_n is the cyclic group of order n . A unitary representation is given by $\exp \frac{2\pi i}{n} a$ ($a = 1, \dots, n$)
- 9) About centres of compact Lie groups see J.P. Serre, Séminaire Sophus Lie 1954-55, Tores Maximaux des Groupes de Lie Compacts, page 23-08.
- 10) This case suggests the consideration of an enlarged six dimensional Minkowski space $\mathcal{M}_E(+, -, -, -, -, -)$ where the little group is defined as conserving a time-like six vector of $\mathcal{M}_E$. One has then to interpret the translation operators along the fifth and sixth axis.
- 11) The $B = 2, S = 0$ octet, if it exists, has probably a mass larger than - or of the order of - two proton masses. This means that the five dimensional multiplet to which it belongs has a wide mass splitting. This already faces us with the problem of the extension of the Poincaré group by internal symmetry groups where the mass splitting appears as an exact formula.
- 12) The conditions of this problem can then be written
 - a) G contains

$$SO_3 \otimes \frac{SU_3}{Z_3}$$

b) G is different from $\bar{G}$ and contains

$$SU_2 \otimes \frac{SU_3}{Z_3}$$

as a covering group of $SO_3 \otimes \frac{SU_3}{Z_3}$;

c) $(G, \bar{G})$ are simple compact Lie groups.

13) This result has been obtained in collaboration with Dr. H. Ruegg. We thank him for permission to quote it.

14) The universal covering group $\bar{G} = SU_6$ has recently been proposed in a paper by F. Gursey and L. Radicati. It seems to present very striking successes in the classification of hadrons. Some further considerations about this group have been given by A. Pais. We thank Professor T.D. Lee for communication of their preprints.