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THE INFLUENCE OF LEADING PARTICLES ON
DIFFRACTION SCATTERING AT VERY HIGH ENERGY

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A B S T R A C T

The influence of leading particles on diffraction scattering of strongly interacting particles at very high energy is investigated in the framework of the model of uncorrelated jets proposed by L. Van Hove. The conditions under which the correction from leading particles becomes important are examined in some detail. It appears that its behaviour depends mainly on the behaviour of the transverse momentum distribution of the leading particles as compared with the transverse momentum distribution of pions.

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I. Recently, Van Hove ¹⁾ has investigated the connection between elastic and inelastic scattering of strongly interacting particles at high energy. Such a connection is a simple consequence of the unitarity of the S matrix. Van Hove considered a model of inelastic collisions, with a basic assumption that the particles in the final state are uncorrelated. He has been able to show that, as a consequence of this model and the assumption that the elastic amplitude is purely imaginary, one obtains an almost exponential diffraction peak in elastic scattering, i.e.,

$$\frac{d\sigma_e}{dt} = \left\{ \frac{d\sigma_e}{dt} \right\}_{t=0} e^{at + bt^2}, \quad a > 0, b > 0, \quad \frac{b^2}{a} \ll 1 \quad (\text{I.1})$$

where $t = - \text{c.m. momentum transfer squared}$.

As is well known, such a behaviour of the elastic scattering has been found in various experiments and seems to be a very fundamental property of elastic scattering at high energies.

Van Hove considered all the particles in the inelastic final state to be identical. This means that he neglected the contributions of the "leading particles", i.e., nucleons or nucleon isobars. The existence of leading particles which take (on the average) about half of the incident energy is rather well established in the energy region we consider ²⁾. It is the purpose of this note to investigate corrections to the diffraction scattering which arise if we take into account the existence of one or two leading particles.

The results we obtain can be summarized as follows. Under very plausible assumptions [which form a slight generalization of those made by Van Hove ¹⁾] we can write the overlap function [Van Hove (1963) ¹⁾] in the form of the product

$$F(k, \theta) = F_\pi(k, \theta) F_N(k, \theta) \quad (\text{I.2})$$

2.

where F_π denotes the overlap function induced by pions, and F_N is the correction from leading particles. The overlap function determines the elastic amplitude through unitarity. To obtain (I.1) one needs

$$F(k, \theta) \sim e^{-A\theta^2} \quad (I.3)$$

for small θ . As shown by Van Hove, F_π does behave for small θ like the exponential :

$$F_\pi = C e^{-\lambda_\pi \frac{\theta^2}{2}} \quad (I.4)$$

We know nothing about the behaviour of F_N . The problem is to find out which of these two factors dominates, i.e., which one is decreasing faster with θ^2 . Our analysis suggests that the answer depends mainly on the behaviour of the average transverse momentum of the leading particles, as compared with the average transverse momentum of pions. If we write

$$\langle k_\perp \rangle_N = \alpha \langle k_\perp \rangle_\pi \quad (I.5)$$

then

- a) if $\lim_{k \rightarrow \infty} \alpha \ll 1$ the nucleon term dominates;
- b) if $\lim_{k \rightarrow \infty} \alpha \sim 1$ both terms are important;
- c) if $\lim_{k \rightarrow \infty} \alpha \gg 1$ the pion term dominates.

One can then ask which of these three cases is realized in nature.

Unfortunately, the answer cannot be stated at present in an unambiguous way. It seems that the experimental data exclude the case a), but both cases b) and c) are possible.

We would like to point out that if case b) is realized one has to find some other explanation of the observed shape of the diffraction peak, which would then require an exponential shape of F_N .

In Section II we state our main assumptions and introduce the notation. Section III contains a brief account of the calculations. In Section IV we compare our results with experimental data at 10 GeV (π^-p collisions), and in Section V we discuss the possible asymptotic behaviour in the very high energy limit.

II. We do not want to explain here all the details of Van Hove's model. The reader is therefore referred to the paper by Van Hove¹⁾. Henceforth it will be quoted (I). Our notation differs as little as possible from that of (I).

We consider the case in which there is only one leading particle. The generalization of our argument to the case of two such particles can be performed in a very straightforward way, and does not change our conclusions.

Our main assumptions can be stated as follows :

- i) the inelastic final state can be represented in the form :

$$|f\rangle = g_1 \dots g_n h |0\rangle \quad (\text{II.1})$$

where

$$g_i = g_i^{(0)} + \int g_i^{(u)}(\vec{k}) A^*(\vec{k}) d^3\vec{k} \quad (\text{II.2})$$

and

$$h = \int h(\vec{k}) B^*(\vec{k}) d^3\vec{k} \quad (\text{II.3})$$

A^* and B^* denote the creation operators of pion and nucleon (or more generally, leading particle), respectively.

4.

- ii) the sum of the average momenta of all particles is equal to the initial energy-momentum vector :

$$\langle k_\mu \rangle_N + \sum \langle k_\mu \rangle_\pi = p_\mu^{(0)} \quad (\text{II.4})$$

- iii) for each particle the longitudinal momentum is much bigger than the transversal one

$$k_{||} \gg k_\perp \quad (\text{II.5})$$

- iv) the longitudinal and transversal momentum distributions are uncorrelated :

$$g_i(\vec{k}) = g_i(k_{||}) g_i(k_\perp) \quad (\text{II.6})$$

$$h(\vec{k}) = h(k_{||}) h_\perp(k_\perp) \quad (\text{II.7})$$

All these assumptions represent simply the generalization of those made in (I) about pions.

In the calculations actually performed, we have also made the assumption [assumption v)] that all functions g_i , h are real. This assumption does not influence our result in any significant way, but it considerably simplifies the calculations.

III. Under assumptions i) and v) the overlap function can be written in the following form, (I) :

$$F(k, \theta^*) = \text{const } D^{-1/2} \prod_i \langle 0 | g_i' g_i'' | 0 \rangle \langle 0 | k' k'' | 0 \rangle e^{\frac{1}{2} \Gamma_{\mu\nu} (\vec{p}^{\mu} - \beta^{\mu})(\vec{p}^{\nu} - \beta^{\nu})} \quad (\text{III.1})$$

where $\Gamma_{\mu\nu}$ is the inverse of the matrix

$$D_{\mu\nu} = d_{\mu\nu} + \sum_i \gamma_{i\mu\nu} \quad (\text{III.2})$$

$$d_{\mu\nu} = \frac{\int d^3\vec{k} \, h'(\vec{k}) h''(\vec{k}) (k_\mu - b_\mu) (k_\nu - b_\nu)}{\int d^3\vec{k} \, h'(\vec{k}) h''(\vec{k})} \quad (\text{III.3})$$

$$\gamma_{i\mu\nu} = \frac{\int d^3\vec{k} \, g_i'(\vec{k}) g_i''(\vec{k}) (k_\mu - \beta_{i\mu}) (k_\nu - \beta_{i\nu})}{|g_i^{(0)}|^2 + \int d^3\vec{k} \, g_i'(\vec{k}) g_i''(\vec{k})} \quad (\text{III.4})$$

$$D = \det D_{\mu\nu} \quad (\text{III.5})$$

$$\beta^\mu = b^\mu + \sum_i \beta_{i\mu} \quad (\text{III.6})$$

$$b^\mu = \frac{\int d^3\vec{k} \, h'(\vec{k}) h''(\vec{k}) k_\mu}{\int d^3\vec{k} \, h'(\vec{k}) h''(\vec{k})} \quad (\text{III.7})$$

$$\beta_{i\mu} = \frac{\int d^3\vec{k} \, g_i'(\vec{k}) g_i''(\vec{k}) k_\mu}{|g_i^{(0)}|^2 + \int d^3\vec{k} \, g_i'(\vec{k}) g_i''(\vec{k})} \quad (\text{III.8})$$

$$g_j^{(0)'} = g_j^{(0)} \quad g_j^{(0)'}(\vec{k}) = g_j^{(0)}(\vec{k}') \quad h'(\vec{k}) = h(\vec{k}') \quad (\text{III.9})$$

$$g_j^{(0)''} = g_j^{(0)} \quad g_j^{(0)''}(\vec{k}) = g_j^{(0)}(\vec{k}'') \quad h''(\vec{k}) = h(\vec{k}'') \quad (\text{III.10})$$

$$\begin{aligned} k'^1 &= k''^1 = k \cos \theta/2 \\ k'^2 &= -k''^2 = k \sin \theta/2 \end{aligned} \quad k'^3 = k''^3 = 0 \quad (\text{III.11})$$

As shown in (I) the last factor in (III.1) becomes constant in the limit $E \rightarrow \infty$ if we assume iii).

The factor $\prod_i \langle 0 | g_i | g_i 0 \rangle$ gives an exponential of type (I.4). Using assumptions iii), iv), v), and formula 3) :

$$\int F(k_{||}) g_j'(\vec{k}) g_j''(\vec{k}) d^3 \vec{k} = \int F(k_{||}) g_j^2(\vec{k}) d^3 \vec{k} - \frac{\theta^2}{2} \int F(k_{||}) p_{||}^2 \psi g_j'(\vec{k}) d^3 \vec{k} \quad (\text{III.12})$$

we can write

$$\prod_i \langle 0 | g_i' g_i'' | 0 \rangle = \text{const } e^{-\lambda_\pi \theta^2/2} \quad (\text{III.13})$$

where

$$\lambda_\pi = N \langle p_{||}^2 \rangle_\pi \langle \psi \rangle_\pi \quad (\text{III.14})$$

$$\langle \psi \rangle_\pi = \frac{1}{N} \sum_i \int \left(\frac{\partial \log g_i(\vec{k})}{\partial p_{||}} \right)^2 g_i^2(\vec{k}) d^3 \vec{k} \quad (\text{III.15})$$

N denotes the average number of pions.

The factor $\langle 0 h' h'' 0 \rangle$ can be estimated in exactly the same way, and gives ⁴⁾ :

$$\langle 0 h' h'' 0 \rangle = \left(1 - \lambda_N \frac{\theta^2}{2}\right) \quad (\text{III.19})$$

$$\lambda_N = \langle p_{||}^2 \rangle_N \langle \psi \rangle_N \quad (\text{III.20})$$

$$\langle \psi \rangle_N = \int \left(\beta \frac{\log k(\vec{k})}{\partial p_{\perp}} \right)^2 h^2(\vec{k}) d^3 \vec{k} \quad (\text{III.21})$$

The analysis of the behaviour of the determinant D is somewhat complicated. We discuss this problem in the Appendix. One can see that for small θ^2 the factor $D^{-\frac{1}{2}}$ can be written as :

$$D^{-\frac{1}{2}} = \text{const} \left(1 + \lambda_N \beta \frac{\theta^2}{2}\right) \quad (\text{III.22})$$

where β is a constant. The value of β depends on the longitudinal momentum distribution of the leading particle. We have found the upper limit for the value of β [see Appendix] :

$$\beta \leq \frac{P_{N \max}^2 - \langle p_{||}^2 \rangle_N}{\langle p_{||}^2 \rangle_N} \quad (\text{III.23})$$

where $p_{N \max}$ is the maximal momentum of leading particles. Taking into account formulae (III.1), (III.13), (III.19), and (III.22) we finally obtain

$$F(k, \theta) = \text{const} e^{-\lambda_N \frac{\theta^2}{2}} \left(1 - \lambda_N (1 - \beta) \frac{\theta^2}{2}\right) \quad (\text{III.24})$$

To see which of the two factors entering (III.24) is decreasing faster with α^2 we have to estimate the ratio :

$$\mu \equiv \frac{\lambda_{\pi}}{\lambda_N} \frac{1}{1-\beta} = \frac{N \langle p_{\parallel}^2 \rangle_{\pi}}{\langle p_{\parallel}^2 \rangle_N} \frac{\langle \psi \rangle_{\pi}}{\langle \psi \rangle_N} \frac{1}{1-\beta} \quad (\text{III.25})$$

If we assume that the shape of the transverse momentum distribution is the same for pions as for leading particles we can write

$$\frac{\langle \psi \rangle_{\pi}}{\langle \psi \rangle_N} \sim \frac{\langle p_{\perp}^2 \rangle_N}{\langle p_{\perp}^2 \rangle_{\pi}} \sim \alpha^2 \quad (\text{III.26})$$

IV. Recent experiments on the $\pi^- p$ inelastic scattering at 10 GeV/c [Fleury et al., Biswas et al., Bardadin et al. ⁵⁾] enable us to estimate the ratio (III.25). We performed only a very rough estimation.

Using the given values of cross-sections for events with different number of prongs, Fleury et al. ⁵⁾, one can find that the average number of charged secondaries is ~ 3.3 . The average number of all particles in the inelastic final state (including neutrals) is therefore ~ 5 . For this reason we calculate the quantities of interest ($\langle p^2 \rangle_N$, $N \langle p^2 \rangle_{\pi}$, β) in the channels with multiplicity 5, i.e., one nucleon and four pions. These reactions were measured by Biswas et al. ⁵⁾. The results are ⁶⁾ :

$$\langle p^2 \rangle_N \sim 2.6 \text{ (GeV/c)}^2 \quad (\text{IV.1})$$

$$\sum \langle p^2 \rangle_\pi \sim 3.3 \text{ (GeV/c)}^2 \quad (\text{IV.2})$$

$$(p_{\max})_N \sim 2 \text{ GeV/c} \quad \alpha \sim 1.1 \quad (\text{IV.3})$$

This gives :

$$\frac{N \langle p^2 \rangle_\pi}{\langle p^2 \rangle_N} \sim 1.3 \quad (\text{IV.4})$$

$$\beta \leq \frac{4 - 2.6}{2.6} \sim 0.5 \quad (\text{IV.5})$$

We estimate therefore :

$$\mu = 1.3 \times 1.2 \times \frac{1}{1-\beta} \sim 1.6 \frac{1}{1-\beta} \quad (\text{IV.6})$$

We do not want to consider these numbers very seriously, because of the great uncertainty of both our approximation and experimental data. We think, however, that they can be treated as a hint that the possibility that the nucleon term is important has to be taken into account.

V. In this Section we would like to say a few words about the possible behaviour of μ in the very high energy limit. Because of the very small amount of experimental data concerning this region, our discussion can be only very qualitative and no definite conclusions can be drawn from it. Nevertheless, it seems to be worth while to discuss what type of measurements are needed to clarify the problem.

As shown by Van Hove, (I), there are rather good reasons to believe that the longitudinal momentum distribution of pions satisfies the condition

$$\frac{\langle p_{||}^2 \rangle_{\pi}}{\langle p_{||} \rangle_{\pi}^2} = a_{\pi} N \quad (V.1)$$

where a_{π} is a constant of the order of 1.

On the other hand, it seems that the most natural assumptions about the longitudinal momentum distribution of leading particles give :

$$\frac{\langle p_{||}^2 \rangle_N}{\langle p_{||} \rangle_N^2} = a_N \quad (V.2)$$

where a_N is a constant of the order of 1. One can see that (V.2) is valid for almost all possible distributions.

Taking into account (V.1) and (V.2) we can write

$$\frac{N \langle p_{||}^2 \rangle_{\pi}}{\langle p_{||}^2 \rangle_N} = \frac{a_{\pi}}{a_N} \frac{(N \langle p_{||} \rangle_{\pi})^2}{\langle p_{||} \rangle_N^2} = a \frac{(Kk)^2}{[(1-K)k]^2} = a \frac{K^2}{(1-K)^2} \quad (V.3)$$

where K is the inelasticity coefficient, k the primary c.m. momentum and a is a constant of the order of unity. It is generally believed that the inelasticity coefficient for cosmic energies is ≤ 0.5 . Therefore it seems that expression (V.3) cannot be big in the very high energy limit.

The next factor we have to take into account is $1/1-\beta$. Clearly, if $\beta \approx 1$ then this factor will play a dominant role, and μ will be very big. Unfortunately, the experimental data at cosmic energies are not precise enough to exclude or confirm this possibility. The assumption $\beta \approx 1$ can be one of the possible ways to avoid the difficulties connected with the leading particle term. However, we do not see any theoretical reason for this assumption and, on the contrary, as mentioned in the preceding Section, the data from 10 GeV π^-p collisions indicate $1-\beta \geq \frac{1}{2}$.

The last factor we want to discuss is $\langle \psi \rangle_\pi / \langle \psi \rangle_N$. The experimental evidence concerning this point, i.e., the measurements of the average transverse momenta of pions and leading particles, is also very poor and we cannot make any definite conclusions. However, it seems to be worth while to discuss some theoretical possibilities.

Suppose that in the inelastic scattering a nucleon creates pions independently of each other and in a random way. Then we expect :

$$\langle p_\perp^2 \rangle_N = \sum_i \langle p_{i\perp}^2 \rangle_\pi = N \langle p_\perp^2 \rangle_\pi \quad (V.4)$$

This means :

$$\frac{\langle \psi \rangle_\pi}{\langle \psi \rangle_N} \sim \frac{\langle p_\perp^2 \rangle_N}{\langle p_\perp^2 \rangle_\pi} \equiv \alpha^2 \sim N \quad (V.5)$$

We see that in this case the pion term becomes more important in the high-energy limit. It is well known that the multiplicity is growing very slowly with energy. Therefore (V.4) gives a rather slow increase of the transversal momentum of nucleons with respect to pions, which cannot be excluded on the basis of the present data.

If, on the other hand, we assume that in the inelastic collisions a small number of pion structures (for example, fire-balls) are formed, then we expect that $\alpha \sim 1$. This is also in agreement with the present data.

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A P P E N D I X

By the definition, and using symmetry properties (I), we get :

$$D = (D_{11} D_{00} - D_{01}^2) D_{22} D_{33} \quad (\text{A.1})$$

Taking into account formulae (III.3), (III.4), and (I.5) one can estimate

$$d_{22} \sim \alpha f_{i,22} \quad d_{33} \sim \alpha f_{i,33} \quad (\text{A.2})$$

Therefore (unless case a) is realized, which does not seem to be true) the factors D_{22} and D_{33} are not important, (I).

To estimate the first factor we observe that the pion term can be neglected in the elements D_{11} , D_{00} , D_{10} of the matrix D_{ik} in comparison with the leading particle term. We have :

$$D_{ik} \approx d_{ik} \quad (\text{A.3})$$

The elements of d_{ik} are not independent because of the connection between energy, momentum and mass. Taking this into account and assuming $p \gg M$, we find that the determinant D can be represented as a product of two factors, each of them having the form :

$$\frac{\int d^3\vec{k} \, h'(\vec{k}) h''(\vec{k}) \phi_i(k_{||})}{\int d^3\vec{k} \, h'(\vec{k}) h''(\vec{k})} \quad (\text{A.4})$$

The corresponding ϕ functions are :

$$\phi_1 = (p - \bar{p})^2 \quad (\text{A.5})$$

$$\phi_2 = \frac{M^4}{4} \left\{ \frac{1}{p} - \left(\frac{1}{p} \right) \right\}^2 \quad (\text{A.6})$$

Taking this into account and using formula (III.9) we get :

$$\frac{\partial \ln(D^{-1/2})}{\partial \theta^2} = \frac{1}{2} \langle p^2 \rangle_N \langle \psi \rangle_N \left(\frac{\delta_1 + \delta_2}{2} \right) \quad (\text{A.7})$$

where

$$\delta_i = \frac{\langle \phi_i p^2 \rangle - \langle \phi_i \rangle \langle p^2 \rangle}{\langle \phi_i \rangle \langle p^2 \rangle} \quad (\text{A.8})$$

One can easily find the upper limit for δ_i . As the functions fulfil the condition

$$\phi_i \geq 0 \quad (\text{A.9})$$

we have the inequality

$$\delta_i \leq \frac{p_{\max}^2 - \langle p^2 \rangle}{\langle p^2 \rangle} \equiv \delta \quad (\text{A.10})$$

where $p_{\max}$ is the maximal value of the momentum of the leading particle.

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- 2) See, e.g., A. Stanghellini, High energy physics, Report delivered at the Siena Conference, 4 October (1963).
- 3) This formula can be obtained under assumptions iii), iv), and v).
- 4) If there are two leading particles this formula becomes

$$\langle 0h_1^*h_1^*0 \rangle \langle 0h_2^*h_2^*0 \rangle = (1 - 2\lambda_N \theta^2/2)$$

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- 6) In the calculations we replaced $p_{//}$ by p , because there are no data concerning $p_{//}$.