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BASIC SU_3 TRIPLETS WITH INTEGRAL CHARGE AND UNIT BARYON NUMBER

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The success encountered by the octet model of SU_3 symmetry ¹⁾ has given increased interest to speculations about the possible existence and properties of hitherto undiscovered particles which would belong to the basic representations 3 and $\bar{3}$ of SU_3 , and would be through strong binding forces the building blocks of mesons and baryons. Up to now the SU_3 multiplets established for mesons belong to the representations 1 and 8 , while representations 1 , 8 and 10 have been found for baryons. Just these representations are obtained in an elegant triplet model of SU_3 symmetry recently proposed by Gell-Mann ²⁾ and Zweig ³⁾, where mesons are given the structure $A \times \bar{A} (3 \times \bar{3} = 1 + 8)$ and baryons the structure $AAA (3 \times 3 \times 3 = 1 + 8 + 8 + 10)$ in terms of one basic triplet A of spin $\frac{1}{2}$ particles. The latter, however, must then be assigned the unusual values $\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}$ for the electric charge Q and $N = \frac{1}{3}$ for the baryon number N .

Our aim is to show that consideration of two basic triplets instead of one allows to eliminate in a simple way the occurrence of fractional Q and N , without losing the elegant structures $3 \times \bar{3}$ for mesons and $3 \times 3 \times 3$ for baryons. We introduce two triplets T and $\bar{T}$ of spin $\frac{1}{2}$ particles, which we call trions. They all have $N = 1$ and are distinguished by a new additive quantum number D . The trions are listed in Tables 1 and 2.

TABLE 1

T-trions ($N=1$, $D=1$, spin $s=\frac{1}{2}$)

STATES	T^+	T^0	T'^0
charge Q	1	0	0
hypercharge Y	$1/3$	$1/3$	$-2/3$
isospin I	$1/2$	$1/2$	0
I_3	$1/2$	$-1/2$	0

2.

TABLE 2

$\textcircled{H}$ -trions ($N=1$, $D=2$, spin $s=\frac{1}{2}$)

STATES	$\textcircled{H}^0$	$\textcircled{H}^+$	$\textcircled{H}^{++}$
Q	0	1	1
Y	$-1/3$	$-1/3$	$2/3$
I	$1/2$	$1/2$	0
I_3	$-1/2$	$1/2$	0

For a given SU_3 multiplet the value of D is related to its mean charge $\langle Q \rangle$ by

$$D = 3 \langle Q \rangle$$

and the corresponding generalized Gell-Mann - Nishijima formula is

$$Q = I_3 + \frac{Y}{2} + \frac{D}{3} .$$

Consequently D is conserved in strong and electromagnetic interactions.

It is natural to assume that all particles observed up to now have the quantum number D equal to zero, or more generally that $D = 0$ characterizes the most stable particles built up from trions. This raises the question of the possible composite particles having $D = 0$. Those among them which are obtained as products of two and three triplets are listed in Table 3.

It is interesting to note that products of more than three trions, when they have $D = 0$, can always be obtained as products of the composite particles in Table 3.

TABLE 3

Number of trions in composite particles	Representations	Baryonic number N	Spin and parity
2	$\overline{T}T = 1+8$	0	0^- or 1^-
	$\overline{\Theta}\Theta = 1+8$	0	0^- or 1^-
3	$\overline{\Theta}TT = 1+8+8+10$	1	$1/2$ or $3/2$
	$\Theta\overline{T}\overline{T} = 1+8+8+\overline{10}$	-1	$1/2$ or $3/2$
spin and parity have been given for s-state binding			

The model here discussed has the following properties :

- 1) non-integral charges and non-integral baryonic numbers are avoided;
- 2) there is no place in the four classes given in Table 3 for the representation 27. (This was also the case in the schemes proposed in Refs. ^{2),3)}.)
- 3) the trions T and Θ are all possible triplets with charges $0, \pm 1$ such that $D > 0$. $\overline{D} > 0$ goes with $N = 1$, $D < 0$ with $N = -1$. This correlation between the signs of D and of N is related to the asymmetry existing between positive and negative charges in the baryon decuplet which contains one particle with charge $+2$ while all other particles have charges $0, \pm 1$.⁷

The occurrence in our model of the third quantum number D besides I_3 and Y suggests the introduction of a simple group of rank three to describe a possible higher symmetry involving all trions and their combinations. Such groups correspond to the three Lie algebras A_3 (group SU_4), B_3 (SO_7) and C_3 (Sp_6). These algebras all contain A_2 (SU_3) as a subalgebra. Hereafter we examine the C_3 case as the simplest example. The small differences shown by A_3 and B_3 are mentioned afterwards. The lowest representations of C_3 are given in Table 4 with their contents in A_2 representations ⁴⁾.

4.

TABLE 4

Representations of C_3	1	6			14				14'			21			
Submultiplets A_2	1	3	$\bar{3}$	1	$\bar{6}$	6	1	$\bar{3}$	8	3	6	1	8	$\bar{6}$	
Z	0	1/3	-1/3	1	1/3	-1/3	-1	2/3	0	-2/3	2/3	0	0	-2/3	

Representations of C_3	56							64						
Submultiplets A_2	10	3	15	$\overline{15}$	$\overline{3}$	$\overline{10}$	8	3	$\overline{6}$	15	$\overline{15}$	6	$\overline{3}$	8
Z	1	1/3	1/3	-1/3	-1/3	-1	1	1/3	1/3	1/3	-1/3	-1/3	-1/3	1

Z is the third additive quantum number which is obtained besides Y and I_3 from the Abelian subalgebra of C_3 . In our model it is related to D and N by

$$D = \frac{3}{2}(N-Z) .$$

Any representation of C_3 can be obtained from products of representations 6 (octahedrons). From the preceding considerations the T-trions and the (N) -trions are to be classified in this representation 6, mesons in the product

$$6_1 \times 6_{-1} = 1_0 + 14'_0 + 21_0 \\ (1) + (8) + (1+8)$$

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About simple Lie groups of rank 3.