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TRANSVERSE MUON POLARIZATION IN NEUTRINO INDUCED INTERACTIONS
AS A TEST FOR TIME REVERSAL VIOLATION

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CM-P00056962

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The experiments of Christenson et al.¹⁾ interpreted as a violation of CP invariance have opened the question as to whether there may be indeed other weak interaction processes which violate CP invariance. If the CPT theorem²⁾ is assumed then a violation of CP invariance implies the corresponding violation of time reversal invariance. Experiments³⁾ studying the β decay of polarized neutrons already performed indicate that only a small violation of time-reversal invariance is permitted in the allowed β transitions. However, should there be time-reversal violations in terms which vanish for small momentum transfers then there exists no $\Delta S = 0$ experiments which explicitly test for possible violations.

In this note we investigate neutrino induced processes of the type

$$\nu + \text{nucleus} \rightarrow \mu^- + \text{all final states} \quad (1)$$

and the elastic process

$$\nu + n \rightarrow \mu^- + p \quad (2)$$

which are especially well suited for the study of weak interactions at appreciable momentum transfers. In particular we examine the possibility of studying the produced muon transverse (i.e., perpendicular to the plane containing neutrino and muon momenta) polarization as a means of detecting a possible violation of time-reversal invariance.

We first show that if time reversal is a valid symmetry operation that the transverse muon polarization vanishes for an arbitrary neutrino induced nuclear process like (1) provided one averages over all strongly interacting particles in the final state and uses an unpolarized target. Thus, a non-zero value for the transverse muon polarization indicates violation of time reversal invariance.

Subsequently an estimate of what the numerical value of such a transverse polarization might be is given by following the suggestion of Cabibbo ⁴⁾ who speculates that the time-reversal violation is due to a 90° phase between first and second class currents ⁵⁾.

Consider now reactions (1), described by an S matrix which to lowest order in weak interactions, all orders in strong interactions and neglecting electromagnetism, is of the form

$$S = 1 + i j_\alpha O_\alpha + iT \quad (3)$$

Of course, $j_\alpha O_\alpha$ and T have non-zero matrix elements only between states of equal energy momentum. j_α is the usual Hermitian lepton current $j_\alpha = \bar{\Psi}_\mu \gamma^\alpha (1 + \gamma^5) \Psi_\nu + \text{h.c.}$ The operator O_α carries the initial nucleus to the system of final strongly interacting particles and includes all possible interactions among the final strongly interacting particles. The operator $1+iT$ is the S matrix for strongly interacting particles in the absence of weak interactions. Thus T is zero between states where one state has leptons different from those present in the other state. The reverse holds for $j_\alpha O_\alpha$.

The cross-section for process (1) where only the outgoing muon is observed is proportional to

$$\begin{aligned} |M|^2 &= \sum_{I, F} \langle \mu | j_\alpha | \nu \rangle \langle \nu | j_\beta | \mu \rangle \langle F | O_\alpha | I \rangle \langle I | O_\beta^\dagger | F \rangle \\ &\equiv L_{\alpha\beta} N_{\alpha\beta} \end{aligned} \quad (4)$$

Averaging over initial and summing over final states is indicated. $L_{\alpha\beta}$ refers only to leptons and $N_{\alpha\beta}$ to the system of strongly interacting particles. The quantity $L_{\alpha\beta}$ is given by ⁶⁾

$$\begin{aligned}
L_{\alpha\beta} = & q_\alpha q'_\beta + q'_\alpha q_\beta - \delta_{\alpha\beta} (q q') + \varepsilon_{\lambda\alpha\kappa\beta} q'_\lambda q_\kappa \\
& - m \{ q_\alpha w_\beta + w_\alpha q_\beta - \delta_{\alpha\beta} (q w) + \varepsilon_{\lambda\alpha\kappa\beta} w_\lambda q_\kappa \}
\end{aligned} \quad (5)$$

q, q' are neutrino and muon four momenta respectively, m is the muon mass and w is a unit space-like vector along the direction of the muon polarization. As a consequence of Lorentz invariance and hermiticity the most general form for the tensor $N_{\alpha\beta}$ is

$$\begin{aligned}
N_{\alpha\beta} &= \sum_{I, F} \langle I | O_\beta^\dagger | F \rangle \langle F | O_\alpha | I \rangle \\
&= A_1 \delta_{\alpha\beta} + A_2 p_\alpha p_\beta + A_3 Q_\alpha Q_\beta + A_4 (p_\alpha Q_\beta + Q_\alpha p_\beta) \\
&\quad + A_5 \varepsilon_{\alpha\beta\sigma\tau} p_\sigma Q_\tau + i A_6 (p_\alpha Q_\beta - Q_\alpha p_\beta)
\end{aligned} \quad (6)$$

p is the four momentum of the initial nucleus, $Q = q' - q$ is the four momentum transfer. The six real scalar quantities $A_1 \dots A_6$ are functions of the only possible invariants Q^2, p^2 and (pQ) .

Unitarity of the S matrix as well as time-reversal invariance has been shown ⁷⁾ to give

$$\langle F, \mu | S | I, \nu \rangle = \langle I', \nu' | S | F', \mu' \rangle \quad (7)$$

where I', ν' and F', μ' are the time reversal initial and final states respectively, for instance, F' is F with all three momenta and spins reversed, and outgoing states changed into ingoing states. Thus instead of (4) we have the time reversed cross-section proportional to

$$|M'|^2 = \sum_{I', F'} \langle \nu' | j_\beta | \mu' \rangle \langle \mu' | j_\alpha | \nu \rangle \langle I' | Q_\beta | F' \rangle \langle F' | Q_\alpha^\dagger | I' \rangle$$

which should equal $|M|^2$ if time reversal invariance holds. The time reversed form $L'_{\alpha\beta}$ of $L_{\alpha\beta}$ may be obtained from the explicitly given form (5) by the rule $\mu \rightarrow \mu'$, $\nu \rightarrow \nu'$. On the other hand the expression for $N_{\alpha\beta}$ (6) is more complicated since Q_α is not Hermitian.

We now note that the unitarity condition $S^\dagger S = 1$ leads in lowest order in weak interactions to

$$Q_\alpha = Q_\alpha^\dagger + i T^\dagger Q_\alpha + i Q_\alpha^\dagger T \quad (8)$$

and, independently

$$i(T - T^\dagger) = -T^\dagger T \quad (9)$$

Furthermore, since the initial nucleus is stable against strong interactions we have

$$T | I' \rangle = T | I \rangle = 0 \quad (10)$$

Using (8) and (10) we have

$$\begin{aligned} \sum_{F'} \langle I' | Q_\beta | F' \rangle \langle F' | Q_\alpha^\dagger | I' \rangle &= \\ &= \sum_{F'} \langle I' | Q_\beta^\dagger (1 + iT) | F' \rangle \langle F' | (1 - iT^\dagger) Q_\alpha | I' \rangle \quad (11) \end{aligned}$$

Inserting a complete set of states between O_β^+ and $(1+iT)$ as well as between $(1-iT)$ and O_α yields with the help of (9) the desired result

$$\begin{aligned} \sum_{F'} \langle I' | O_\beta | F' \rangle \langle F' | O_\alpha^+ | I \rangle &= \\ &= \sum_F \langle I' | O_\beta^+ | F \rangle \langle F | O_\alpha | I' \rangle \end{aligned}$$

we see that in $N'_{\alpha\beta}$ we have the same operator taken between the state $|I'\rangle$ whereas for $N_{\alpha\beta}$ with respect to $|I\rangle$. Therefore, $N'_{\alpha\beta}$ is derived from $N_{\alpha\beta}$ with three momenta reversed, thus $A'_i = A_i$.

Applying the result to the term

$$i \varepsilon_{\lambda\alpha\kappa\beta} w_\lambda q_\kappa (p_\alpha Q_\beta - Q_\alpha p_\beta) = 2 M_I (\vec{w} \cdot \vec{q} \times \vec{q}')_{Lab}$$

M_I being the mass of the initial state, yields immediately that this term goes to the negative of itself under the rule $\nu \rightarrow \nu'$, $\mu \rightarrow \mu'$, $I \rightarrow I'$ (i.e., $\vec{q} \rightarrow -\vec{q}$, $\vec{q}' \rightarrow -\vec{q}'$, $\vec{w} \rightarrow -\vec{w}$) and hence must vanish if time reversal holds. All other terms are invariant under the transformation and do not occasion any perpendicular polarization of the muon.

If time reversal invariance is violated then A_6 may be non-vanishing and would give rise to a transverse polarization $\vec{P}_\perp$ of amount

$$\vec{P}_\perp = 2 m M_I (\vec{q} \times \vec{q}') A_6 / |M|^2$$

The transverse polarization can be determined by the up-down asymmetry of the decay electron with respect to the $\vec{q}, \vec{q}'$ plane.

In order to give some estimate of the possible order of magnitude of such a transverse polarization we follow the suggestion of Cabibbo ⁴⁾ that the second class currents could be 90° out of phase with the first class currents and thus constitute an explicit example of time reversal violation. Consider the elastic process (2). Taking for the first class current the usual one ⁸⁾ we have for the matrix element of the heavy current ⁹⁾

$$J_\alpha = F(Q^2) \left(\bar{u}_p \left\{ \gamma^\alpha - \frac{\mu}{2M} \sigma_{\alpha\lambda} Q_\lambda + A \frac{Q_\alpha}{m} + \lambda \gamma^\alpha \gamma^5 + \frac{iB}{2M} \sigma_{\alpha\lambda} \gamma^5 Q_\lambda + i \frac{\ell}{m} Q_\alpha \gamma^5 \right\} u_n \right)$$

The terms with A and B constitute the second class current. Squaring this we obtain for the coefficients A_1 - A_6 as in (6) :

$$A_1 = 4\lambda^2(pQ + 2M^2) + 2Q^2(1+\mu)^2$$

$$A_2 = 8(1+\lambda^2) + 2(B^2 + \mu^2)Q^2M^{-2}$$

$$A_3 = 4A^2(pQ + 2M^2)m^{-2} + (B^2 + \mu^2)(pQ)M^{-2} - 4\mu - 2\mu^2 - 8\lambda\ell Mm^{-1} + 4\ell^2(pQ)m^{-2}$$

$$A_4 = -4(1+\lambda^2) - 2(B^2 + \mu^2)(pQ)M^{-2}$$

$$A_5 = -8\lambda(1+\mu)$$

$$A_6 = -2(B\ell - A\mu)Q^2M^{-1}m^{-1} + 8AMm^{-1} - 4B\lambda$$

where M is the nucleon mass.

From μ capture we may obtain a lower limit ¹⁰⁾ for the scalar coefficient A . The result is that A must be smaller than 0.2, thus giving rise to negligible effects. The transverse polarization for two choices for B and several neutrino energies and muon neutrino angles in the lab. system are given in the Table. We see that for a large domain of energy and angle the polarization can be considerable.

Conversations with J.B. Adams, C. Bouchiat and T. Ericson are gratefully acknowledged. One of the authors (S.M.B.) wishes to thank Professor V.F. Weisskopf and Professor L. Van Hove for the hospitality extended to him at CERN.

$\theta \backslash E_\nu$	B = 3.71					B = 6				
	500	750	1000	1500	2000	500	750	1000	1500	2000
0	0	0	0	0	0	0	0	0	0	0
10	-1.3	-0.8	-0.1	1.4	3	-2	-1.2	-0.2	2	4
20	-0.3	2.4	5.2	10	14	-0.5	3.5	7.1	13	16
30	3	8.2	13	19	23	4.4	11	16.5	23	27
40	7	14	19	26	30	10	19	25	32	35
50	11	18	23	30	33	16	25	31	37	41
60	14	21	26	31	34	20	29	35	41	44
70	16	22	26	31	34	23	32	37	42	45
80	16.5	22	26	30	32	24	32	37	42	45
90	16.5	21	25	28	30	25	32	36	40	43
100	16	20	23	25	27	24	30	34	37.5	40
110	14.5	18	20	22	24	22	28	31	34	36
120	13	16	17.5	19	20	20	25	27	30	31
130	11	13.5	15	16	17	17	21	23	25	26.5
140	9	11	12	13	13.5	14	17	19	20.5	21.5
150	7	8.2	9	10	10	11	13	14	15.5	16
160	4.5	5.5	6	6.5	7	7.4	9	9.5	10.5	11
170	2.3	2.7	3	3.2	3.4	3.7	4.5	5	5	5.5

T A B L E

E_ν in MeV, θ in degrees,

average polarization in %.

θ is the angle between muon and neutrino momentum in the lab. system. E_ν = neutrino-energy in the lab. system.

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