CM-P00056985

BOUND STATES AND RENORMALIZATION PROPERTIES ⁺)L. Bertocchi ^{*)}

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A B S T R A C T

We discuss, both in potential scattering and in quantum field theory, the connection between the bound state problem and the renormalizability of the theory. In particular, it is shown that a meaning can be given to the bound state condition also in the framework of perturbation theory.

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⁺⁾ Research reported in this document has been sponsored in part by the Air Force Office of Scientific Research OAR through the European Office, Aerospace Research, United States Air Force.

1. INTRODUCTION

Recently a large amount of interest has been devoted to the study of singular potentials at small distances, both for the non-relativistic potentials and for the relativistic Bethe-Salpeter equation. The principal aim, common to all of these investigations, is to understand some characteristic features of the problem and to try to use them as a hint to investigate the fundamental difficulties of quantum field theory and its divergences. Thus particular attention has been paid to the treatment of the Bethe-Salpeter equation (we always mean in ladder approximation) in the presence of singular potentials at the origin. In fact, the Bethe-Salpeter equation is a typical derivation of quantum field theory and contains some of the difficulties proper to the complete problem. It gives a covariant description of the two body problem and, on the other hand, it presents some characteristics of simplicity, which make the problem tractable practically ¹⁾⁻⁸⁾.

The difficulties due to the presence of singular potentials manifest themselves, for instance in the bound state problem, in the fact that the integral equations to be handled are no more of the Fredholm type, with all the traces divergent. As extensively discussed in Refs. ^{1),2)}, the reason of these divergences can be identified in an improper treatment of the problem at small distances, so that a careful analysis in this region makes it possible to recast the equations in such a way that no more infinities are present.

Moreover, it has been understood that the bound state condition can be obtained putting $Z = 0$ ⁹⁾, where Z is the renormalization constant relative to the vertex between the quantized interacting fields and the bound state external field, considered as a fictitious elementary particle. Owing to this fact, in the discussion of the singular potentials problem, it is possible to recognize a connection between the infinities of the integral Bethe-Salpeter equation and the divergences (in the perturbative expansion) of Z .

In this note we want to analyze the problem from the point of view of the renormalization constants, both in potential scattering and in the relativistic field theory. In particular it will be shown how the presence of divergent (or, equivalently, cut-off dependent) terms in the perturbative expansion of Z does not prevent the possibility of giving an unambiguous meaning to the bound state condition $Z = 0$.

Our procedure will be the following : first, we shall discuss the general relation between the renormalization constants and the usual treatment in perturbation theory. Secondly, using arguments deriving from the renormalizability of the theory we shall be able to give a precise meaning to the bound state problem. Finally, we shall discuss how to generalize our results to relativistic quantum field theory - in this way we shall be able to recognize in a deeper way the analogies and the differences existing between the general theory and the, more or less, simple models considered up to now.

2.

RENORMALIZATION CONSTANTS IN POTENTIAL SCATTERING

In order to obtain a better understanding of the bound state problem we begin by recalling the definition of the vertex renormalization constant in potential theory ¹⁰⁾. We start from the wave function $\psi(\vec{k}, \vec{k}_0) \equiv \psi_E(\vec{k})$ which, in momentum space, describes the relative motion of two equal masses particles interacting through a potential V (of strength f). $\vec{k}_0$ is the relative momentum and $k_0^2 = E$ the total energy in the c.m. system ($m = 1$). For a bound state of energy $-B$, we can write the integral equation

$$\psi_E(\vec{k}) = \frac{N_0}{E - k^2} + \frac{1}{E - k^2} \int V(\vec{k}, \vec{k}') \psi_E(\vec{k}') d\vec{k}'$$

$$E = -B < 0 \quad (2.1)$$

where we have explicitly introduced from the beginning an inhomogeneous term. This corresponds to assume the existence of a fictitious external field X (of spin zero for simplicity) and to introduce a local interaction between this field and the two particles. This can be simulated in configuration space with the addition in the wave equation of a δ like term, whose strength is measured by the unrenormalized coupling constant N_0 . If the potential is well behaved at large momenta N_0 is given by

$$N_0 = \lim_{k^2 \rightarrow \infty} (E - k^2) \psi_E(\vec{k}) \quad (2.2)$$

The renormalized coupling constant N , which represents the strength at the pole, is

$$N = \lim_{k^2 \rightarrow -B} (-B - k^2) \psi_E(\vec{k}) \quad (2.3)$$

Finally we introduce the renormalization constant

$$Z = N_0 / N \quad (2.4)$$

Equation (2.4) is the non-relativistic version of the quantum field theoretical relation

$$g_0 = Z g \quad (2.4')$$

which defines the renormalization of the interaction with the external field.

In co-ordinate space N_0 represents the coefficient of the singular part of the wave function at $r = 0$, and N the coefficient of the asymptotic normalization. Thus from the point of view of the regularity of the wave function the bound state condition amounts to requiring $N_0 = 0$. Conversely if we follow our field theoretical analogy this is equivalent to imposing that the external field X be a true bound state: in so doing, X is no more "elementary" and it cannot participate in a primitive interaction. Since N must, of course, be different from zero, the bound state condition can be written as

$$Z(B, m, f) = 0 \quad (2.5)$$

which gives the desired relation between B, m, f *).

It is useful to use Eq. (2.1) to continue $Z(E)$ for $E > 0$. We will write it in the scattering region as

$$\psi_E^{(\pm)}(\vec{r}) = \frac{N_0}{E \pm i\epsilon - k^2} + \frac{1}{E \pm i\epsilon - k^2} \int V(\vec{r}, \vec{r}') \psi_E^{(\pm)}(\vec{r}') d\vec{r}' \quad (2.6)$$

*) For higher spin "external fields" the corresponding quantities $N_{0\ell}, N_\ell, Z_\ell$ can be introduced going to the angular momentum representation and performing the limits on the wave function $\psi_\ell(k^2, E)$. In the following we will always consider the s wave case, but the extension to other ℓ of the results of this section is straightforward.

In the continuous region the denominator $E - k^2$ can now vanish in the integration domain and the behaviour in the neighbourhood of the poles is fixed by the presence of the $\pm i\varepsilon$ (corresponding to outgoing and incoming waves respectively). We thus introduce the two constants

$$N^{(+)} = \lim_{k^2 \rightarrow E} (E - k^2) \psi_E^{(+)}(\vec{k}) = N(E + i\varepsilon) = Z^{(+)} N_0 = Z(E + i\varepsilon) N_0 \quad (2.7)$$

$$N^{(-)} = \lim_{k^2 \rightarrow E} (E - k^2) \psi_E^{(-)}(\vec{k}) = N(E - i\varepsilon) = Z^{(-)} N_0 = Z(E - i\varepsilon) N_0 \quad (2.7')$$

The interest in this continuation ^{*)} lies in the fact that the final state theorem enables us to derive an expression for the S matrix in terms of the vertex functions $Z^\pm$. To get this we have to use the unitarity and the time reversal invariance properties of the theory which give

$$N^{(+)}(E) = R(E) e^{i\delta} \quad (2.8)$$

$$N^{(-)}(E) = R(E) e^{-i\delta} = N^{(+)*}(E) \quad (2.8')$$

where $R(E)$ is a real function and δ the scattering phase shift. Then

$$S(E) = e^{2i\delta} = N^{(+)}(E) / N^{(-)}(E) \quad (2.9)$$

*) We notice that when we continue $Z^\pm$ back from $E > 0$ to the $E < 0$ region, $Z^\pm$ have to be defined in the physical ($\text{Im} \sqrt{E} > 0$) and unphysical ($\text{Im} \sqrt{E} < 0$) sheet, respectively. In the plane $k = \sqrt{E}$ the physical sheet corresponds to the upper half plane, the unphysical sheet to the lower half plane.

and using Eq. (2.4)

$$S(E) = \frac{N_0 Z(E - i\epsilon)}{N_0 Z(E + i\epsilon)} = \frac{Z^{(-)}(E)}{Z^{(+)}(E)} \quad (2.10)$$

Equation (2.10) looks more familiar when we realize (see Ref. ¹⁰) that in potential scattering Z coincides with the Jost function. However, the fact that Eq. (2.10) can be derived as a consequence of unitarity and time reversal will allow, under some circumstances, its generalization to quantum field theory. From Eq. (2.10) we can also see directly that the zeros of Z correspond to the poles of the S matrix.

3. SINGULAR POTENTIALS

We turn now to the relation between the properties of the Z function and the behaviour of the potential $V(r)$ at small distances. In Refs. 1), 2) we have classified the potentials according to the small distance behaviour

$$V(r) \sim \frac{r^\beta}{r^\beta}, \quad r \rightarrow 0 \quad V(k) \sim \frac{k^\beta}{k^{3-\beta}}, \quad k \rightarrow \infty \quad (3.1)$$

into three classes, I, II, III, corresponding to $\beta \leq 2$. We shall not discuss now the interesting possibility that the behaviour of $V(r)$ for small r contains polynomials in $\log r$, which will be considered in the following.

For class I the vertex function is finite at every order of its perturbation expansion [as constructed by iterating Eq. (2.6)]. For class II theories the vertex function is logarithmically divergent in each order, whereas for class III every term in the expansion is divergent and the degree of divergence increases order by order. It is also interesting to discuss the behaviour of the S matrix expansion, obtained from the Lippmann and Schwinger equation

$$T(\vec{k}, \vec{k}_0) = V(\vec{k}, \vec{k}_0) + \int T(\vec{k}, \vec{k}') \frac{1}{k_0^2 - k'^2 \pm i\epsilon} V(\vec{k}', \vec{k}_0) d\vec{k}' \quad (3.2)$$

in the three cases. For class I the S matrix is of course finite and the Born approximation is dominant at large k . For class II we stress the very important fact that the S matrix is still finite at every order and all the successive approximations have the same, large k behaviour as the Born approximation. In our reasoning this property will characterize the renormalizable interactions: namely the S matrix is finite while the expansion of the vertex function Z needs the introduction of a cut-off. Finally, for class III, even if some first terms of the expansion might be finite (depending on the degree of singularity of the potential), starting from a certain order all the other terms are divergent.

From now on we will limit ourselves to theories of class II.

We want to discuss how it is possible to give a meaning to the $Z = 0$ condition when the perturbation expansion of Z contains terms which are all divergent. First of all, we have to introduce a cut-off λ (larger than all the considered masses) ^{*)} which amounts to regularize the potentials at distances smaller than $1/\lambda$. In this way one considers Z as a function of the parameters, by which the vertex function is built up, and of the cut-off. In this way the bound state condition seems now to be dependent on the cut-off λ . We want, however, to show on a very general ground that the roots of the equation

$$Z(E, \lambda) = 0 \quad (3.3)$$

are indeed λ independent.

A first hint in this direction is obtained by observing that the finiteness of the S matrix and the relation (2.10)

$$S(E) = \frac{Z^{(-)}(E)}{Z^{(+)}(E)}$$

require necessarily that in the power expansion of the ratio of the Z functions such cancellations happen so that all the terms of the resulting series of S are finite. This has to imply some sort of factorization of the λ dependence of Z .

In order to prove in a general manner this factorization we shall use the fact that there is some freedom in the choice of the point where we perform the subtraction. In other words the regularization procedure is independent of the values of the masses which means that the ratio

$$\frac{Z(E_2, \lambda)}{Z(E_1, \lambda)} = R(E_2, E_1) \quad (3.4)$$

*) Namely the masses on which the Z function depends, which are m and E .

is λ independent ^{*)}. The previous relation can be verified term by term by means analogous to the techniques of Dyson.

Differentiating with respect to λ we get

$$\frac{dZ(E, \lambda)}{d\lambda} \cdot \frac{1}{Z(E, \lambda)} = \frac{dZ(E_1, \lambda)}{d\lambda} \cdot \frac{1}{Z(E_1, \lambda)} \quad (3.5)$$

which means that the quantity

$$\frac{dZ(E, \lambda)}{d\lambda} \cdot \frac{1}{Z(E, \lambda)} = C(\lambda) \quad (3.6)$$

is E independent. Thus we obtain

$$Z(E, \lambda_2) = Z(E, \lambda_1) \exp \left\{ \int_{\lambda_1}^{\lambda_2} C(\lambda) d\lambda \right\} \quad (3.7)$$

Equation (3.7) allows us to conclude that (since $C(\lambda)$ is E independent) the roots of $Z(E, \lambda)$ for a given λ are indeed independent of the value of the cut-off. In this way, owing to the renormalizability of the theory, it is possible to have a meaningful (cut-off independent) definition of the bound states.

Let us discuss the meaning of Eq. (3.3) in perturbation theory. We see that if we use for $Z(E, \lambda)$ its n -th order perturbative approximation, the roots $E_n(\lambda)$ are still λ dependent. This dependence, however, becomes weaker and weaker as n increases. In other words, we get a sequence of solutions

$$E_1(\lambda), E_2(\lambda), \dots, E_n(\lambda)$$

which depend on λ but for $n \rightarrow \infty$ tend to a λ independent value. This observation suggests the possibility of still using improved versions of perturbation theory, in order to get approximate solutions for some bound state problems ¹¹⁾.

*) It is interesting to observe that Eq. (2.10) is a particular case of Eq. (3.4), when $E_{2,1} = E + i\varepsilon$. It might be of importance to look for the possibility of deriving Eq. (3.4) starting from the finiteness of S (obtained without the introduction of a cut-off) and using some analyticity properties.

Our previous discussion is rather general and has not made any use of very specific properties of the potential. In the case in which the potential is of the form

$$V(r) \sim f/r^2$$

one obtains, on simple dimensional grounds, that $C(\lambda)$ has the form

$$C(\lambda) \sim S/\lambda \quad (3.8)$$

where S is a parameter which depends only on the strength f of the potential and on the angular momentum of the bound state. Thus Eq. (3.4) becomes

$$\frac{Z(E, \lambda_2)}{Z(E, \lambda_1)} = \left(\frac{\lambda_2}{\lambda_1} \right)^S \quad (3.9)$$

or

$$Z(E, \lambda) = \psi(E) (\lambda)^S \quad (3.10)$$

The physical meaning of the previous result is connected with the fact that the behaviour of $Z(E, \lambda)$ for large λ corresponds to the small distances behaviour of its Fourier transform, which is controlled by the potential. Then for the considered f/r^2 potential the singularity at the origin is of the Fuchsian type, that is the vertex behaves as a power^{*)}.

It is of interest at this point to consider potentials which can be still considered of class II, but whose behaviour at the origin is more complicated than a simple power. Thus let us discuss potentials of the form

*) If we look for the large E behaviour of $\psi(E)$ we find an analogous power form. This can be compared with analogous results found in the quantum field theoretical problem by M. Gell-Mann and F.E. Low¹²⁾.

$$V(r) \sim f \frac{(\ln r)^\gamma}{r^2}, \quad r \rightarrow 0 \quad (3.11)$$

whatever ^{*)} γ .

Following our definition the corresponding theory is still of the renormalizable type, because the presence of the logarithmic terms does not modify the previous conclusions of the integrals, i.e., that the terms of the perturbation series for Z and S are, in every order, logarithmically divergent and finite respectively. Equation (3.4) is still valid and in the comparison between the series of the Z functions the λ dependence disappears term by term. In this way the conclusions on the λ independence of the bound state condition still hold, however it is no longer possible to extract a power behaviour for the vertex function. In other terms at the origin the wave function has not a Fuchsian type singularity, although its more singular part can be still factorized. More precisely the regular and irregular wave functions at $r \sim 0$ are respectively

$$u_{\pm}(r) \sim \exp \left\{ \pm \frac{\sqrt{f}}{1+\gamma/2} (\ln r)^{1+\gamma/2} \right\} \quad \gamma > 0$$

There is, however, a fact which seems to differentiate rather strongly these potentials from the simple f/r^2 case. Arguments have been recently ¹³⁾ given which show that the S matrix has an essential singularity at $f = 0$, for potentials of the form (3.11).

This means that although all the terms of the perturbation expansion of S are finite, the series is not convergent and probably is an asymptotic series. Thus our conclusions on the λ independence of the bound state determination can be generalized to this case in the amount that the function $R(E_2, E_1)$ can be determined from its asymptotic series. Very likely this can be done only for certain values of the potential strength, for instance for those which make repulsive the leading part of the potential at small distances.

*) That this problem is not a merely academic one can be realized when we remember that, for instance, the α^2 vacuum polarization potential has the same form ($\gamma = 1$).

4. QUANTUM FIELD THEORY

We want now to discuss the relation between the condition $Z = 0$ and the bound state problem in quantum field theory. This connection has been first introduced in field theory through the analysis of the bound state condition in some simple soluble models, as for instance the Lee model ¹⁴⁾ and can be qualitatively understood by noticing that the vanishing of the vertex renormalization constant between the deuteron (considered as a bound state of the proton-neutron pair), and the proton and neutron fields means that a bare deuteron must not appear in the Lagrangian.

On the other hand it is still possible to express the scattering amplitude as a ratio of two renormalization constants :

$$S(E) = \frac{Z^{(-)}(E)}{Z^{(+)}(E)} \quad (4.1)$$

this equation, which is deduced on the basis of the final state theorem ¹⁵⁾, holds for values of the energy E below the inelastic threshold (where necessary, analytic continuation is understood).

This property gives a good plausibility argument for considering the zeros of $Z^{+}(E)$ as possible bound states. However, the situation is much less clear in this case, since we do not know the radius of convergence of the power series expansion of $Z^{+}(E)$ and $Z^{-}(E)$.

From a general point of view one can distinguish between two different programmes in the problem of getting masses and coupling constants from the zeros of the renormalization constants.

A first, less ambitious and perhaps provisional approach, is to consider still some of the particles appearing in nature as fundamental, in the sense that the corresponding fields do appear in the Lagrangian, and to obtain the remaining particles as bound states. In this case, the possible candidates for composite

particles can still be considered as external, unquantized fields, and the corresponding renormalization constants are on a different footing as the ones of the propagators and vertex functions involving only the fundamental fields, which are the only ones which can appear in the internal part of a Feynman diagram. In this approach, one imposes the condition $Z = 0$ only for the renormalization constants of the composite particles, and one determines only the masses and coupling constants of the bound states.

A more general programme ¹⁶⁾, which is somewhat analogous to the bootstrap mechanism in dispersion theory, makes no distinction between elementary and composite particles ; all the particles existing in nature are treated on the same footing, and one equates to zero all the renormalization constants, obtaining in this way all the masses and coupling constants.

We shall not now discuss the feasibility of this programme which leads, in principle, to a theory which goes far beyond the concepts of the conventional quantum field theory ; our aim is to limit ourselves, as a tentative scheme, to the less ambitious programme of the first kind, which can be still treated and understood in the framework of the old-fashioned quantum field theory ^{*)}.

Let us now discuss the situation when the theory is renormalizable, and the renormalization constants are logarithmically divergent (class II).

In Refs. ^{1),2)} we have shown, in some examples, that the Bethe-Salpeter ladder equation has indeed well-defined and finite roots, which can be obtained by taking into account correctly the behaviour of the "wave function" at small distances.

*) Looking at an example taken from electrodynamics, in the first scheme, one considers the positronium as a bound state of the electron-positron fields, and one sets the renormalization constant of the positronium vertex equal to zero ; in the second scheme, one would impose also that the charge renormalization constant must vanish.

This result can be understood as follows : one is considering the renormalization constant coming from the ladder graphs

Fig. 1

where the only infinities come from the vertex with the external field ^{*)}. The analogous scattering graphs

Fig. 2

would indeed be finite.

We see therefore that, from the point of view of infinities, no new features are introduced with respect to the singular potential problem.

Using the same kind of arguments, one can prove that the roots of $Z^+(E, \lambda)$ are indeed λ independent, and that

$$\frac{Z(E, \lambda_1)}{Z(E, \lambda_2)} = \left(\frac{\lambda_1}{\lambda_2} \right)^S \quad (4.2)$$

^{*)} We do not consider therefore, in this case, the $\lambda \phi^4$ interaction.

In the full field theory, however, one encounters a new kind of infinities, connected with the propagators and vertex functions of the quantized fields.

One can visualize this essential difference in a simple ladder case by saying that there we have both singular potentials and infinite coupling constants in the Hamiltonian.

Let us now discuss the meaning of the bound state problem in this more general framework. Let us consider the scattering matrix S and the renormalization constant Z_{ext} of the interaction with an external field, in which all the internal renormalizations have already been performed. Therefore, those quantities will depend on the physical masses and coupling constants related to the elementary particles and on the mass and angular momentum of the bound state ; moreover, Z_{ext} will only be meaningful if one introduces an external cut-off λ (with logarithmic cut-off dependence), whereas the S matrix, which in the elastic interval, is given by

$$S(E) = \frac{Z^{(-)}(E, \lambda)}{Z^{(+)}(E, \lambda)} \quad (4.3)$$

will be finite and therefore λ independent.

The renormalizability of the external vertex, together with the property that the renormalization procedure is indeed independent of the values of all the masses, would still lead to a ratio

$$\frac{Z(E_2, \lambda)}{Z(E_1, \lambda)} = R(E_2, E_1) \quad (4.4)$$

which is λ independent.

Therefore, as in the corresponding problem of section 3, one gets

$$Z(E, \lambda_2) = Z(E, \lambda_1) \exp \left\{ \int_{\lambda_1}^{\lambda_2} C(\lambda) d\lambda \right\} \quad (4.5)$$

from which one deduces that the bound state condition is independent of λ .

In this general case $C(\lambda)$ is no longer of the simple form S/λ , as in formula (4.2). The reason is that the homogeneity arguments, which are used ¹⁾⁻⁸⁾ in order to study the small distance behaviour, do not hold in this case, since the elementary local interaction has only a formal meaning and can be understood as a logarithmic function of the cut-off.

In the case in which internal loops are neglected, so that the unrenormalized constants are finite, those homogeneity arguments hold again ¹²⁾, and so the power behaviour (4.2) holds in the same way as for the $1/r^2$ potential (or the Bethe-Salpeter case) ^{*)}. On the other hand, the presence of internal loops of the kind in Fig. 3

Fig. 3

leads to a complicated dependence on $\log \lambda$, such that we have almost no control on the sum of the series. We are here in a situation similar to that we have encountered with potentials of the form

$$V(r) \sim \frac{1}{r^2} (a + b \log r + c (\log r)^2 + \dots)$$

*) In the case of quantum electrodynamics, owing to the Ward identity, it is sufficient to neglect the vacuum polarization loops; see Ref. ¹²⁾.

The comparison with potential scattering therefore makes it plausible that our solutions have an essential singularity in the coupling constant, and that the perturbative expansion has a meaning only as an asymptotic expansion^{*)}.

We see that the difficulties coming from the large λ behaviour of our solutions do not arise from complicated crossed diagrams, but from the physical effects coming from the renormalizations of the vertices and of the propagators of the "elementary" particles. Therefore, it is likely that this difficulty is due to the rather artificial distinction between elementary and composite particles, and might be overcome on more general grounds, using schemes which might go beyond the usual rules of quantum field theory.

^{*)} This argument is completely independent of those based on the number of the n -th order diagrams ; for instance, the series in Fig. 3 (uncrossed ladder with internal loops) already gives very probably an asymptotic expansion.

RIASSUNTO

Viene discussa, sia in teoria di potenziale che in teoria dei campi, la connessione tra il problema degli stati legati e la rinormalizzabilità della teoria. In particolare si mostra come attribuire un significato alla condizione di stati legati anche nell'ambito di una teoria perturbativa.

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